Low-Bias Transport in Graphene: an introduction

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acknowledgments

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A set of notes to accompany this lecture is available. The notes provide derivations for all of the equations presented in this lecture, as well as additional discussions for $T_L > 0\,\text{K}$, the role the the graphene quantum capacitance, and derivations of scattering rates. See:

D. Berdebes, T. Low, and M.S. Lundstrom, “Lecture notes on Low bias transport in graphene, July 2009.”
1) Introduction and Objectives
2) Theory
3) Experimental approach
4) Results
5) Discussion
6) Summary
Graphene is a one-atom-thick planar carbon sheet with a honeycomb lattice. 

Graphene has an unusual bandstructure that leads to interesting effects and potentially useful electronic devices.

source: CNTBands 2.0 on nanoHUB.org
objectives

• Describe the experimental techniques commonly-used to characterize low-bias conductance of graphene.

• Show some typical results.

• Analyze the results and discuss the general features of low-bias transport in graphene and how they are related to carrier scattering.
1) Introduction and Objectives
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We will use a very simple description of the graphene bandstructure, which is a good approximation near the Fermi level.

We will refer to the $E_F > 0$ case, as “n-type graphene” and to the $E_F < 0$ case as “p-type graphene.”

$$E(k) = \pm \hbar \nu_F k = \pm \hbar \nu_F \sqrt{k_x^2 + k_y^2}$$

$$\nu(k) = \nu_F \approx 1 \times 10^6 \text{ cm/s}$$

$$D(E) = 2|E|/\pi \hbar^2 \nu_F^2$$
low-bias transport theory

\[ I = \frac{2q}{h} \int_{-\infty}^{+\infty} T(E) M(E) \left( f_1 - f_2 \right) dE \]

\[ G = \frac{I}{V} = \frac{2q^2}{h} \int_{-\infty}^{+\infty} T(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]

\[ f_0 (E) = \frac{1}{1 + e^{(E-E_F)/k_B T}} \]

\[ T(E) \equiv \frac{\lambda (E)}{(\lambda (E) + L)} \]

\[ M(E) = W \frac{|E|}{\pi \hbar \nu_F} \]
expected results: $G$ vs. $E_F$ at $T_L = 0$K

$$G(0K) = \frac{2q^2}{h} T(E_F) M(E_F)$$

$$\frac{\partial f_0(E)}{\partial E} = \delta(E - E_F)$$

$$M(E) \propto |E|$$

$E_F > 0$

$E_F = 0$

$E_F < 0$
expected results: $G$ vs. $n_S$ at $T_L = 0K$

\[ G = \frac{2q^2}{h} T(E_F) M(E_F) \]

\[ n_S(E_F) = \frac{1}{\pi} \left( \frac{E_F}{\hbar \nu_F} \right)^2 \propto E_F^2 \]

\[ M(E_F) \propto E_F \propto \sqrt{n_S} \]

\[ G \propto \sqrt{n_S} \]
expected results: $T_L > 0K$

$G(T_L > 0K) = \frac{2q^2}{h} \langle T(E_F) M(E_F) \rangle$

$G_s(E_F = 0) > 0$

$T > 0K$

$T = 0K$

$E_F > 0$

$E_F = 0$

$E_F < 0$

$M(E) \propto |E|$
some key equations \((T = 0K)\)

\[
G(0K) = \frac{2q^2}{h} T(E_F) M(E_F)
\]

\[
M(E_F) = W \frac{2E_F}{\pi \hbar \nu_F}
\]

\[
T(E_F) = \frac{\lambda(E_F)}{(\lambda(E_F) + L)}
\]

\[
G(0K) = \frac{2q^2}{h} \frac{\lambda(E_F)}{\lambda(E_F) + L} W \frac{2E_F}{\pi \hbar \nu_F}
\]

\[
G = G_s \frac{W}{L}
\]

\[
G_s(0K) = \frac{2q^2}{h} \lambda_{app} \left( \frac{2E_F}{\pi \hbar \nu_F} \right)
\]

Describes the conductance of the conduction \((E > 0)\) or valence \((E < 0)\) bands.

(For \(T > 0\), the total conductance is the sum of the two.)

\[
\frac{1}{\lambda_{app}} = \frac{1}{\lambda(E_F)} + \frac{1}{L}
\]

\(G_s\) is the “sheet conductance” or conductivity, \(\sigma\)
When $E_F > 0$, graphene is strongly degenerate and:

$$G_S(E_F) = \left( \frac{2q^2k_B T_L}{\pi^2 \hbar^2 \nu_F} \right) \left\langle \lambda_{app} \right\rangle F_0 \left( \frac{E_F}{k_B T_L} \right) \approx \frac{2q^2}{\hbar} \lambda_{app} \left( E_F \right) \left( \frac{2E_F}{\pi \hbar \nu_F} \right)$$

$T_L > 0K$ result $\approx T_L = 0K$ result

$$n_S = \left( \frac{2}{\pi} \right) \left( \frac{k_B T_L}{\hbar \nu_F} \right)^2 F_1 \left( \frac{E_F}{k_B T_L} \right) \approx \frac{1}{\pi} \left( \frac{E_F}{\hbar \nu_F} \right)^2$$
questions

• How is $G$ vs. $E_F$ (or $G$ vs. $n_S$) measured experimentally?

• How do the results compare to theory?

• What do the results us about scattering in graphene?

1) Introduction and Objectives
2) Theory
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gate-modulated conductance in graphene

1) The location of the Fermi level (or equivalently the carrier density) is experimentally controlled by a “gate.”

2) In a typical experiments, a layer of graphene is place on a layer of SiO₂, which is on a doped silicon substrate. By changing the potential of the Si substrate (the “back gate”), the potential in the graphene can be modulated to vary $E_F$ and, therefore, $n_S$. 
Typically, Cr/Au or Ti/Au are used for the metal contacts.

The thickness of SiO$_2$ is typically 300nm or 90nm, which makes it possible to see a single layer of graphene.
“Temperature-Dependent Transport in Suspended Graphene”

measurements

At a fixed temperature:

\[ G(V_G) \text{ or } R(V_G) \]

At a fixed gate voltage:

\[ G(T_L) \text{ or } R(T_L) \]

Frequently the sheet conductance or sheet resistance is reported (and this is usually referred to as the ‘conductivity’ or the ‘resistivity.’)

\[ G = G_S \left( \frac{W}{L} \right) \]

\[ R = R_S \left( \frac{L}{W} \right) \]
using a gate voltage to change the Dirac point (or $E_F$)

\[ V_G = V_G - V_{NP} \]

\[ V_G' = V_G - V_{NP} \]

Back gate (doped Si)

\[ \Delta V \]

\[ I \]

\[ E(k) \]

\[ V_G > 0 \]

\[ V_G = 0 \]

\[ V_G < 0 \]
gate voltage - carrier density relation

If the oxide is not too thin (so that the quantum capacitance of the graphene is not important), then:

\[ qn_S = C_{ins} V_G \]

\[ C_{ins} = \frac{\varepsilon_{ins}}{t_{ins}} \]
outline

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sheet conductance vs. $V_G$

\[ G = G_S W/L \]

\[ G_S(E_F) \approx \frac{2q^2}{h} \lambda_{app}(E_F) \left( \frac{2E_F}{\pi \hbar \nu_F} \right) \]

\[ n_S = C_{ox} V_G \approx \frac{1}{\pi} \left( \frac{E_F}{\hbar \nu_F} \right)^2 \]

\[ \lambda_{app}(E_F) = \frac{G_S \left( 2q^2 / h \right)}{2 \sqrt{n_S / \pi}} \]

mean-free-path ($V_G = 100V$)


$G_S \approx 3.0 \text{ mS}$

$n_s \approx 7.1 \times 10^{12} \text{ cm}^{-2}$

$E_F \approx 0.3 \text{ eV}$

$\lambda_{app}(0.3 \text{ eV}) \approx 130 \text{ nm}$

$\lambda(0.3 \text{ eV}) \ll L$
mean-free-path ($V_G = 50V$)


- $G_S \approx 1.5 \text{ mS}$
- $n_s \approx 3.6 \times 10^{12} \text{ cm}^{-2}$
- $E_F \approx 0.2 \text{ eV}$
- $\lambda_{app}(0.2 \text{ eV}) \approx 90 \text{ nm}$
- $\frac{\lambda(0.2 \text{ eV})}{\lambda(0.3 \text{ eV})} \approx 0.69$
- $\frac{0.2 \text{ eV}}{0.3 \text{ eV}} \approx 0.67$
- $\lambda(E_F) \propto E_F$
Since, $G_S \sim n_S$, we can write:

$$G_S \equiv n_S q \mu_n$$

and deduce a mobility:

$$\mu_n \approx 12,500 \text{ cm}^2 / \text{V-sec}$$

Mobility is constant, but mean-free-path depends on the Fermi energy (or $n_S$).
$V_G = 0$


$G_S \approx 0.16 \text{ mS}$

$n_s = C_{ox} V_G \approx 0 ?$

$\lambda_{app} = \frac{G_S}{(2q^2/h)2\sqrt{n_s/\pi}}$

$\lambda_{app} \rightarrow \infty ?$

$\left( T_L = 0 \text{ K} \right)$
electron-hole puddles


The effect of potassium doping on the charge-carrier transport in graphene was studied by J.-H. Chen, C. Jang, S. Adam, M. S. Fuhrer, E. D. Williams, and M. Ishigami. In nominally undoped samples, the graphene mobility $G_S$ vs. the sheet carrier density $n_S$ is non-linear. As doping increases, $G_S$ vs. $n_S$ becomes more linear, the mobility decreases, and the normal process (NP) shifts to the left.

nominally undoped sample

\[ t_{ox} = 300 \text{ nm} \]
\[ \lambda \ll L \]
\[ T = 20K \]

\[ \lambda_{app} = \frac{G_S / \left( \frac{2q^2}{h} \right)}{2 \sqrt{n_{S}/\pi}} \approx 164 \text{ nm} \]

\[ \lambda \ll L \]

Away from the conductance minimum, the conductance decreases as $T_L$ increases (or resistivity increases as temperature increases).

\[ T_L < 100 K : \quad R_S \propto T_L \]

(acoustic phonon scattering - intrinsic)

\[ T_L > 100 K : \quad R_S \propto e^{\hbar \omega_0 / k_B T_L} \]

(optical phonons in graphene or surface phonons at SiO$_2$ substrate)

phonons and temperature dependence

\[ R_S = \frac{1}{G_S} \propto \frac{1}{\lambda} \propto N_\beta \]

\[ N_\beta = \frac{1}{e^{\hbar \omega (\beta)/k_B T_L} - 1} \]

**Acoustic phonons:**

\[ \hbar \omega < k_B T_L \]

\[ N_\beta \approx \frac{k_B T_L}{\hbar \omega} \]

\[ R_S \propto T_L \]

**Optical phonons:**

\[ \hbar \omega_0 \approx k_B T_L \]

\[ N_\beta = \frac{1}{e^{\hbar \omega_0/k_B T_L} - 1} \]

\[ R_S \propto \frac{1}{e^{\hbar \omega_0/k_B T_L} - 1} \]
unannealed vs. annealed suspended graphene

\[ G_S \propto \sqrt{n_S} \]

\[ \lambda_{app} \approx 1300 \text{ nm} \]

expected from ballistic theory

\[ G_S = n_S q \mu_n + G_{res} \]

about mobility

\[ G_S(E_F) \approx \frac{2q^2}{h} \lambda_{app}(E_F) \left( \frac{2E_F}{\pi \hbar \nu_F} \right) \]

\[ G_S(E_F) \propto \lambda_{app}(E_F) \sqrt{n_S} \]

\[ G_S \equiv n_S q \mu_n \]

\[ \mu_n \propto \frac{\lambda_{app}(E_F)}{\sqrt{n_S}} \]

**Case 1):**

\[ \lambda_{app} \propto E_F \propto \sqrt{n_S} \]

\[ G_S \propto n_S \]

\[ \mu_n \text{ constant} \]

**Case 2):**

\[ \lambda_{app} \text{ constant} \]

\[ G_S \propto \sqrt{n_S} \]

\[ \mu_n \propto \frac{1}{\sqrt{n_S}} \]
Experimental summary: graphene on SiO$_2$

1) Low conductance samples often show $G_S \sim n_S$ (away from the minimum)

2) Higher conductance samples are frequently non-linear ($G_S$ rolls off at higher $n_S$)

3) $G_S(T)$ decreases with temperature (“metallic”) for large $n_S$

4) $R_S \sim T_L$ for $T_L < 100$K and superlinear for $T_L > 100$K

5) Best mobilities for graphene on SiO$_2$ are $\sim 30,000$ cm$^2$/V-s at $T_L = 5$K

6) Asymmetries between $+V_G$ and $-V_G$ are often seen.
experimental summary: suspended graphene

1) Before annealing $G_S \sim n_S$ (away from the minimum)

2) After annealing, $G_S$ increases and $G_S$ vs. $n_S$ becomes non-linear

3) After annealing, $G_S$ is close to the ballistic limit

4) Best mean-free-paths are $\sim 1 \mu m$ at $T_L = 5K$

5) $G_S$ decreases with $T_L$ for large $n_S$ but increases with $T_L$ near the Dirac point.
outline

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conductance and scattering

\[ G(0K) = \frac{2q^2}{h} \frac{\lambda(E_F)}{\lambda(E_F) + L} W \frac{2E_F}{\pi \hbar \nu_F} \]

\( \lambda(E) \) is the mean-free-path (technically, the mfp for “backscattering”), which is determined by the dominant scattering processes.
scattering

\[ \frac{1}{\tau(E)} \]  
scattering rate per sec

typically computed from FGR

\[ \lambda(E) \propto v_F \tau(E) \]  
mean-free-path for backscattering
scattering

\[ \lambda(E) = \frac{\pi}{2} \nu_F \tau_m(E) \quad \text{(elastic or isotropic scattering)} \]

For many scattering mechanisms (e.g. acoustic phonon, point defect), the scattering rate is proportional to the density of final states:

\[ \frac{1}{\tau(E)} \propto D(E) \propto E \quad \tau(E) \propto E^{-1} \]

The energy-dependent mean-free-path is:

\[ \lambda(E) \propto \frac{1}{E} \]

What does this type of scattering do to the conductance?
effect of short range / ADP scattering

Assume $T_L = 0$ K and diffusive transport (just to keep the math simple)

$$G_S = \frac{2q^2}{h} \lambda \left( E_F \right) \left( \frac{2E_F}{\pi \hbar \nu_F} \right) \lambda \left( E_F \right) \propto \frac{1}{E_F}$$

$G_S = \text{constant!}$

For short range or ADP scattering, $G_S$ is constant.

long range (charged impurity) scattering

For screened or unscreened charged impurity scattering, the mfp is proportional to energy.

Random charges introduce random fluctuations in $E(k)$, which act as scattering centers.

High energy electrons don’t “see” these fluctuations and are not scattered as strongly.
effect of charged impurity scattering

Assume $T_L = 0$ K and diffusive transport (just to keep the math simple)

$$G_S = \frac{2q^2}{h} \lambda(E_F) \left( \frac{2E_F}{\pi \hbar \nu_F} \right) \lambda(E_F) \propto E_F$$

$$G_S \propto n_S \quad (\mu_n \text{ constant})$$

For charged impurity scattering, $G_S$ vs. $n_S$ is linear.


comment on linear $G$ vs. $n_S$

The observation of a linear $G(n_S)$ characteristic is frequently taken as experimental evidence of charged impurity scattering, but…

Theoretical work shows that strong, neutral defect scatter can lead to a linear $G$ vs. $n_S$ characteristics…


Even more recent experimental work on intentionally damaged graphene bears this out…

the energy-dependent mfp

Mobility is not always the best way to characterize the quality of a graphene film, but mean-free-path is always a well-defined quantity. We can extract the mean-free path vs. energy from measured data.

\[ G_S(0K) = \frac{2q^2}{\hbar} \lambda_{app}(E_F) \left( \frac{2E_F}{\pi \hbar \nu_F} \right) \]

\[ n_S(0K) = \frac{1}{\pi} \left( \frac{E_F}{\hbar \nu_F} \right)^2 \]

\[ \lambda_{app}(E_F) = \frac{G_S(V_g)/(2q^2/\hbar)}{2\sqrt{n_S(V_g)/\pi}} \]

\[ \frac{1}{\lambda_{app}(E_F)} = \frac{1}{\lambda(E_F)} + \frac{1}{L} \]

The apparent mfp is the shorter of the actual mfp and the sample length.
“Temperature-Dependent Transport in Suspended Graphene”

suspended, annealed

“Temperature-Dependent Transport in Suspended Graphene”
$\lambda_{app}(E_F) = \frac{G_S(V_g)/(2q^2/h)}{2\sqrt{n_S(V_g)/\pi}}$

\[
\frac{1}{\lambda_{app}(E_F)} = \frac{1}{\lambda(E_F)} + \frac{1}{L}
\]

suspended, unannealed

linear $G_S$ vs. $n$ suggests charged impurity scattering.

$T = 40 \, K$


analysis complicated by large residual resistance.
minimum and residual conductance


\[ G_{\text{res}} \approx G_{\text{min}} \]

\[ G_{\text{res}} \approx 14 \frac{q^2}{h} \]

\[ G(n_s) = G_{\text{res}} + (q\mu_1)n_s \]
suspended, unannealed

\[ \lambda_{app} (E_F) = \left( \frac{G_S(V_g) - G_{res}}{2q^2/h} \right) \left( \frac{2\sqrt{n_S(V_g)}}{\pi} \right) \]
We have discussed $V_g (n_S) > 0$, but by symmetry, the same thing should occur for p-type graphene ($E_F < 0$).

If the mfp is small and constant, then $G$ is also proportional to $\sqrt{n_S}$, but the magnitude is less than the ballistic limit.
general picture of \( G_S \) vs. \( n_S \) (diffusive)

Short range or acoustic phonon scattering.

Charged-impurity (long-range) scattering. Constant mobility.

Result is a combination of charged impurity and phonon scattering.

Non-zero residual resistance commonly observed.
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The general features of the graphene conductance vs. gate voltage are readily understood (but still being discussed).

Data can be analyzed by extracting the mean-free-path for backscattering and relating it to the underlying scattering mechanisms.

More sophisticated theoretical treatments include screening, remote, polar phonons, etc.

Actual experiments are frequently non-ideal (e.g. not symmetrical about $V_{NP}$, non monotonic behavior, variations due to sample state, uncertainties in $W$ and $L$, etc.

But the material presented here gives a general framework and starting point for analyzing experimental data.
minimum and residual conductance


\[
G_{\text{res}} \approx 14 \frac{q^2}{h} \quad \text{and} \quad G(n_s) = G_{\text{res}} + (q \mu_1) n_s
\]
suspended, unannealed

linear $G_S$ vs. $n$ suggests charged impurity scattering.

Expect $\sim \lambda |E|$

$T = 40K$


analysis complicated by large residual resistance.