From Macro Down to Micro and Nano
- Scaling Issues

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Course Website: www.nanoHUB.org
Compass.uiuc.edu
How things behave in small scale?

- E.g. Ants vs. Human

<table>
<thead>
<tr>
<th>Action</th>
<th>Ants</th>
<th>Humans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift 10 times of its own weight</td>
<td>(Barely 1 times)</td>
<td></td>
</tr>
<tr>
<td>Do not injure when falling</td>
<td>(easily injured)</td>
<td></td>
</tr>
<tr>
<td>Some can fly</td>
<td>(we wish)</td>
<td></td>
</tr>
<tr>
<td>(Dead near fire)</td>
<td>We cook</td>
<td></td>
</tr>
<tr>
<td>(Dry cleaning)</td>
<td>We take showers</td>
<td></td>
</tr>
<tr>
<td>(? If they are smart?)</td>
<td>We use tools</td>
<td></td>
</tr>
</tbody>
</table>

Problem of Direct Scaling

- We don’t have a good intuition of microscale physics. Our common sense in the macro-world often get flawed in the microworld

- What shall we do?
  - Dimensional analysis could reveal “How things behave” in the microscopic world without full scale analysis.
  - In fact, you might have exercised these in many thermal/fluidic problems!
Scaling at Micro/Nanoscale

- Classical cases

- Departure from continuum

Scaling Physics

- Most physical quantities (force, torque, power, etc) scales differently with dimension L

- Assuming geometric similarity
  - Surface force $\propto L$ (size)
  - Heat transfer, Diffusion $\propto L^2$ (area)
  - Compression/elongation stiffness $\propto L^2$ (area)
  - Mass $\propto L^3$ (volume)
Why dimensional analysis?

- Intuition tends to perceive the micro/nanoscale objects as geometrically similar counterparts of those in macroworld → hinders conceptual level designs

- As scale reduces, some physical quantities become dominant while others become negligible

Classical Mechanical Scaling Laws

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Scaling Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration</td>
<td>$L^{-1}$</td>
</tr>
<tr>
<td>Energy density</td>
<td>$L^0$</td>
</tr>
<tr>
<td>Characteristic time</td>
<td>$L$</td>
</tr>
<tr>
<td>Friction force</td>
<td>$L^2$</td>
</tr>
<tr>
<td>Mechanical power</td>
<td>$L^2$</td>
</tr>
<tr>
<td>Torque</td>
<td>$L^3$</td>
</tr>
<tr>
<td>Energy/work</td>
<td>$L^3$</td>
</tr>
</tbody>
</table>

A flying bee
- The diameter of a tall tree must increase as $3/2$ of its height ($L^{3/2}$)

- Young and small trees appear slender, and old and tall ones appear squat or stunted

Also applies to weight lifting and even to production of nails

Scaling of Nails

Common nails arranged by size from 60 penny (6 inches) to 2 penny (1 inch).

Nail diameter vs. nail length on a log-log plot, showing the allometric formula $d = 0.07/2/3$. A broken line of slope 1.0, representing strict isometry, is also shown.

(On Size and Life by McMahon and Bonner)
Jumping:

- The work done in leaping up is proportional to the mass and height of the jump
  \[ \frac{W}{M} \sim gh \]

- Power density is almost invariant with size in different animals:
  \[ \frac{W}{M} \sim L^0 \]

- Thus \( h \) tends to be constant irrespective of the animal

Flying and Rotating:

- Flying (birds fly from 10.8 to 20.7 m/sec):
  - Wing length \( \sim l \sim M^{1/3} \) and wing area \( \sim l^2 \sim M^{2/3} \)
  - The characteristic speed for flying varies as \( l^{1/2} \) or \( M^{0.17} \)
  - Drag/lift forces are given by \( L = \frac{1}{2} C_L \rho A v^2 \). This expression has an order of \( 2 + 2v \)
  - Of key importance is the lift-to weight ratio (divide by \( l^3 \)) which is of the order \( 2v^{-1} \)
  - Since the lift-to-weight ratio should be invariant with scale to achieve flight a zero order scaling law is needed thus \( v \) must be \( 1/2 \) to achieve sufficient lift (same result as above)

- Rotation:
  - Mass moment of inertia \( (I) = \int r^2 dm \). This is a fifth order expression
  - A small motor will reach top speed in a fraction of a second, large motors may require seconds to reach full speed
Relative Significance of Forces

e.g. Absolute deflection of a beam (width H, length L) due to its own weight

\[ \delta = \frac{3}{2} \rho g \left( \frac{L}{E} \right)^2 L^2 \]

Percent deflection (Keep L/H the same)

\[ \frac{\delta}{L} = \frac{3}{2} \rho g \left( \frac{L}{H} \right)^2 L \propto L \]

The relative deformation of 2 um long beam would be 1/1000 of the 2mm beam!
- In microscale, structures appear to be stiffer against inertia forces

Surface Force

- Adhesion, surface tension

A thermodynamic property

\[ F = 2 \gamma l \]

- \( dG = \gamma dA \)
- Unit: J/m^2 or N/m
Pressure in Small Bubbles/Droplets

Laplace-Young Equation relates the pressure drop to the surface tension and the dimension of the bubble/droplets:

\[ \Delta P = \frac{2\gamma}{r} \] \hspace{1cm} \text{droplets} \\
\[ \Delta P = \frac{4\gamma}{r} \] \hspace{1cm} \text{bubbles} \\

The relative pressure inside a 50nm vesicle could exceed 400kPa (~4 atm)!

Body Force vs. Surface Force

A water strider floating on a pond without worrying gravity

Bond Number

\[ Bo = \frac{\text{gravity viscous}}{\gamma} = \frac{\rho g L^3}{\gamma} = \frac{\rho g L^2}{\gamma} \]
Quiz: a bridge for cells

Assuming we need to construct a 1 μm thick silicon bridge for the cells to travel between two of their villages, 500μm apart, on top of a drop of water between the two villages. We want to construct a strong bridge that would not collapse easily. Which type of bridge among the followings is the most suitable?

http://www.pbs.org/wgbh/buildingbig/bridge/basics.html

Scaling in Fluidics

Reynolds Number:

\[ \text{Re} = \frac{UL\rho}{\mu} = \frac{U^2L}{\mu} \]  

In typical cases of micro/nanofluidics

\[ \rho = 1g/cm^3, L = 10^{-6}m, U = 0.01m/s, \mu = 10^{-3} Pa.s \]

The flow is highly laminar!
Consequence of Microfluidics

Incredibly High Flow Resistance

\[ R = \frac{8 \mu L}{\pi r^4} \propto L^{-3} \]

Viscous Drag

\[ C_d \sim 1/Re \]

Poor Mixing

No turbulent mixing
Slow diffusion at L-L interface only


Stopping Distance at Low Re

• Can a Fly Catch a fish like Pelican?

• A viscous drag would prevent that!

\[ (3\pi \mu Dv)l = \frac{1}{6} (\rho - \rho_0) \pi D^3 v^2 \]

• Stopping Distance

\[ \sim (Re/18)D \]

A fly could only dive in a few centimeters;
A cell would only slide at a fraction of the diameter (<1μm)
Swimming in Low Re Flow

- It’s much like swimming in a slurry
  - Your body move forward on the thrust stroke but go backward at equal distance at return stroke: effect of reversibility

Without diffusion, a vesicle released by the cell would be taken back by an reverse motion of the membrane!

Ciliary Propulsion

A flexible propeller has to be used
Thermal scaling (Classical)

- Heat capacity $\sim L^3$
  - assuming that the specific heat capacity is to remain constant

- Heat dissipation (by conduction, convection and radiation) $\sim L^2$

- Biot Number representing the heat resistance:
  $$B = \frac{hL}{\kappa}$$
  This implies that micro/nanoscale object can be heated faster and cooled faster

- Peclet Number showing the ratio between convection to conduction:
  $$Pe = \frac{\rho C_p v L}{\kappa}$$
  This implies that convective heat transfer become negligible compared to conduction

Animal Size

Small mammals must keep on eating to stay warm ((heat loss $\sim L^2$ and heat generation (through eating) is $\sim L^3$))—insects avoid this problem by staying cold blooded.
Mass Transport

Diffusion time
\[ \sim L^2 \]

- Bacteria has the fastest response to chemical gradient!

Another note on mass transport

- Dragonfly uses air filled tubes instead of blood to transport oxygen to muscles
Classical Wear Life

- Unlubricated system
  - Constant interfacial stress
  - Constant speed
  \[ \Rightarrow \text{Erosion rate} = \text{constant} \]

- Wear life \( \propto \) thickness/erosion rate \( \propto L \)

- 1-cm part \( \Rightarrow \) 10 years
- 1-nm part \( \Rightarrow \) 30 seconds!

*Fortunately, that never happened*

Nanoscale Friction

- Telescoping nanotube segments
- No wear or fatigue
- \text{van der Waals energy-based retraction force}

20 nm

John Cumings, A. Zettl, Science 2000
• Because of non-linear effects electrostatic devices can be operated in air without breaking down (operation on the left side of the Paschen curve).
• New physics and chemistries to be explored.

Additional Reading

• Madou Chapter 9, “Scaling, Actuators, and Power in Miniaturized Systems”