1) For parabolic energy bands, the 2D density of states is

\[ D_{2D}(E) = \frac{m^*}{\pi \hbar^2} \Theta(E - \varepsilon_1) \]

Assume a non-parabolic band described by the so-called Kane dispersion,

\[ E(k)\left[1 + \alpha E(k)\right] = \frac{\hbar^2 k^2}{2m^*(0)} \]

and

1a) derive the density of states

1b) use a figure to explain why the DOS changes from the parabolic case

2) For a nonparabolic energy band described by the Kane dispersion, derive the corresponding \( M(E) \)
HW 4

(a) \[ NkdE = D(E)dE \]

\[ 2\pi kdk = \frac{D(E)dE}{4\pi^2} \]

\[ D(E) = \frac{1}{\pi} \frac{kdk}{dE} \]

\[ D(E) = \frac{m^2(0)}{\pi\hbar^2} \left(1 + 2\alpha E\right) \]

\[ E + \alpha E = \frac{\hbar^2 k^2}{2m} \]

\[ \frac{dE}{dk} \left(1 + 2\alpha E\right) = \frac{\hbar^2}{m} \frac{dE}{d\alpha E} \]

\[ \frac{d\alpha E}{dE} = \frac{\alpha E}{\hbar^2} \left(1 + 2\alpha E\right) \]

(b) Any given range of energy, \(dE\), will map into a larger range of \(k\) (\(\alpha\) vs. \(\alpha\) above) and, therefore, contain more states.

\[ E \quad \frac{\hbar^2 k^2}{2m} \]

\[ \text{non-parabolic} \]

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HW 4

\[ M(E) = \frac{W}{\lambda_B(E)/2} \]  \hspace{1cm} (1)

\[ E(1+\alpha E) = \frac{\hbar^2 k^2}{2m^*(0)} \]

\[ k = \sqrt{2m^* E(1+\alpha E)} = \frac{2\pi}{\lambda_B} = \frac{\pi}{\lambda_B/2} \]

\[ \frac{1}{\lambda_B/2} = \sqrt{\frac{2m^* E(1+\alpha E)}{\pi\hbar}} \]

Use in (1)

\[ M(E) = \frac{W}{\pi\hbar} \sqrt{2m^* E(1+\alpha E)} \] \checkmark