Fundamentals of Nanoelectronics

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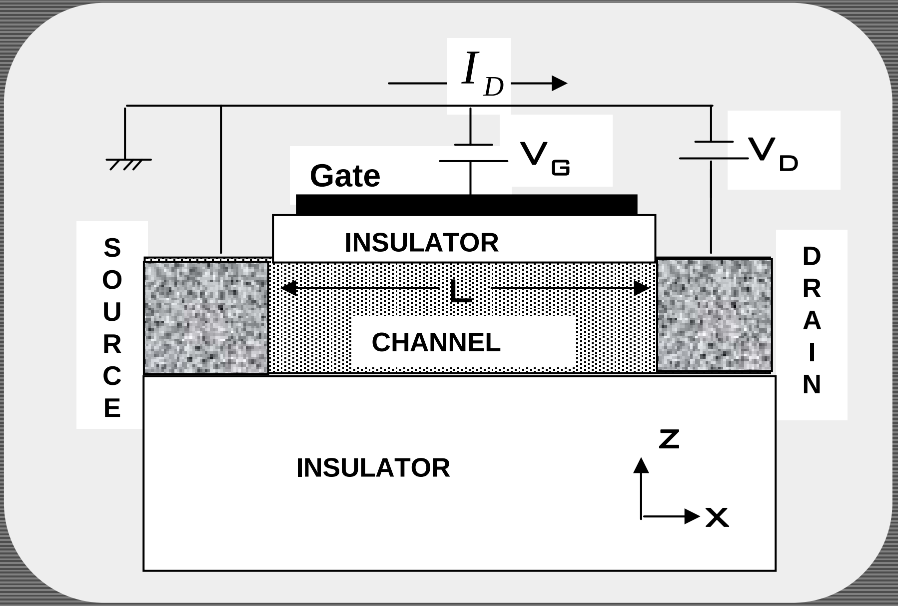
Lecture 30: Quantum Capacitance

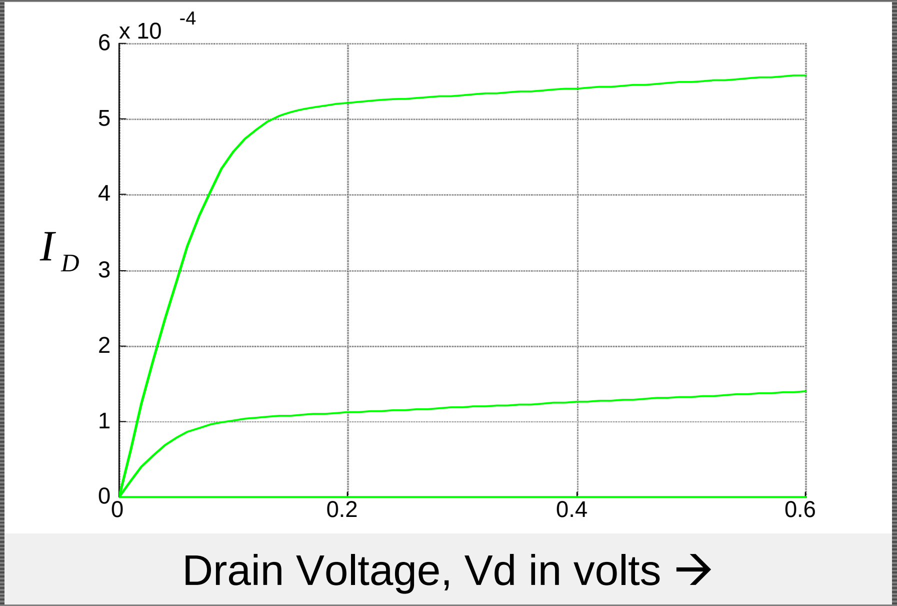
Ref. Chapter 7.3

Network for Computational Nanotechnology

NCN
nanoHUB
online simulations and more

Preview





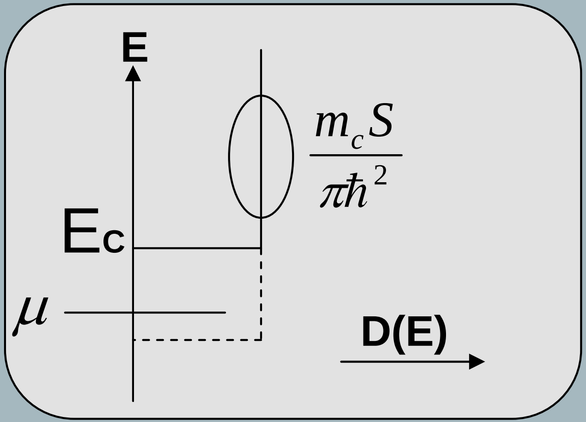
- In general we are interested in the current voltage characteristics of a transistor. How today we want to consider a simpler, namely the equilibrium problem where the drain voltage is 0.
- We want to know how the electron density in the channel changes as the gate voltage changes.

DOS / Electron Density

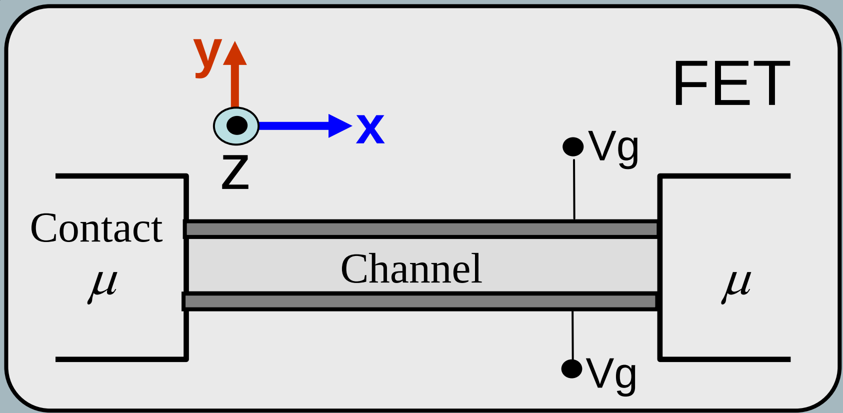
- The density of states in a 2D can be derived using the following assumptions.
- Assume the energy levels are describe by an effective mass equation:

$$E = E_c + \frac{\hbar^2 k^2}{2m_c}$$

- Also assume that the channel is so narrow in the z direction that only the lowest band is occupied.
- DOS is:



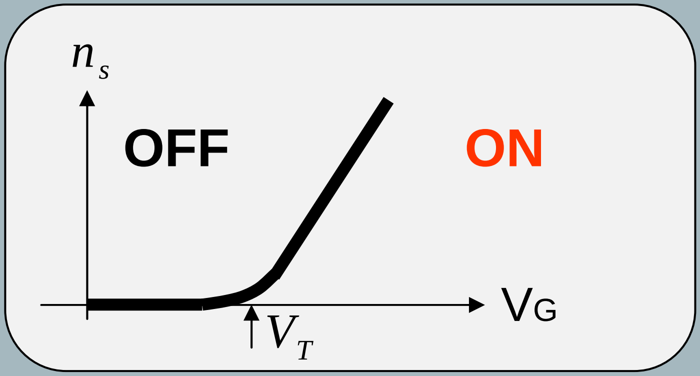
- A positive voltage makes the energy levels to go down.



- Number of electrons "N" can found as:

$$N = \int dE D(E) F(E) = \frac{m_c S}{\pi\hbar^2} \int_{Ec}^{E(top)} dE f(E)$$

- Electrons density has the form:



Electron Density vs. Gate Voltage (slope)

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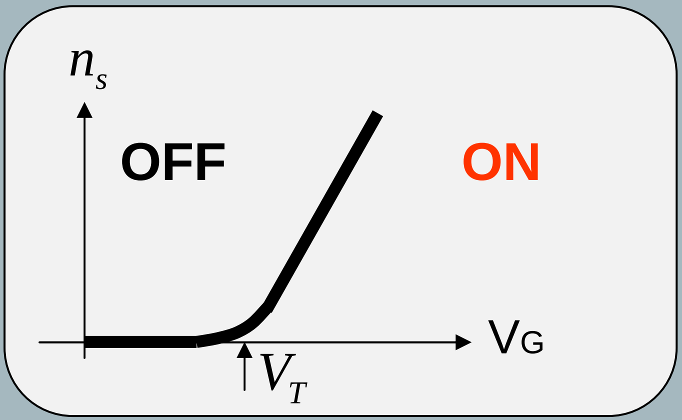
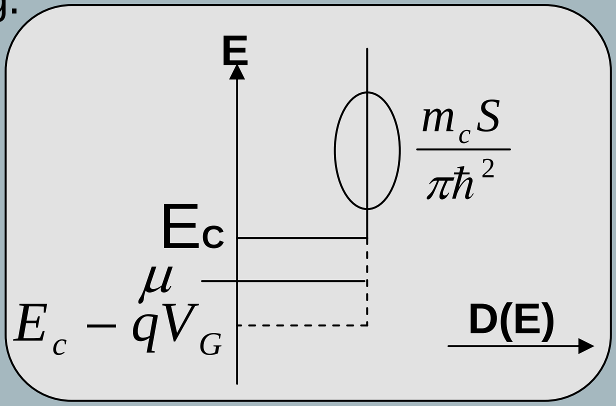


- Next question we want to address is that at what rate does the electron density increase in the on state.
- Note that we are lowering DOS by applying Vg.



$$N = \int dE D(E) F(E) = \frac{m_c S}{\pi\hbar^2} \int_{Ec}^{E(top)} dE f(E)$$

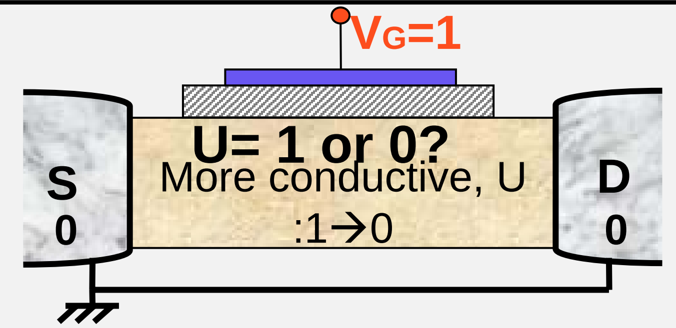
$$n_s = \frac{N}{S} = \frac{m_c}{\pi\hbar^2}\left(\mu - \left(E_c - qV_G\right)\right) \Rightarrow$$

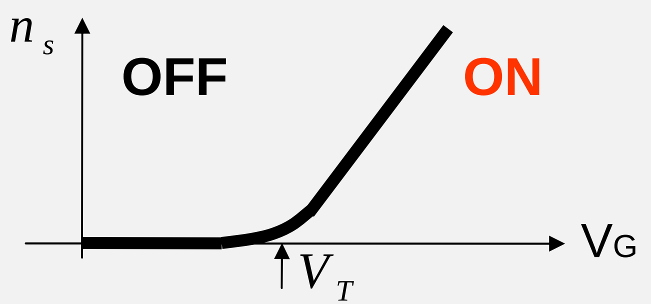
$$\frac{d(qn_s)}{dV_G} = \frac{m_c}{\pi\hbar^2} q^2$$

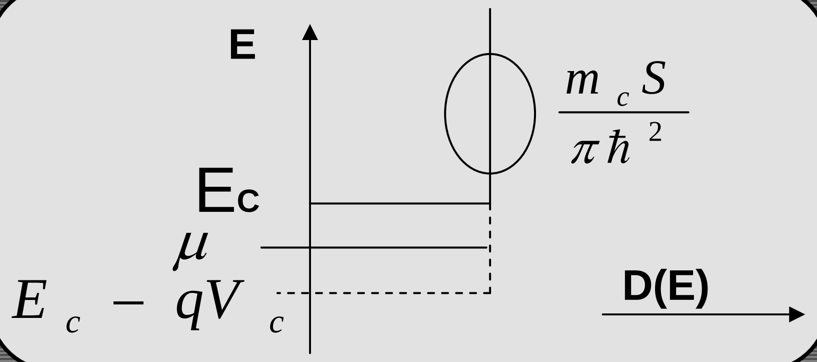
- Although one might expect the above to be the answer, the correct slope is actually smaller than what is calculated above. Why?

Importance of the Potential in the Channel

- We assumed that if we apply Vg to the gates, the potential in the channel we'll be Vg. As the result we assumed that the conduction band edge goes down by this amount.
- This was correct if there were no carriers in the channel. If there were lots of electrons in the channel then it would behave more like a conductor and since it is connected to regions of 0 potential, it tends to have a 0 potential.
- So we are actually dealing with two limits. One is the case where the channel is an insulator. In this case its potential will be exactly as the gate's potential. The other is where the channel is a conductor and its potential will be the same as contacts.
- We need to know the exact potential in the channel to get the correct answer.







Electron Density vs. Gate Voltage (slope)

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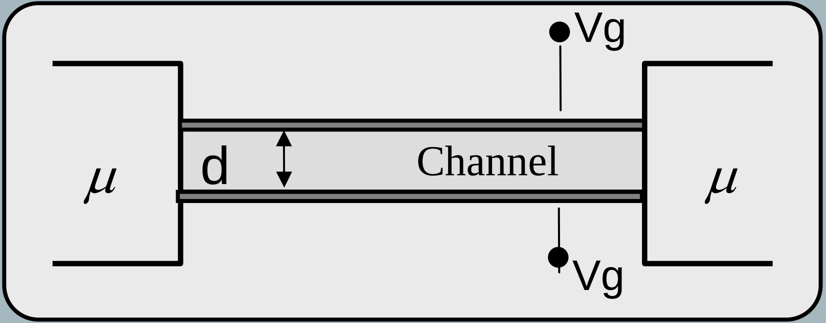
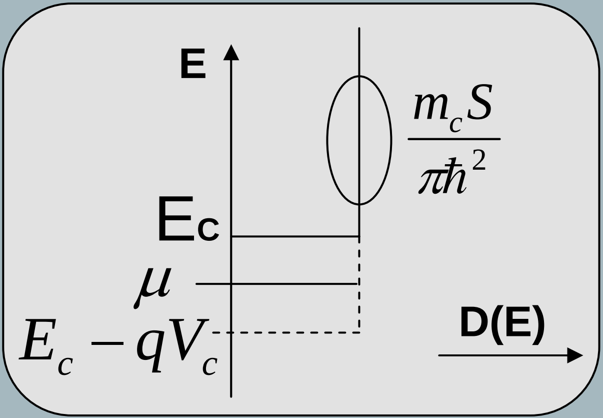
- We expect the potential in the channel to be the gate voltage minus the potential due to electrons in the channel.



$$V_c = V_G - \frac{qn_s}{C_E} \quad \text{where} \quad C_E = \frac{\varepsilon}{d}$$

$$\left.\begin{aligned} \frac{d(qn_s)}{dV_G} &= \frac{d(qn_s)}{dV_c}\frac{dV_c}{dV_G} \\ \frac{d(qn_s)}{dV_c} &= \frac{m_c}{\pi\hbar^2}q^2 \end{aligned}\right\} \frac{d(qn_s)}{dV_G} = \frac{q^2 m_c}{\pi\hbar^2}\left(1-\frac{1}{C_E}\frac{d(qn_s)}{dV_G}\right) = C_Q - \frac{C_Q}{C_E}\frac{d(qn_s)}{dV_G} \Rightarrow$$



$$\boxed{\frac{d(qn_s)}{dV_G} = \frac{C_Q C_E}{C_Q + C_E}}$$

$$C_Q = q^2 \frac{m_c}{\pi\hbar^2}$$

Two Capacitances in Series

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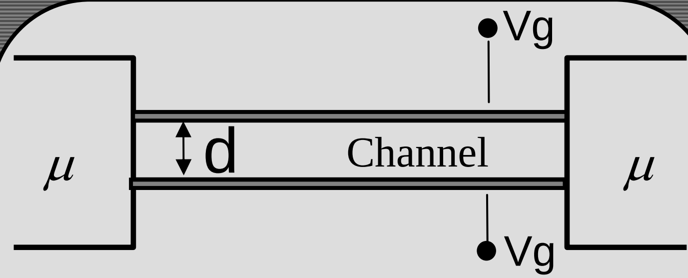
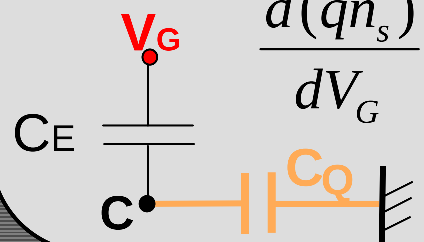
LOW DOS

- Low DOS results in low quantum capacitance. For a small capacitance, since caps behave as inverse resistors (conductors), conductance is low (open) and the entire voltage appears across C_Q.





$$\frac{d(qn_s)}{dV_G} = \frac{C_Q C_E}{C_Q + C_E}$$

HIGH DOS

- For high DOS, the capacitor is large, hence more metallic; acts as a short, node C becomes 0. And there will be low voltage drop across C_Q.

$$C_Q = q^2 DOS$$

INSULATOR — **METAL**

- This is now becoming an important issue in the small devices. In the past Cins was small relative to C_Q; hence effect of C_Q wasn't significant. Consider the equation for Cins: $Cins = \varepsilon_{ins}/d$. Over the years, d has become smaller. Through making d smaller (Hence increasing C_{ins}) people were able to control the channel. How ever at some point Cins will become big relative to C_Q. C_Q will then have more influence and the channel can't really be controlled by Cins. It is the Quantum Capacitance that controls now.

Ratio of CE Over CQ

- The ratio of electrostatic capacitance to the quantum capacitance can be written as:

$$\frac{C_E}{C_Q} = \frac{\varepsilon}{d} \times \frac{\pi\hbar^2}{m_c q^2} = \frac{\pi\varepsilon\hbar^2}{m_c q^2}\frac{1}{d}$$

- Notice that the first factor above is the same as Bohr radius except for a multiplicative factor: Bohr radius

$$a_0 = \frac{4\pi\varepsilon\hbar^2}{mq^2} \approx 0.53\times10^{-10}\,m$$

- Considering dielectric constant and the effective mass, the **effective** Bohr radius can be written as:

$$a_0^* = \frac{4\pi\varepsilon\hbar^2}{m_c q^2} \approx 50a_0$$

- So

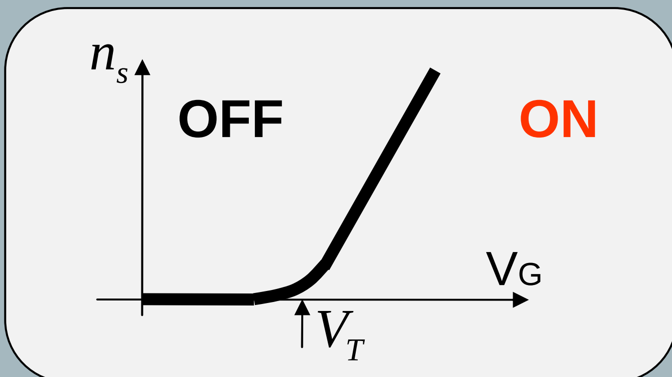
$$\frac{C_E}{C_Q} = \frac{a_0^*}{4d}$$

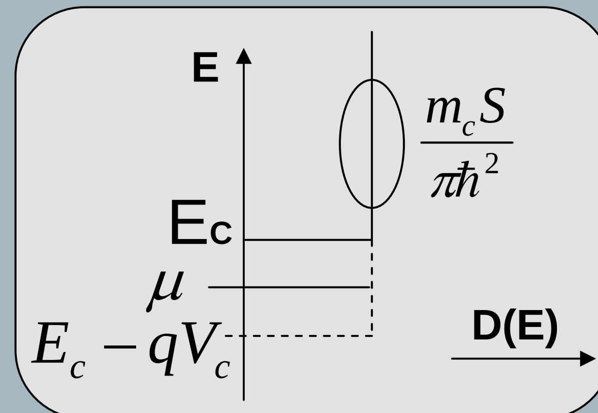
- This is now becoming an important issue in the small devices. In the past CE was small relative to CQ; hence effect of CQ wasn't significant. Consider the equation for CE: $Cins = \varepsilon_{ins}/d$

Over the years, d has become smaller. Through making d smaller (Hence increasing CE) people were able to control the channel. How ever at some point CE will become big relative to CQ. CQ will then have more influence and the channel can't really be controlled by CE. It is the Quantum Capacitance that controls now.

Threshold Voltage and Quantum Considerations

- Do we have to consider quantum physics for determination of threshold voltage?





- We calculated the slope to be: $$\frac{d(qn_s)}{dV_G} = \frac{C_Q C_E}{C_Q + C_E}$$
- We know: $$qV_T = (E_c - \mu)$$
- We don't have to consider CQ because, before the threshold voltage VT is reached, the density of states is essentially 0 and we are left with a channel that acts as an insulator. This causes the potential in the channel to be the same as the potential in the gate (there are no carriers in the channel to change this potential.