Fundamentals of Nanoelectronics

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Lecture 38: Ohm's Law

Ref. Chapter 9.4

Network for Computational Nanotechnology

NCN nanoHUB online simulations and more

Coherent Transport

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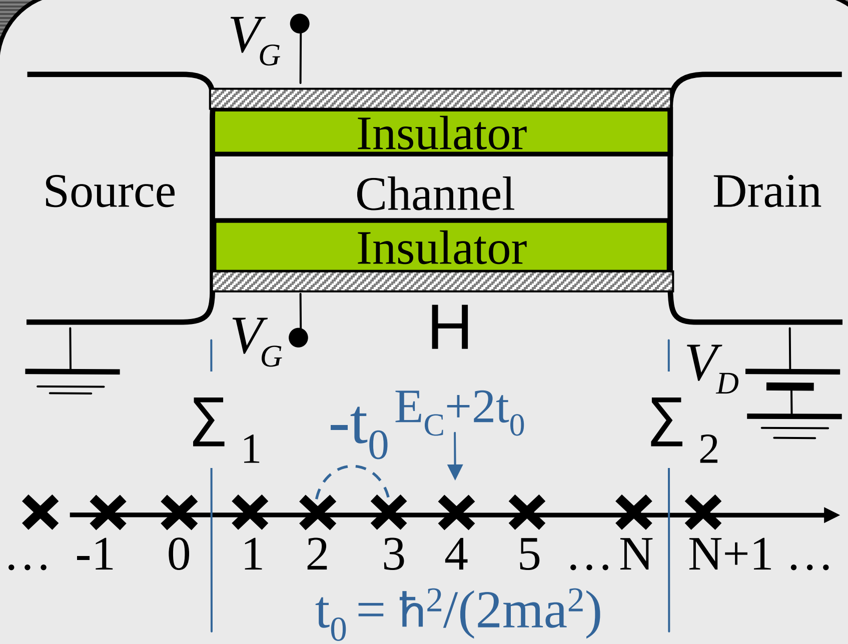


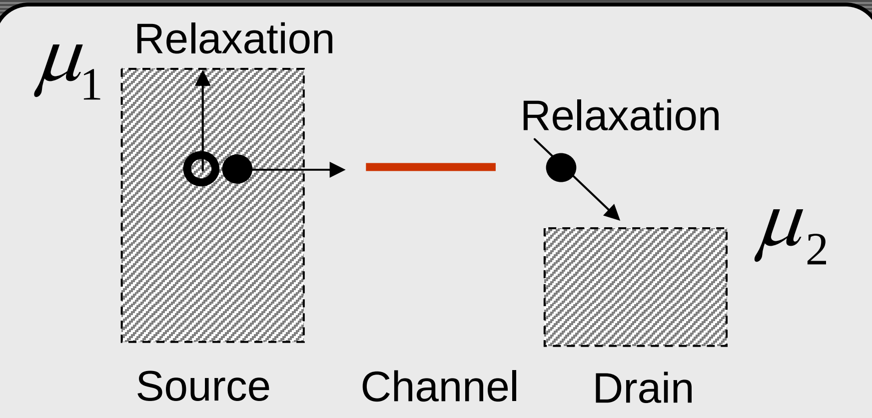
$$I = \frac{2q}{h}\int dE\,\bar{T}(E)\big(f_1(E) - f_2(E)\big)$$

$$\bar{T}(E) = Trace\left(\Gamma_1 G \Gamma_2 G^+\right)$$

$$\Gamma_1 = i(\Sigma_1 - \Sigma_1^+),\ \Gamma_2 = i(\Sigma_2 - \Sigma_2^+)$$

$$G(E) = \left(EI - H - \Sigma_1 - \Sigma_2\right)^{-1}$$

- In the last few lectures we've been discussing coherent transport where electrons go through the channel without loosing energy or dissipating heat. As the figure below shows once an electron gets onto to the drain contact it looses its energy and relaxes down whereby it generates heat. On the other hand the hole left in drain floats up to the top. Describing these processes are very difficult. We've bypassed them by stating that certain forces keep holes and electrons in equilibrium with the Fermi levels in the contacts.



What We Expect From Ohm's Law

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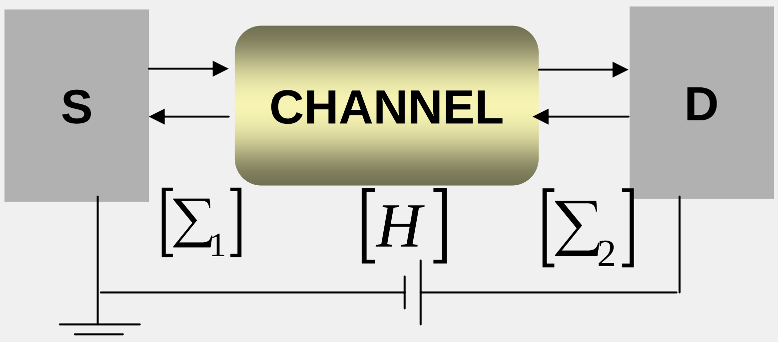

- What we've discussed thus far in this class was current flow through a small device using the Hamiltonian and the self energy matrices.
- Today we want to see how one eventually gets Ohm's law as the device gets larger and larger.
- Transmission is given by:

$$\bar{T} = MT\,; \quad M \propto A$$

- When you think of a conductor that has length L and area A, Ohm's law predicts that **conductance ~ A/L**.
- For our expression of current, we know that the number of modes M will increase as the area is increased. So the cross sectional dependence can be seen easily.
- What is harder to see is that: $T \propto 1/L$
- So if the probability of an electron getting form one side of the device to the other side is inversely proportional to the length of the conductor, then we get the same result as Ohm's law. For short devices, there is no scattering and T=1. How ever for longer devices, electron wave function is more spread over the channel; hence an electron might get thrown backwards to the source once it has gotten into the channel.

Transmission Through Two Scatterers

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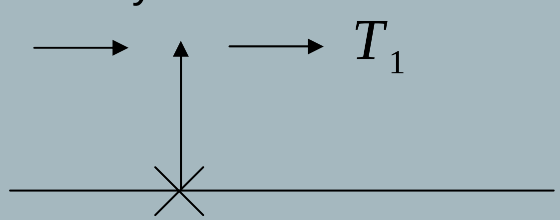
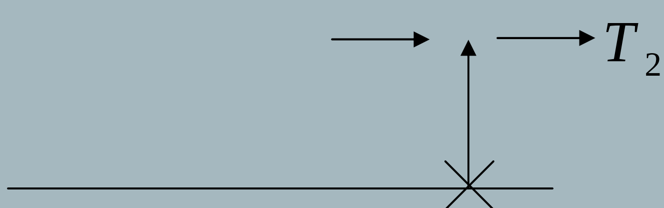
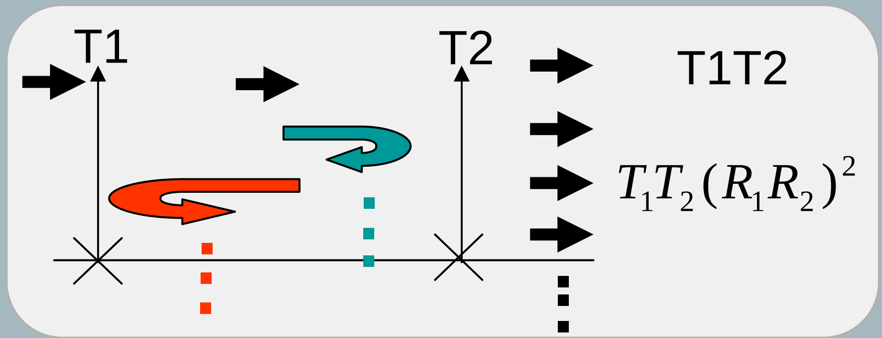
- Under certain conditions: $T = \dfrac{\Lambda}{L+\Lambda}$
- Λ is called the mean free path and it can be defined as:
  - **A conductor that has a length of Λ, has a transmission probability of ½.**
- The question now is that where is the above expression for T is coming from.
- To investigate this consider a conductor with two different scatterers with transmission probabilities of T1 and T2 respectively:





- Thinking of electrons as particles, for transmission through 2 scatterers we have:



$$T = T_1T_2\left(1 + x + x^2 + \ldots\right); \quad x = R_1R_2$$

- This is the Geometric series:

$$T = \frac{T_1T_2}{1-R_1R_2}; \qquad T_1 + R_1 = 1$$

$$T_2 + R_2 = 1$$

$$\frac{1}{T} = \frac{1-(1-T_1)(1-T_2)}{T_1T_2} = \frac{T_1+T_2-T_1T_2}{T_1T_2}$$

$$\Rightarrow T = \frac{T_1T_2}{T_1+T_2-T_1T_2}$$

Transmission Versus Length

- Let's write 1/T instead of T:

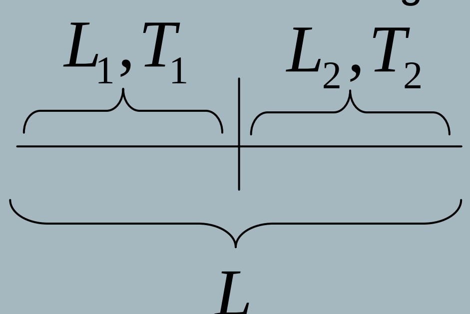
$$T = \frac{T_1 T_2}{T_1 + T_2 - T_1 T_2}$$

$$\Rightarrow \frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2} - 1$$

- Now the question is how transmission is related to the length of a long conductor.

$$T(L) = ?$$

- Think about the wire with length L:



$$\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2} - 1$$

- Our function must satisfy the above relationship. If you separate your conductor into two sections of equal length we should have:

$$\frac{1}{T(L)} = \frac{1}{T(L_1)} + \frac{1}{T(L_2)} - 1$$

- The following equation for T will satisfy the above relationship

$$T = \frac{\Lambda}{L + \Lambda}$$

- To check we do:

$$\frac{L_1 + L_2 + \lambda}{\lambda} = \frac{L_1 + \lambda}{\lambda} + \frac{L_2 + \lambda}{\lambda} - 1$$

100% Transmission in the Presence of Scatterers

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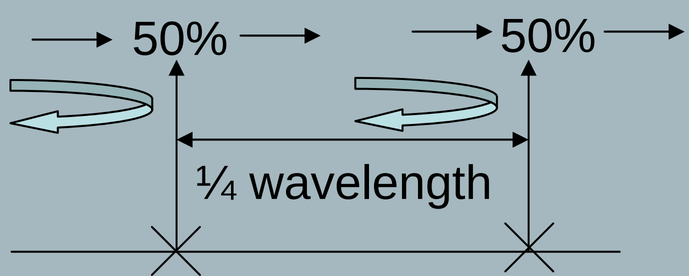
- This picture is not as easy if we think of electrons as waves and our relationship will not compensate for all the physics that is inherent in the problem.

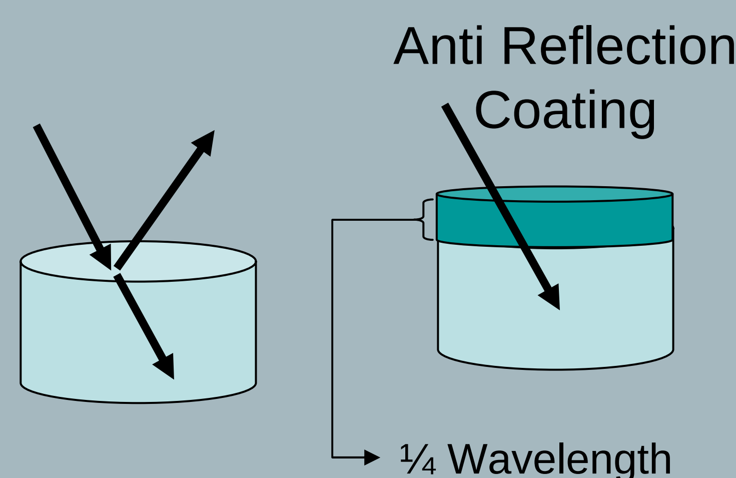


- Consider the case were we have two scatterers; from particle point of view and using the relationship that we've derived for transmission, transmission should reduce by a factor of 2 when we go from one to two scatterers.

$$T = \frac{T_1 T_2}{1 - R_1 R_2} = \frac{0.25}{1 - 0.25} = \frac{1}{3}$$

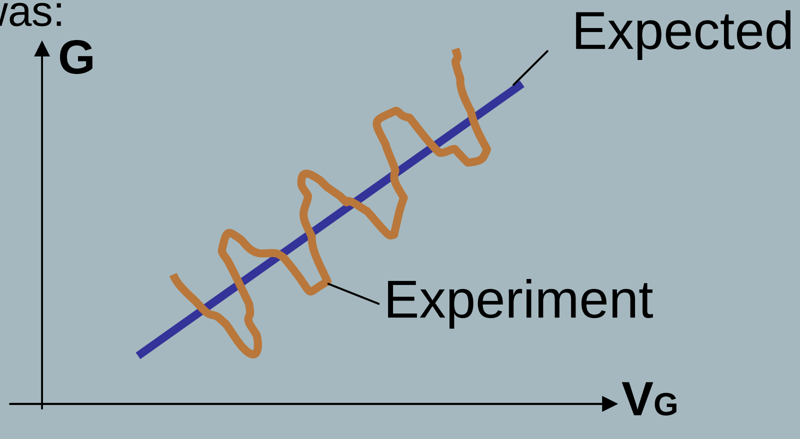
- How ever this is not the case.

- Looking at the problem Quantum mechanically, it is interference between the waves that changes the picture. For example if the distance between the two scatterers is quarter of a wavelength, the two reflections cancel each other and this way we can even get more transmission (Constructive interference). In this case transmission will be 100% and there would be no reflection. This technique has been used to eliminated reflection from lenses:



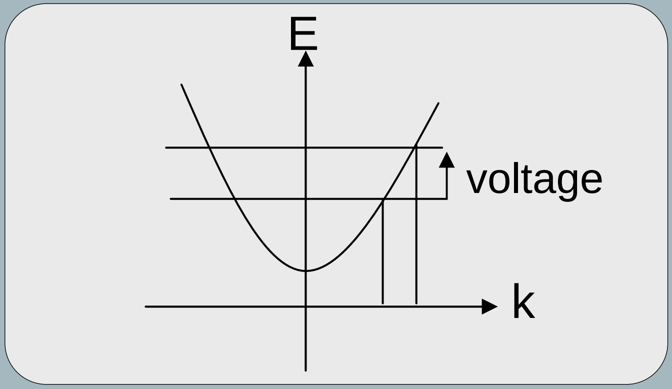
Conductance Fluctuations

• In 1980’s people started experimenting with conductors that were small enough to have only a few scatterers. Notice that for long devices with many scatterers, Ohm’s law follows automatically. However what people discovered for small conductors was:



• To understand this consider a device which has some impurities that cause scattering.

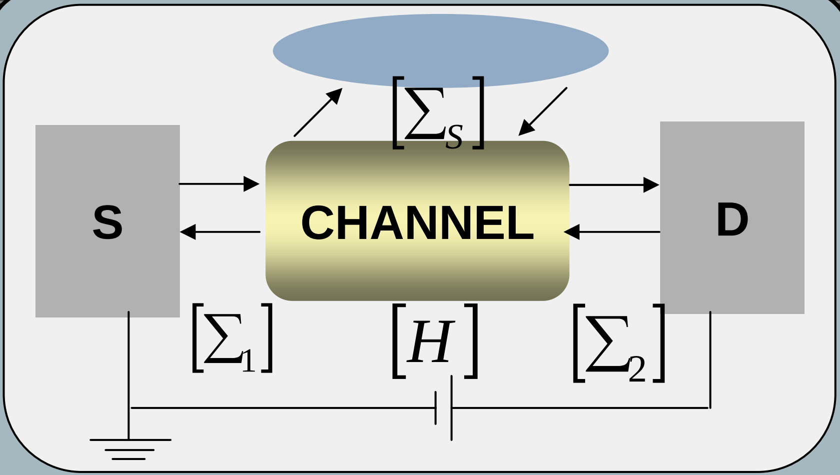
• Then consider the dispersion relation:



• Increasing the gate voltage causes greater number of electrons. This in turn is reflected in the E-k diagram with a higher Fermi level. As the result the k (wave vector) of the electrons close to Fermi energy changes; Which means the wavelength of the electrons are changing.

• What happens is that for some wavelengths there will be constructive interference and for some there will be destructive interference . As the result there are fluctuations in the conductance like the ones drawn on the figure.

The Conceptual Scattering Contact (Buttiker Probe)



- Scattering processes can be taken into account in an indirect way by assuming a conceptual (not real) scattering contact. Electrons get in and get out of this contact with their phases randomized.
- Remember the old expression for current:

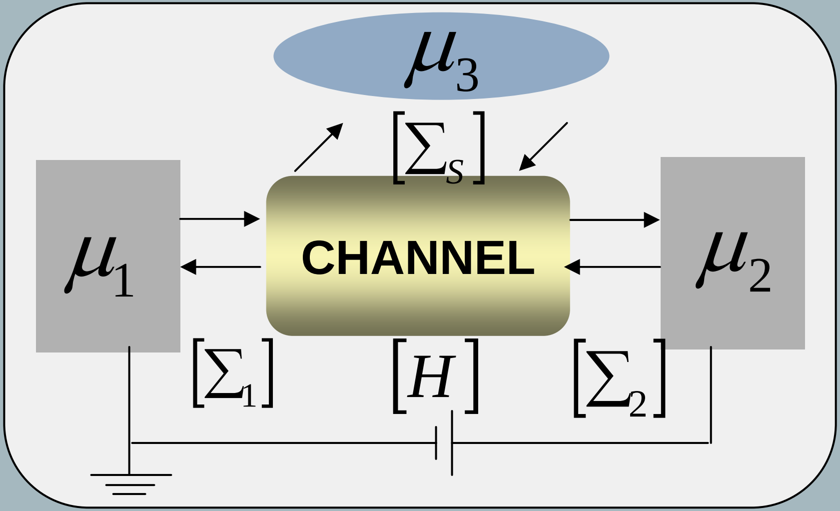
$$I = \frac{2q}{h}\int dE\,\overline{T}(E)\left(f_1(E) - f_2(E)\right)$$

- When the third contact is introduced the new equation for current at a terminal can be written as:

$$I_i = \frac{2q}{h}\int dE \sum_j \overline{T}_{ij}(E)\left(f_i - f_j\right)$$

- The possible transmissions for the three terminal device is shown by the arrows in the figure on the left.

Scattering Processes



• Notice that we do know the where the Fermi Levels μ 1 and μ 2 lie; however we don't know the position of μ 3. The way we can find it is by knowing that the current going to the third contact is 0. So we adjust μ 3 in way to get I3=0.

$$I_3 = \frac{2q}{h} \int dE \sum_j \overline{T}_{3j}(E)\left(f_3 - f_j\right) = 0$$

• This is probably the easiest way to include the scattering processes into the model. Notice that till now we've been considering very small devices for which the transport was ballistic and there was no scattering present. The method above is useful for larger devices.