ECE-656: Fall 2009

Lecture 7:
2 and 3D Resistors

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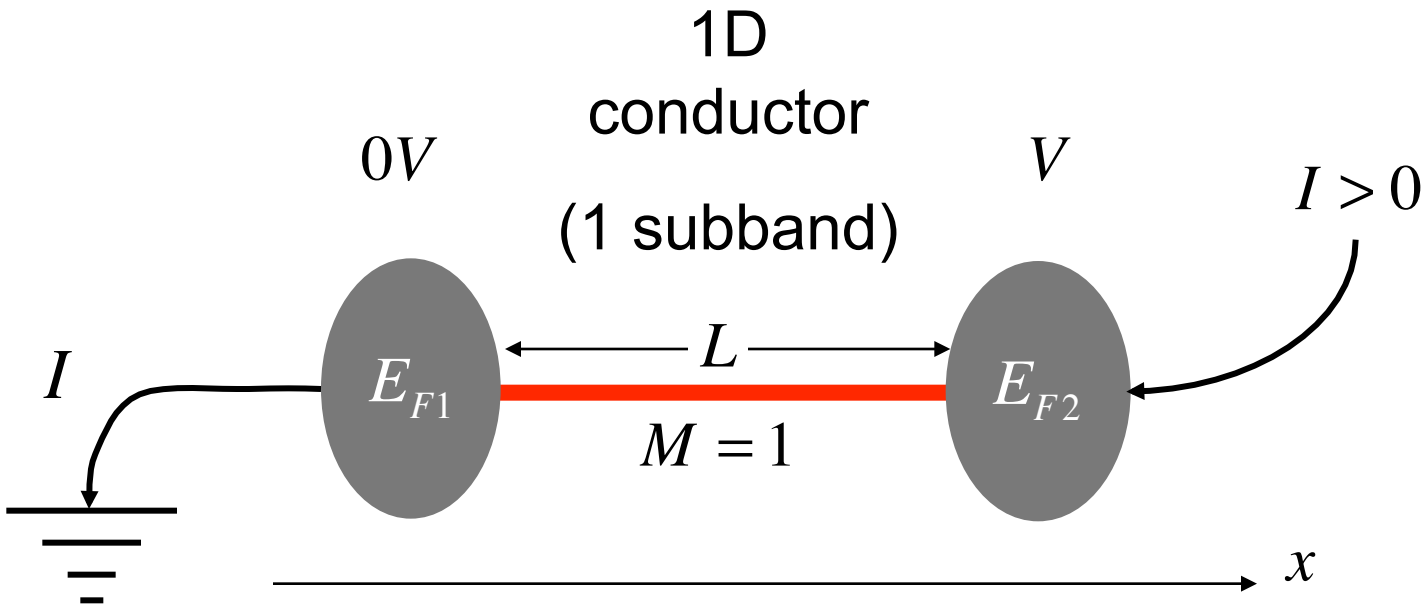
1D resistors



$$T = 0\text{K}: \quad G = \frac{2q^2}{h} T(E_F)$$

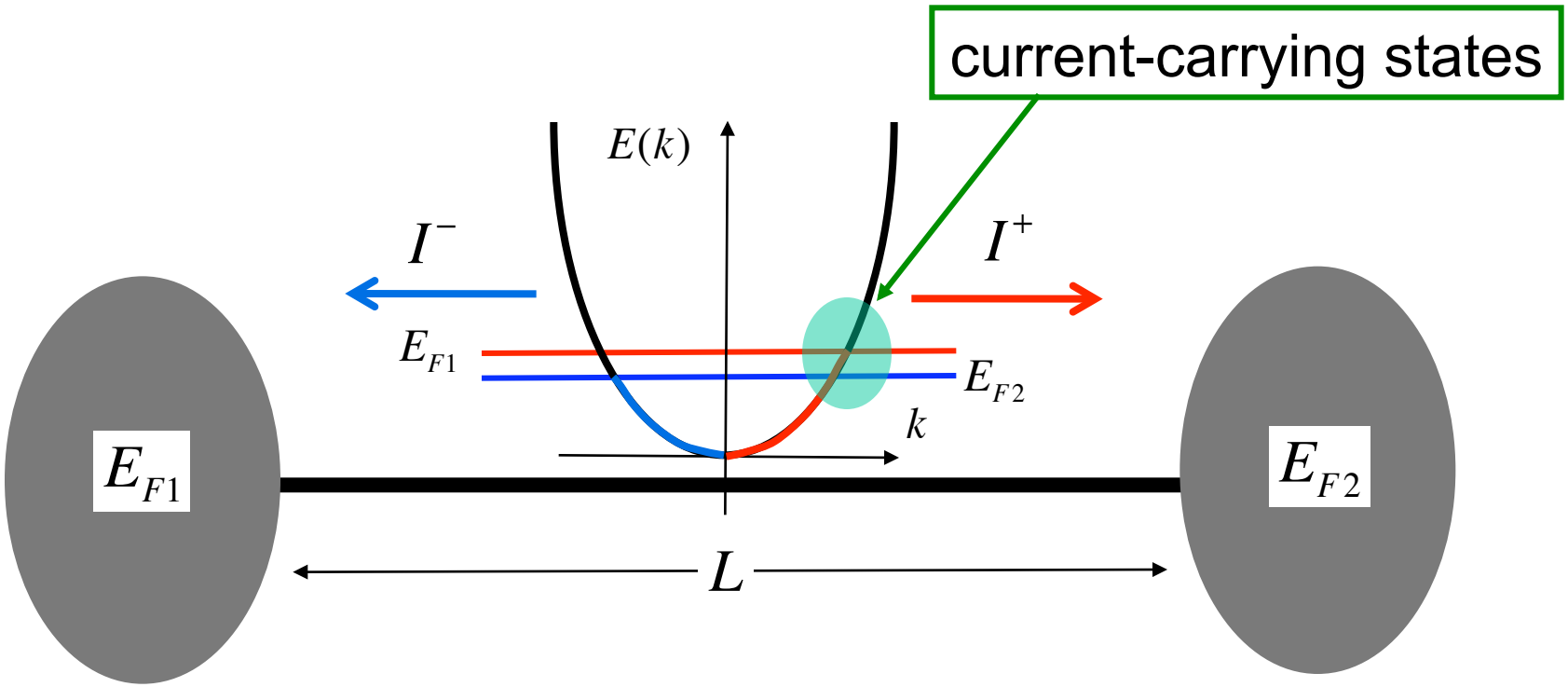
$$T > 0\text{K}: \quad G = \frac{2q^2}{h} \langle T(E) \rangle \mathcal{F}_{-1}(\eta_F)$$

$$\langle T(E) \rangle = \frac{\int_{\varepsilon_1}^{+\infty} T(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}{\int_{\varepsilon_1}^{+\infty} \left(-\frac{\partial f_0}{\partial E} \right) dE}$$

outline

1) **Another view of the same problem**
2) 2D resistors
3) Discussion
4) 3D resistors
5) Summary

1D resistor: k-space treatment



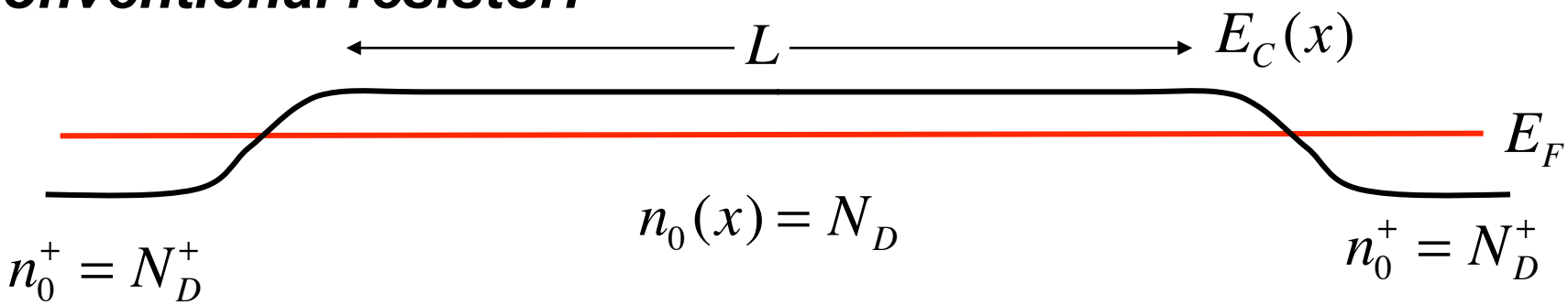
$$I^- = \frac{1}{L}\sum_{k<0} q\upsilon_x f_0(E_{F2})$$

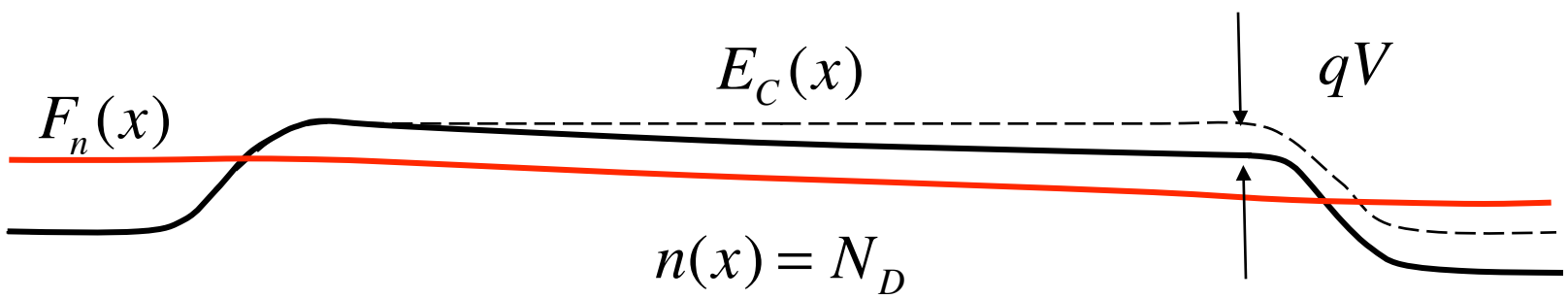
$$I = I^+ - I^-$$

$$I^+ = \frac{1}{L}\sum_{k>0} q\upsilon_x f_0(E_{F1})$$

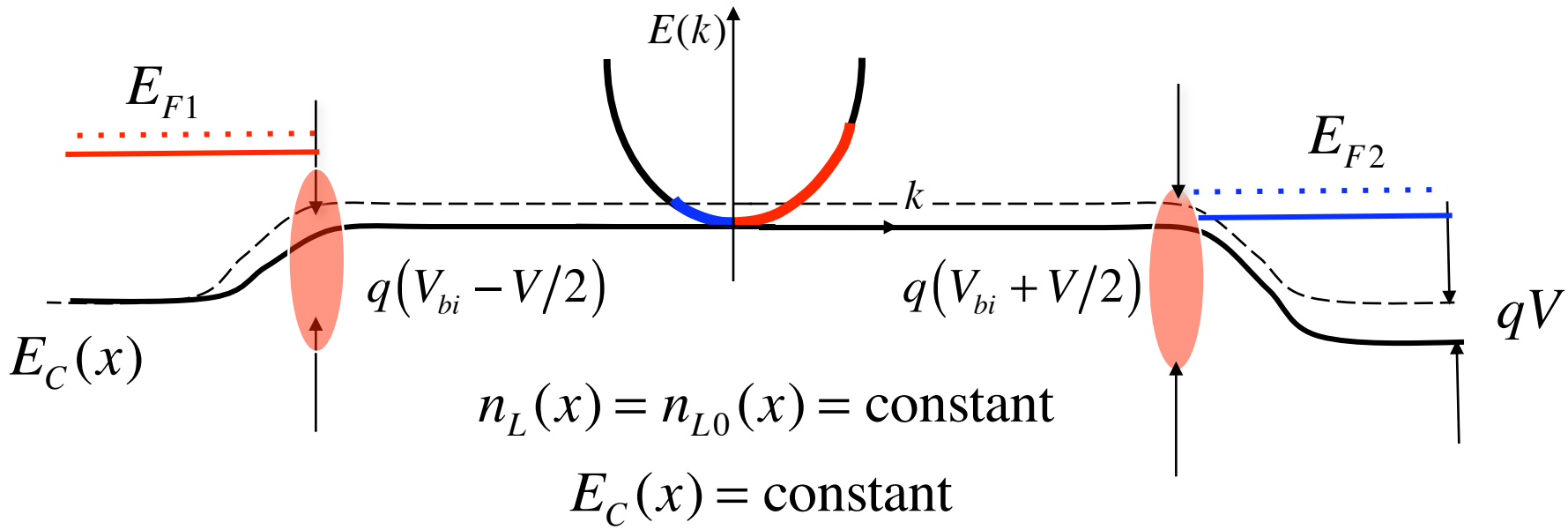
e-band diagram: conventional resistor

conventional resistor:



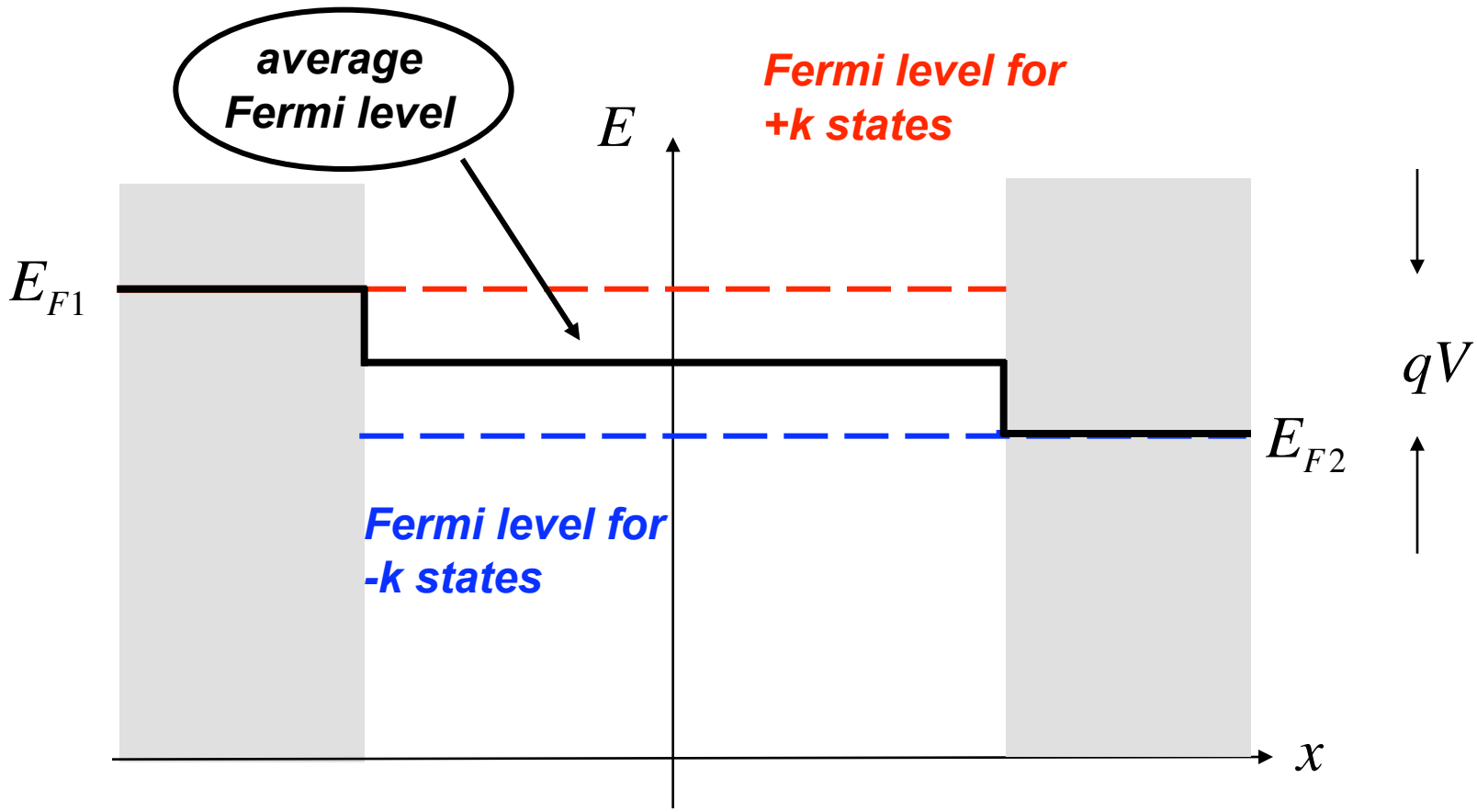


why is the e-band diagram constant?



The potential inside the resistor must be constant in order to maintain the electron density constant at its equilibrium value.
(Note: the figure is exaggerated for emphasis. Under near-equilibrium condiutions, $n^+ \approx n^-$)

another view



S. Datta, *Electronic Conduction in Mesoscopic Systems*, Chpt. 2, Cambridge, 1995

outline

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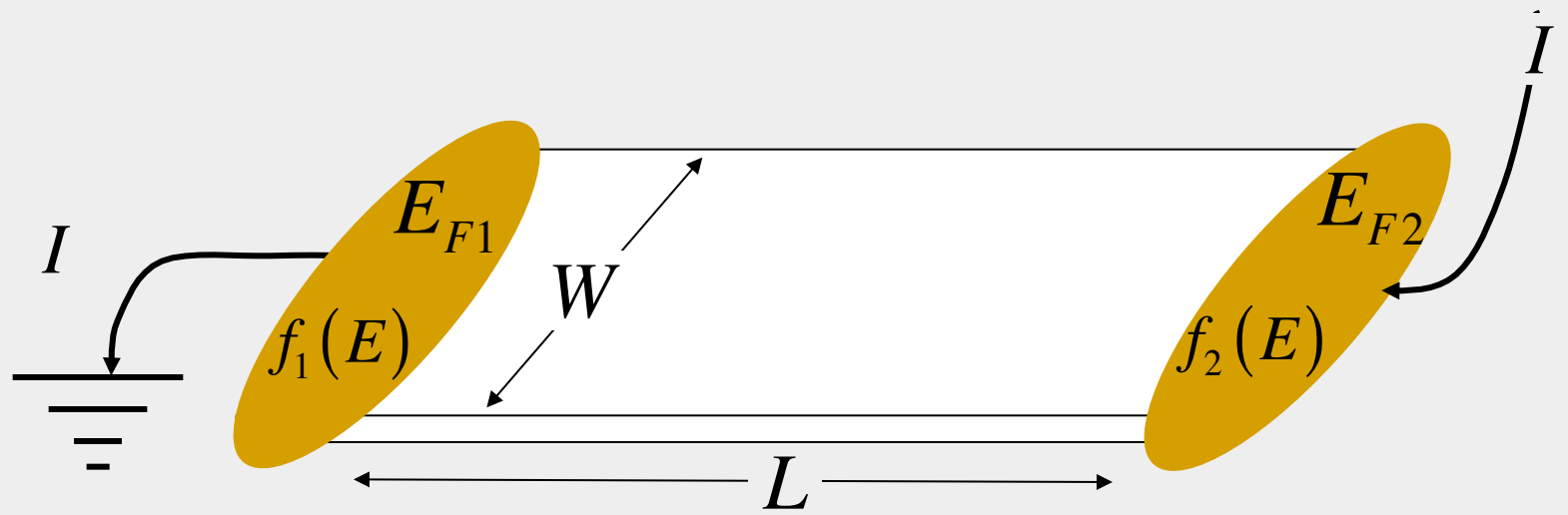
review: modes in 2D



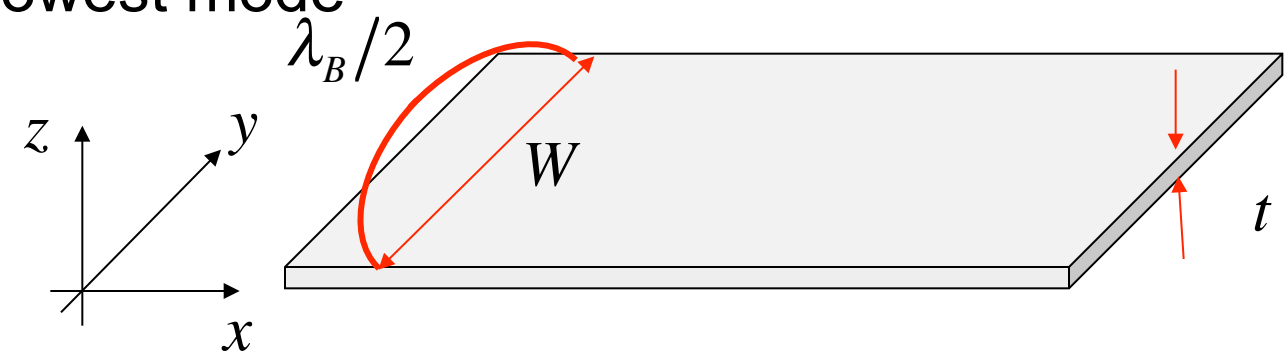
$$M_{2D}(E) = W g_V \frac{\sqrt{2m^*(E-\varepsilon_1)}}{\pi\hbar} = g_V \frac{W}{(\lambda_B/2)} \propto W \qquad \left(E(k) = \hbar^2 k^2 / 2m^*\right)$$

$$G_{2D} \propto W$$

waveguide modes

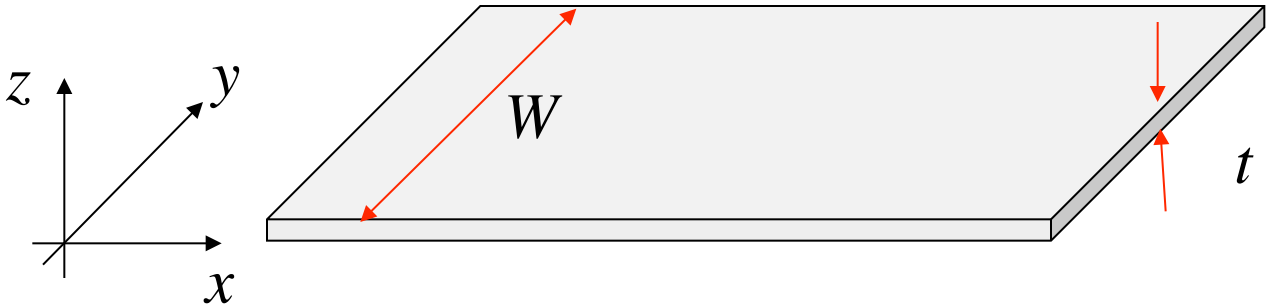
Assume that there is **one** (or a few) subbands associated with confinement in the z-direction. **Many** subbands associated with confinement in the y-direction.

lowest mode



***M* = # of electron half wavelengths that fit into *W*.**

more on modes (T = 0K)



$$M_{2D}(E_F) = W\frac{\sqrt{2m^*(E_F - \varepsilon_1)}}{\pi\hbar}$$

depends on bandstructure

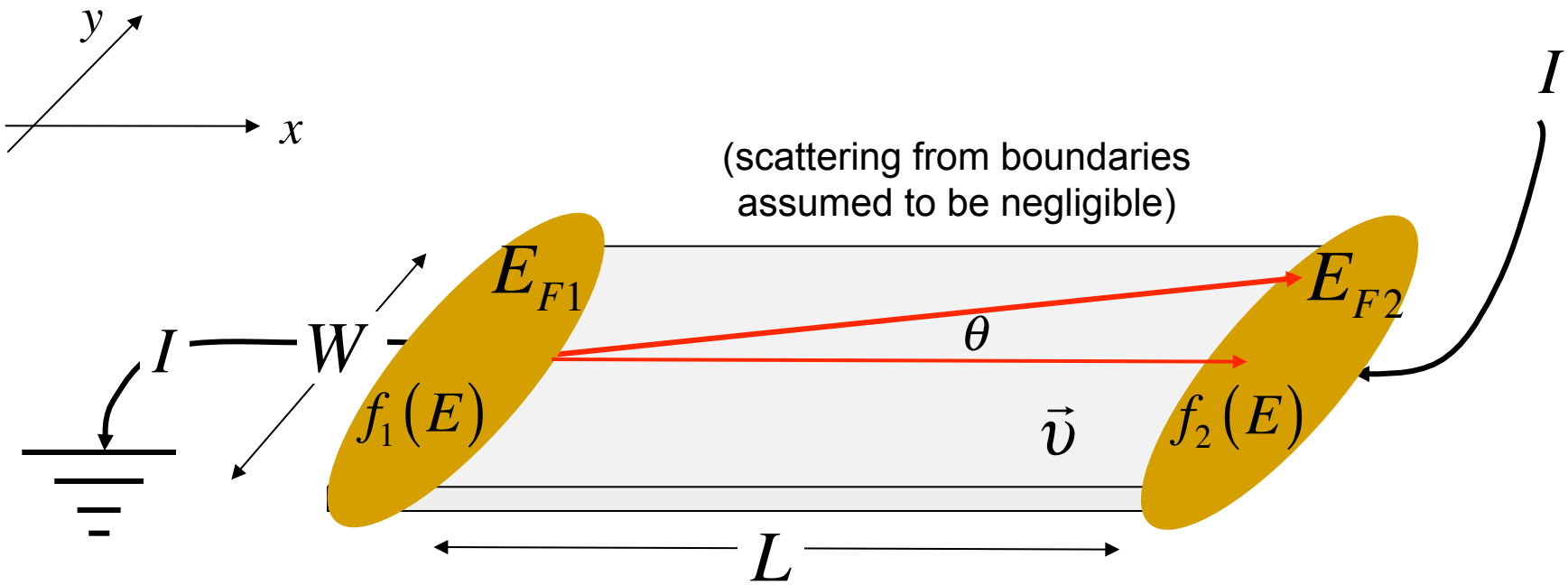
$$M_{2D}(E_F) = \frac{W}{(\lambda_F/2)} = \frac{W k_F}{\pi}$$

$$n_S = \frac{\pi k_F^2}{(2\pi)^2} \times 2 \rightarrow k_F = \sqrt{2\pi n_S}$$

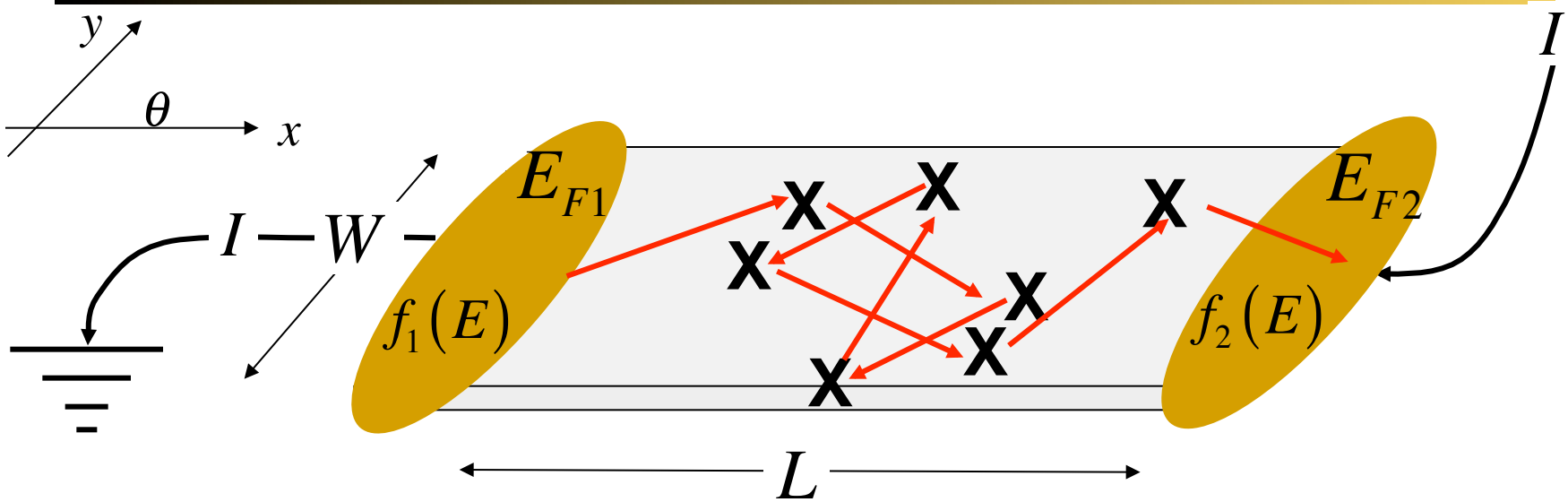
$$M_{2D}(E_F) = W\sqrt{\frac{2n_S}{\pi}}$$

depends only on n_S

ballistic transport in 2D



diffusive transport in 2D



- Electrons undergo a random walk as they go from left to right contact.
- Some terminate a contact 1, and some at contact 2.
- The average distance between collisions is the mfp, λ
- "Diffusive" transport means $L \gg \lambda$
- We expect that the diffusive transit time will be much longer than the ballistic transit time.

conductance in 2D

$$G = \frac{2q^2}{h}\left(\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right) \qquad M_{2D}(E) = Wg_V \frac{\sqrt{2m^*(E-\varepsilon_1)}}{\pi\hbar}$$

$$T = 0\mathrm{K}: \quad G_{2D} = \frac{2q^2}{h}T(E_F)M(E_F) \qquad T(E) = \frac{\lambda(E)}{\lambda(E)+L}$$

$T = 0\,\mathrm{K}$

i) ballistic: $G_{2D} = \frac{2q^2}{h}M(E_F) \propto W$

ii) diffusive: $G_{2D} = \frac{2q^2}{h}\frac{\lambda(E_F)}{L}M(E_F) \propto \frac{W}{L}$

$$G_{2D}(\mathrm{diff}) = G_{2D}(\mathrm{ball}) \times \lambda/L$$

conductance in 2D (ii)

$$G = \frac{2q^2}{h}\left(\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right) \qquad M_{2D}(E) = Wg_V \frac{\sqrt{2m^*(E-\varepsilon_1)}}{\pi\hbar}$$

$$T > 0\text{K}: \quad G_{2D} = \frac{2q^2}{h}\frac{\lambda_0}{\lambda_0 + L}W\sqrt{\frac{m^* k_B T}{2\pi\hbar^2}}\mathcal{F}_{-1/2}(\eta_F)$$

2D diffusive conductance (T = 0K)

$$G_{2D} = \frac{2q^2}{h}\frac{\lambda(E_F)}{L}M(E_F)$$

$$G_{2D} = \sigma_S \frac{W}{L}$$

$$\lambda(E_F) = \frac{\pi}{2}\upsilon(E_F)\tau(E_F)$$

$$\sigma_S = \frac{2q^2}{h}\lambda(E_F)M(E_F)/W$$

$$\sigma_S = n_S q \mu_n$$

$$\mu_n = q\tau(E_F)/m^*$$

$$\sigma_S = q^2 D_{2D}(E_F) D_n(E_F)$$

$$D_n(E_F) = \upsilon^2(E_F)\tau(E_F)/2$$

conductance in 2D: summary

1) The ballistic G is independent of the length of the resistor but proportional to W, because $M(E) \sim W$.

2) The diffusive G is proportional to (W/L) because $T(E) \sim 1/L$

3) The number of transverse modes, $M_{2D}(E)$, depends on the *bandstructure*.

4) The number of transverse modes, $M_{2D}(n_S)$, depends only the carriers density (T = 0K).

outline

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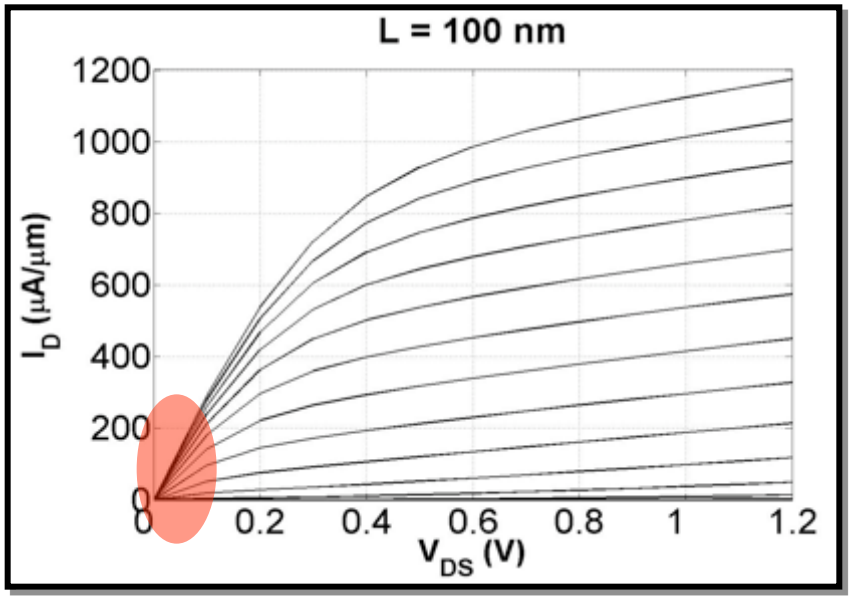
example: nanoscale FETs



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$R_{TOT} = \frac{V_{DS}}{I_D} = R_{SD} + R_{CH}$$

$$R_{SD} \approx 200\ \Omega\text{-}\mu\text{m}$$

Question: How close is the channel resistance to the ballistic resistance?

a “back of the envelope” calculation

Si N-MOSFET (unstrained):

$$G_{ch} = T_{lin} G_B = \frac{\lambda_0}{\lambda_0 + L} G_B$$

$$T_{lin} = \frac{\lambda_0}{\lambda_0 + L} \approx 0.20$$

$$G_{ch} \approx 0.20 G_B \rightarrow R_{ch} \approx 5 \times R_B$$

$$\mu_n \approx 260 \text{ cm}^2/\text{V-s}$$

$$D_n = \frac{\upsilon_T \lambda_0}{2}$$

$$\upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}} = 1.2 \times 10^7 \text{ cm/s}$$

$$\left(m^* = 0.19 m_0\right)$$

$$\lambda_0 \approx 14 \text{ nm}$$

$$\lambda_0 << L = 60 \text{ nm}$$

III-V FETs

InGaAs HEMT:

$$G_{ch} = T_{lin} G_B = \frac{\lambda_0}{\lambda_0 + L} G_B$$

$$T_{lin} = \frac{\lambda_0}{\lambda_0 + L} \approx 0.8$$

$$G_{ch} \approx G_B \rightarrow R_{ch} \approx R_B$$

$$\mu_n \approx 10{,}000 \text{ cm}^2/\text{V-s}$$

$$D_n = \frac{\upsilon_T \lambda_0}{2}$$

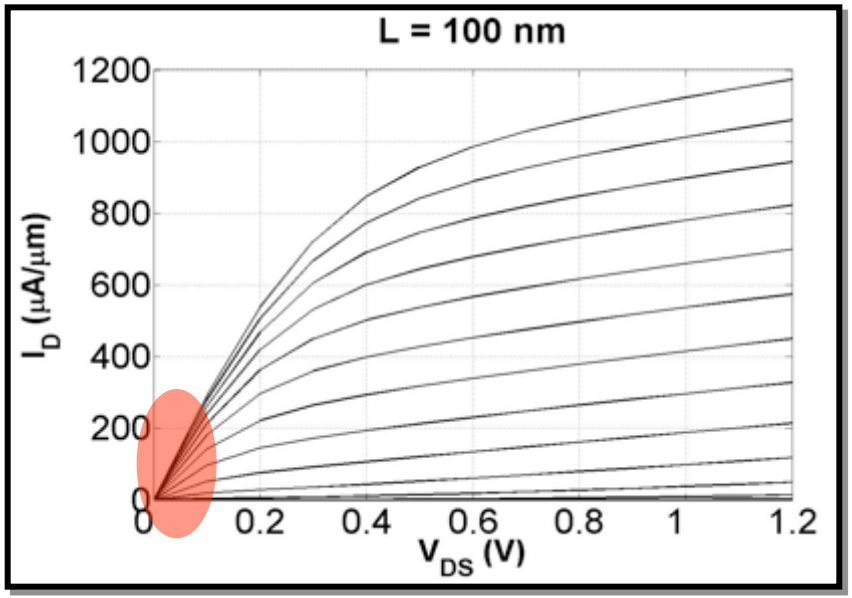
$$\upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}} = 2.7 \times 10^7 \text{ cm/s}$$

$$\left(m^* = 0.041 m_0\right)$$

$$\lambda_0 \approx 200 \text{ nm}$$

$$\lambda_0 > L$$

a second example about nanoscale FETs



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$M_{2D}(E_F) = W g_V \frac{\sqrt{2m^*(E_F - \varepsilon_1)}}{\pi\hbar}$$

$$M_{2D}(k_F) = \frac{W}{(\lambda_F/2)} = \frac{W\,k_F}{\pi}$$

$$n_S(k_F) = g_V \frac{k_F^2}{2\pi}$$

$$M_{2D}(k_F) = W\sqrt{\frac{2n_S}{\pi g_V}}$$

Question: How many transverse modes are there in a 1 micron wide FET?

number of conducting channels in a FET

$$M_{2D}(k_F) = W\sqrt{\frac{2n_S}{\pi g_V}}$$

Silicon n-MOSFETs:

$n_S \approx 1\times10^{13}\ \mathrm{cm}^{-2}$

$g_V = 2$

$M_{2D} \approx 180/\mu\mathrm{m}$

InGaAs HEMTs:

$n_S \approx 2\times10^{12}\ \mathrm{cm}^{-2}$

$g_V = 1$

$M_{2D} \approx 110/\mu\mathrm{m}$

graphene

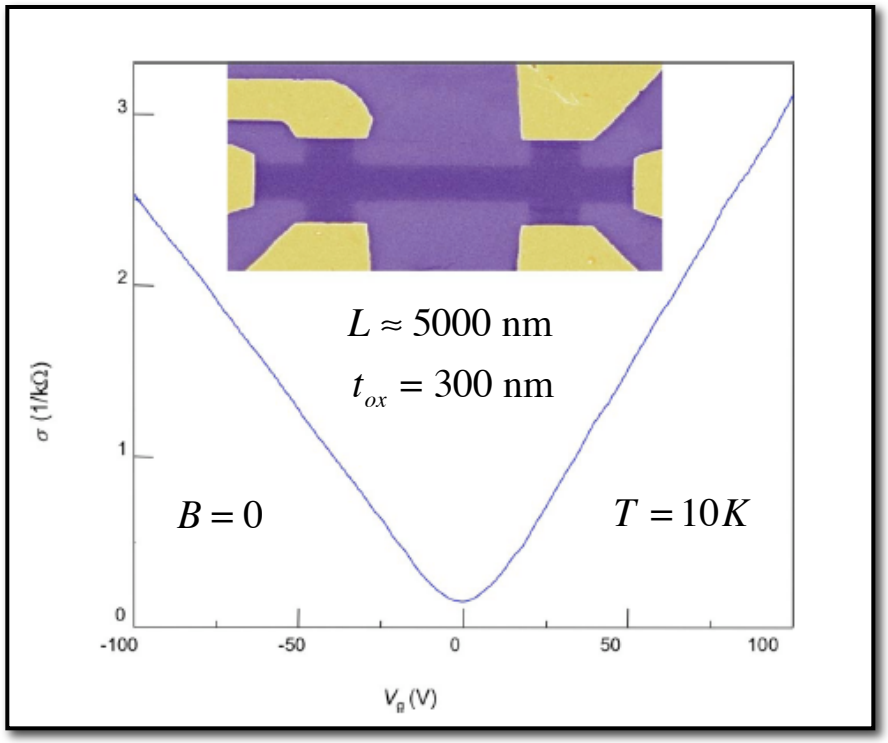
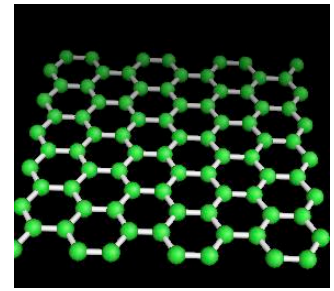
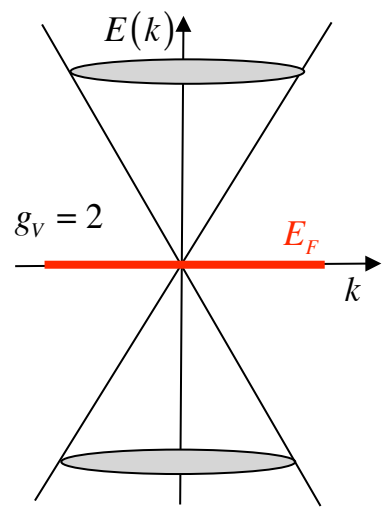


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," Rev. of Mod. Phys., **81**, 109, 2009.



CNTBands 2.0 (nanoHUB.org)



$$E(k) = \pm\hbar\upsilon_F k$$

$$\upsilon(k) = \upsilon_F$$

$$\upsilon_F \approx 1\times10^8 \text{ cm/s}$$

$$D(E) = \frac{2|E|}{\pi\hbar^2\upsilon_F^2}$$

$$M(E) = \frac{W2|E|}{\pi\hbar\upsilon_F}$$

questions

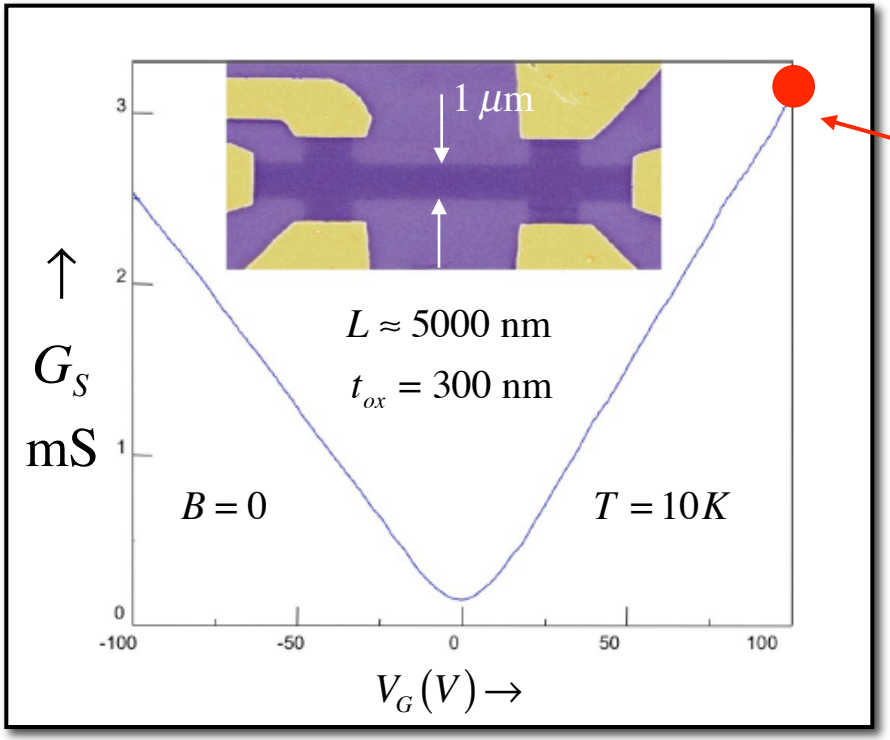


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," Rev. of Mod. Phys., **81**, 109, 2009.

$$\sigma_S \approx 3.0 \text{ mS}$$

$$n_S \approx 7.1 \times 10^{12} \text{ cm}^{-2}$$

1) How close to the ballistic conductance?

2) What is the mfp?

analysis

$$G_{2D} = \frac{2q^2}{h}\frac{\lambda(E_F)}{L}M(E_F) = \sigma_S \frac{W}{L}$$

$$G_B = \frac{2q^2}{h}M(E_F)$$

$$n_S(E_F) = \frac{1}{\pi}\left(\frac{E_F}{\hbar \upsilon_F}\right)^2 \rightarrow E_F = 0.3\,\text{eV}$$

$$M(E_F) = W\,2E_F/\pi\hbar\upsilon_F$$

$$\sigma_S(\text{meas}) = (\lambda/L)\sigma_S(\text{ball})$$

$$\lambda(0.3\text{ eV}) \approx 130\text{ nm}$$

$$\sigma_S(\text{meas}) \approx \sigma_S(\text{ball})/40$$

For more about the conductance of graphene, see: “Low-bias transport in graphene,” by M.S. Lundstrom and D. Berdebes, NCN@Purdue 2009 Summer School, nanoHUB.org

mobility of graphene?

$$G_{2D} = \frac{2q^2}{h}\frac{\lambda(E_F)}{L}M(E_F) = n_S\, q\, \mu_n \frac{W}{L}$$

$$\mu_n = \frac{2q}{h}\frac{\lambda(E_F)M(E_F)/W}{n_S}$$

$$M(E) = W\, 2E_F/\pi\hbar\upsilon_F$$

$$\lambda(E) = \frac{\pi}{2}\upsilon_F\tau(E_F)$$

$$n_S(E_F) = \frac{1}{\pi}\left(\frac{E_F}{\hbar\upsilon_F}\right)^2$$

$$\mu_n = \frac{q\tau(E_F)}{\left(E_F/\upsilon_F^2\right)}$$

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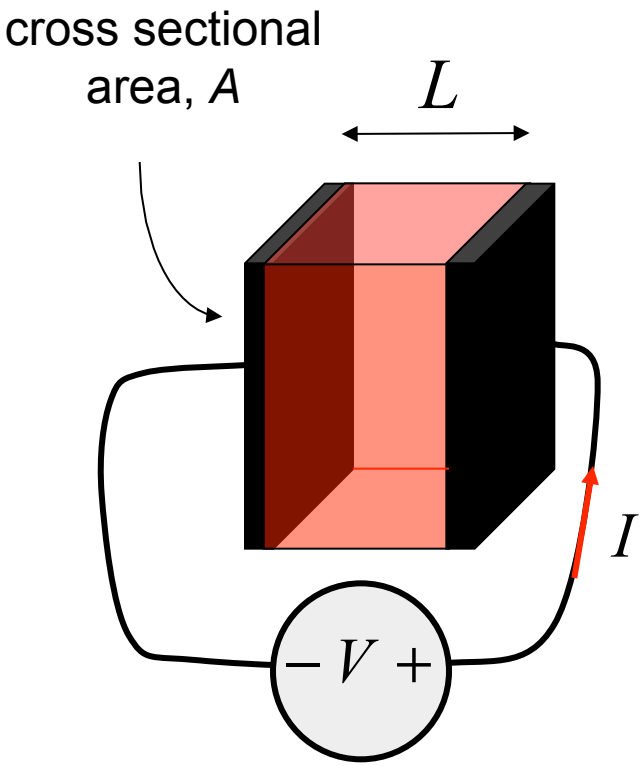
$M(E)$ in 3D



If the cross-sectional area is large, then there is no quantum confinement, and electrons are free to move in 3D.

$$D_{3D}(E) = AL\frac{m^*\sqrt{2m^*(E-E_C)}}{2\pi^2\hbar^3}$$

$$M_{3D}(E) = A\frac{m^*}{2\pi\hbar^2}(E-E_C)$$

$$E(k) = E_C + \frac{\hbar^2 k^2}{2m^*}$$

resistors in 3D

$$G = \frac{2q^2}{h}\left(\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right)$$

$$M_{3D}(E) = A\frac{m^*}{2\pi\hbar^2}(E - E_C)$$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

$$G_B \propto A$$

$$G_{diff} \propto \frac{A}{L} = \sigma\frac{A}{L}$$

$$G_{diff} = nq\mu_n\frac{A}{L}$$

$$G_{3D} = \left(\frac{2q^2}{h}\right)\left(\frac{m^* k_B T}{2\pi\hbar^2}\right)\left(\frac{\lambda_0}{\lambda_0 + L}\right)A\mathcal{F}_0(\eta_F)$$

$$G_{3D}(0\text{K}) = \frac{2q^2}{h}M_{3D}(E_F)T(E_F)$$

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Landauer formula (low bias)

$$G = \frac{2q^2}{h}\left(\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right)$$

- $M(E)$ depends on bandstructure
- $T(E)$ depends on scattering physics
- $G = n\,q\,\mu_n$ in the diffusive limit
- The term, $\left(-\partial f_0/\partial E\right)$ comes from approximating $\left(f_1 - f_2\right)$ for low bias.

Landauer formula (cont.)

$$G = \frac{2q^2}{h}\left(\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right)$$

- works in 1D, 2D, or 3D for any bandstructure
- $G_{2D} \sim W$ because $M_{2D} \sim W$
- $G_{3D} \sim A$ because $M_{3D} \sim A$
- $G_{diff} \sim 1/L$ because $T_{diff} \sim 1/L$
- G_B is independent of L and represents an upper limit to G (lower limit to R)

questions

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