Lecture 4
Stick Percolation and Nanonet Electronics

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1) Stick percolation and nanonet transistors
2) Short channel nanonet transistors
3) Long channel nanonet transistors
4) Transistors at high voltages
5) Conclusions
lecture 3 vs. lecture 4

\[ G \sim \sigma_{row} p^L \frac{W}{L} \]

\[ G \sim \sigma_{row} \frac{W}{L^\alpha} \]
how to make nanonet transistors?

**Advantages**
- Highly crystalline CNT / SiNW by high temperature process
- Plastic, glass or organic substrate:
  - Low temperature final step
- Transparent and conducting

Solution Process

- T > 500 C
- T < 200 C

Nanotransfer Printing

- T < 200 C

CNT / SiNW / ZnO

Rogers et al. Nature Mat. 2006
why do we make nanonet transistors?

Ideally ...

In practice ...

flexible electronics?

... better if ...

... lot of interest in this topic.
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short channel nanonetwork transistors


short channel stick percolation

Fan Diagram:
Collect all the sticks to one point
number of bridging sticks

Generalized Buffon Needle Problem

If needle is curved, use chord length
I-V characteristics: ballistic transport

\[ I_D^{(i)} = \frac{q}{\pi \hbar} \int_{E_b(V_G)}^{\infty} dE \left[ f_0(E_{FS}) - f_0(E_{FD}) \right] \equiv f(V_G, V_D) \]

\[ I_D = \sum_{i=1}^{N} \int_{0}^{L_S-L_C} D_x dx \frac{2}{\pi} \theta_{\text{max}} f = f \frac{2D_c L_S}{\pi} \left[ \sqrt{1 - \left( \frac{L_C}{L_S} \right)^2} - \frac{L_C}{L_S} \cos^{-1} \frac{L_C}{L_S} \right] \]

Electrical Part  Geometrical Part
ballistic transport and length-scaling ....

\[ I_D = \int (V_G, V_D) \times \frac{2D_C L_S}{\pi} \left[ \sqrt{1 - \left( \frac{L_C}{L_S} \right)^2} - \frac{L_C}{L_S} \cos^{-1} \frac{L_C}{L_S} \right] \]

length dependence even for ballistic transport, nothing to do it with mobility!
I-V characteristics: with scattering

\[ I_D^{(i)} = \frac{q}{\pi \hbar} \int_{E_b(V_G)}^{\infty} dE \left[ \frac{\lambda}{\lambda + L^{(i)}} \right] \left[ f_0(E_{FS}) - f_0(E_{FD}) \right] \]

\[ \approx \frac{\langle \lambda \rangle}{\langle \lambda \rangle + L^{(i)}} \frac{q}{\pi \hbar} \int_0^{\infty} dE \left[ f_0(E_{FS}) - f_0(E_{FD}) \right] \]

\[ I_D = \sum_{1}^{N} \frac{\langle \lambda \rangle}{\langle \lambda \rangle + L^{(i)}} \times \bar{f} = \bar{f} \times \sum_{1}^{N} \frac{\langle \lambda \rangle}{\langle \lambda \rangle + L_c / \cos(\theta)} \]
**I-V characteristics: with scattering**

\[
I_D = f \times \sum_{1}^{N} \frac{\langle \lambda \rangle}{\langle \lambda \rangle + L_C / \cos(\theta)}
\]

\[
\frac{I_D}{f} = \frac{2D_C}{\pi b^2} \left[ b g_B \left( \frac{L_C}{L_S} \right) - \cos^{-1} \frac{L_C}{L_S} + \frac{2(bL_C / L_S + 1)}{\sqrt{b^2 - 1}} \tanh^{-1} \frac{(b - 1) \tan(\theta / 2)}{\sqrt{b^2 - 1}} \right]
\]

**Electrical Part**

\[
I_D = \int (V_G, V_D) \xi \left( \frac{L_C}{L_S}, D_C L_S \right) \sim \sigma_0 \frac{W}{L^\alpha}
\]

**Geometrical Part**

Experimental verification of fan-diagram in the appendix ...
.... a remarkable formula

\[ I_T = f(V_D, V_G) \times \xi \left( \frac{L_C}{L_S}, D_C L_S^2 \right) \]

Scaling variables...

Classical Transistor Theory (1950s -- )

Classical percolation Theory (1970s -- )
lecture 3 vs. lecture 4

\[ G \sim \sigma_{\text{row}} p^L \frac{W}{L} \]

\[ G = \frac{2q^2}{\pi^2 \hbar} D_c L_s \left[ \sqrt{1 - \left( \frac{L_c}{L_s} \right)^2} - \frac{L_c}{L_s} \cos^{-1} \frac{L_c}{L_s} \right] \]
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long channel nanonets: numerical model

\[ \sum_i \nabla^2 \Phi_i + \frac{\rho_i}{\varepsilon} = \frac{d^2 \Phi_i}{ds^2} + \frac{\rho_i}{\varepsilon} + \sum_{j \neq i} \frac{\Phi_j - \Phi_i}{\lambda_{ij}^2} - \frac{\Phi_i - V_G}{\lambda^2} = 0 \]

\[ J_{n,i} = qn \mu E - qD \frac{dn}{ds} \]

\[ \sum_i \frac{dJ_{n,i}}{ds} - \sum_{i \neq j} c_{ij}^n (n_i - n_j) = 0 \]
don’t try this at home: use nanohub

- Analytical solution not possible
- Self consistent numerical DD-Poisson solver
- Solve for hundreds of configuration
- Solve for various biases

Simulator at  www.nanohub.org as ‘NanoNET’
the end of Ohm’s law ...

\[ I_D = f(V_D, V_G) \times \xi \left( \frac{L_S}{L_C}, D L_S^2 \right) \]

\[ \frac{G}{\sigma_0} = \frac{W_{ef}}{L_c} \]
geometrical scaling function

\[ I_D = f(V_D, V_G) \times \xi \left( \frac{L_C}{L_S}, D C L_S \right) \]

\[ = A (V_G - V_{th}) V_D \times \xi \left( \frac{L_C}{L_S}, D C L_S \right) \]

\[ G = \frac{I_D}{V_D} \propto \xi \left( \frac{L_S}{L_C}, D L_S^2 \right) \approx \frac{1}{L_S} \left( \frac{L_S}{L_C} \right)^{m(DL_S^2)} \]

Do you see ohm’s law at high D?
$G \sim \sigma_{row} p^{L} \frac{W}{L}$

$G \sim \sigma_{row} \frac{W}{L^\alpha}$

$G = \frac{2q^2}{\pi^2 \hbar} D_c L_S \left[ \sqrt{1 - \left( \frac{L_c}{L_S} \right)^2} - \frac{L_c}{L_S} \cos^{-1} \frac{L_c}{L_S} \right]$
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nonlinear I-V of nanononet transistors
voltage scaling in theory …

\[ I_D = f(V_D, V_G) \times \xi \left( \frac{L_C}{L_S}, DL_S^2 \right) \]

\[ \frac{I_{DL}}{f(V_{DL}, V_G)} = \xi \left( \frac{L_C}{L_S}, DL_S^2 \right) \frac{I_{DH}}{f(V_{DH}, V_G)} \]

… scales exactly, as anticipated from short channel formula
characterizing one transistor is sufficient ...

... and in practice!

\[ I_D = \xi \left( \frac{L_S}{L_C}, DL_S^2 \right) f (V_D, V_G) \]

Hur et al. JACS, 2005
Pimparkar et al. EDL, 2007

\[ \xi \left( \frac{L_S}{L_C}, DL_S^2 \right) \approx \left( \frac{L_S}{L_C} \right)^{m(DL_S^2)} \]

\[ f (V_G, V_D) = (V_G - V_{TH}) V_D - \beta V_D^2 \]
* Metallic CNTs short transistors and must be eliminated.
M-CNT Content: heterogeneous percolation

![Graph showing the relationship between conductivity and density with a linear scale for conductivity and a log scale for density. The graph includes a line with markers for experimental data and a line for numerical data. The graph also shows a series of images illustrating the percolation process.]
‘striping’ control of percolation threshold

\[ \rho^* = \frac{4.26^2}{\pi L_s^2} \]

\[ \rho^* = \frac{4.26^2}{\pi \left( \frac{L_s}{k} \right)^2} \]

Depending on the ckt, shorten the tubes by a factor k!
(*see lecture 3 for the theory of \( p_c \) shift)
Striping for improved on-off ratio
Illustrated the power of stick percolation model by analyzing results of short and long-channel nanonet transistors.

Nanonet transistors have become a testbed for the new theory of “Nonlinear percolation”. This new theory helped design of experiments, optimization of transistors, interpretation of device performance.

You will get additional insight into the problem when you do the HW using the Nanonet simulator at nanohub.org
alignment and asymmetric percolation

With more alignment

Av. Path length $\downarrow$ $\rightarrow$ $I_D \uparrow$
No. of paths $\downarrow$ $\rightarrow$ $I_D \downarrow$

Trade-off for optimal alignment
percolation of quasi-aligned sticks

Random orientation

Quasi-aligned

\[ N_{C,R} \approx \frac{4}{\pi \left( \frac{L_s}{2} \right)^2} \]

Fine for small \( q \)

\[ N_{C,\theta} \approx \frac{4}{\pi \left( \frac{L_s \sin(\theta)}{2} \right)^2} \]

\[ L_s \sin(\theta) / 2 \]
excluded volume for aligned stick ....

\[ A_{\theta_i, \theta_j} = L_s L_s \sin(\theta_i - \theta_j) \]

\[ P(\theta_j) = \frac{1}{2\theta} \]

\[
\langle A_{ex} \rangle_{\theta} = \int_{-\theta}^{\theta} d\theta_i P(\theta_i) \int_{-\theta}^{\theta} d\theta_j P(\theta_j) \times A_{\theta_i, \theta_j}
\]

\[
= \frac{L_s^2}{4\theta^2} \left[ 4\theta - 2\sin(2\theta) \right] \sim \frac{2L_s^2}{\pi} \sin\theta
\]

\[
\langle A_{ex} \rangle_{\theta} N_{c,\theta} \sim 1.8 \sim \langle A_{ex} \rangle_{R} N_{c,R}
\]

\[
\frac{N_{c,\theta}}{N_{c,R}} \sim \frac{1}{\sin(\theta)}
\]

Balberg, PRB, 1984
alignment shifts of percolation threshold

- Alignment shifts of percolation threshold
- Conductivity

Threshold Ratio vs alignment angle (rad)

Aligned vs Random

Conductivity vs alignment angle (rad)

Random vs quasi-aligned

$\frac{L_c}{L_s} = 8$

$I_{ON}$ (mA) vs $\theta_{avg}$
experimental verification

θ = 5

θ = 15

θ = 25

θ = 35

θ = 45

A

B

C

D

E

Pimparkar et al. MRS Spring meeting 2007
conclusions

Illustrated the power of stick percolation model by analyzing results of short and long-channel nanonet transistors.

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Appendix
scaling relationship: simple argument

\[ I_D(x) = W(x)q n \mu \frac{dV}{dx} \]

\[ \int_0^{L_c} \frac{I}{W(x)} \, dx = \int_0^{V_D} C_{ox}(V_G - V_T) \mu \, dV \]

\[ I_D = \frac{A}{L_S} \xi \left( \frac{L_S}{L_C}, DL_S^2 \right) \times f(V_G, V_D) \]

\[ I = \frac{\mu C_{ox}}{\int_0^{L_c} W(x)^{-1} \, dx} \left[ (V_G - V_T) V_D - V_D^2 / 2 \right] \]

\[ = \frac{A}{L_C} \xi(p_i, L_C) \times f(V_G, V_D) \]
metallic and semiconducting CNTs

* Metallic CNTs shorts transistors and must be eliminated.
electrical burning of metallic nanotubes

230 nm

$V_G$ (V)

$V_B$

$I$ (mA)

On current ($\mu$A)

Burn Pulse, $V_B$ (V)

90 nm

170 nm

230 nm

40
Interpretation ..

\[ L_B = \frac{V_B}{I_{CRIT}} \]
interpretation …

\[ L_B = \frac{V_B}{I_{CRIT}} \]
alignment of short nanonet transistors...

Random network is close to optimal!