

# ECE-656: Fall 2009

## Lecture 20: Transmission and Backscattering

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### transmission and mean-free-path

Transmission is a key parameter in the Landauer approach to carrier transport. Transmission is related to the mean-free-path *for back-scattering*. The mfp for backscattering is related to the microscopic scattering processes.

Questions:

- 1) Why is  $T(E) = \lambda(E)/(\lambda(E)+L)$ ?
- 2) How is  $\lambda(E)$  related to  $S(k, k')$ ?
- 3) How can  $\lambda$  be estimated experimentally?

## outline

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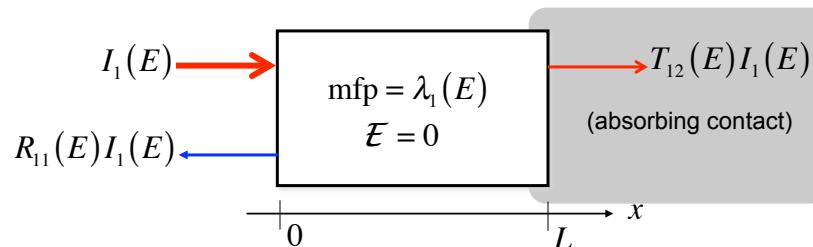
- 1) Transmission and mfp
- 2) MFP and carrier scattering
- 3) Extracting mean-free-paths from experiments

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## transmission across a field-free slab

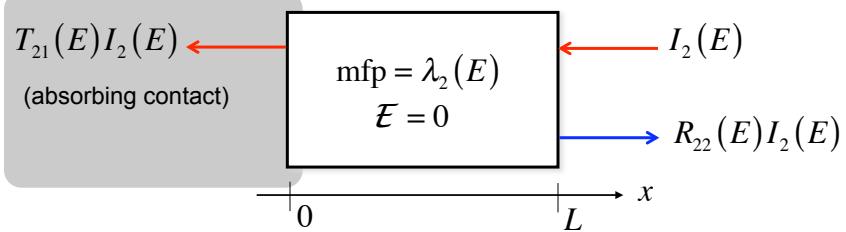
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Consider a flux of carriers injected from the left into a field-free slab of length,  $L$ . The flux that emerges at  $x = L$  is  $T$  times the incident flux, where  $0 < T < 1$ . The flux that emerges from  $x = 0$  is  $R$  times the incident flux, where  $T + R = 1$ , assuming no carrier recombination-generation.

How is  $T$  related to the mean-free-path for backscattering within the slab?

## injection from the right

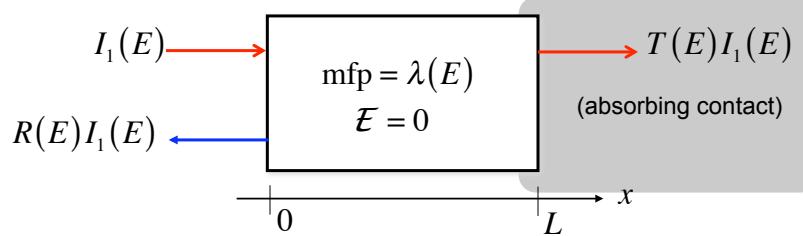


In general, there could be injection from both the left and the right contacts.

$$\text{For elastic scattering: } T_{12}(E) = T_{21}(E) = T(E)$$

$$\text{Near equilibrium: } T_{12}(E) \approx T_{21}(E) \approx T(E)$$

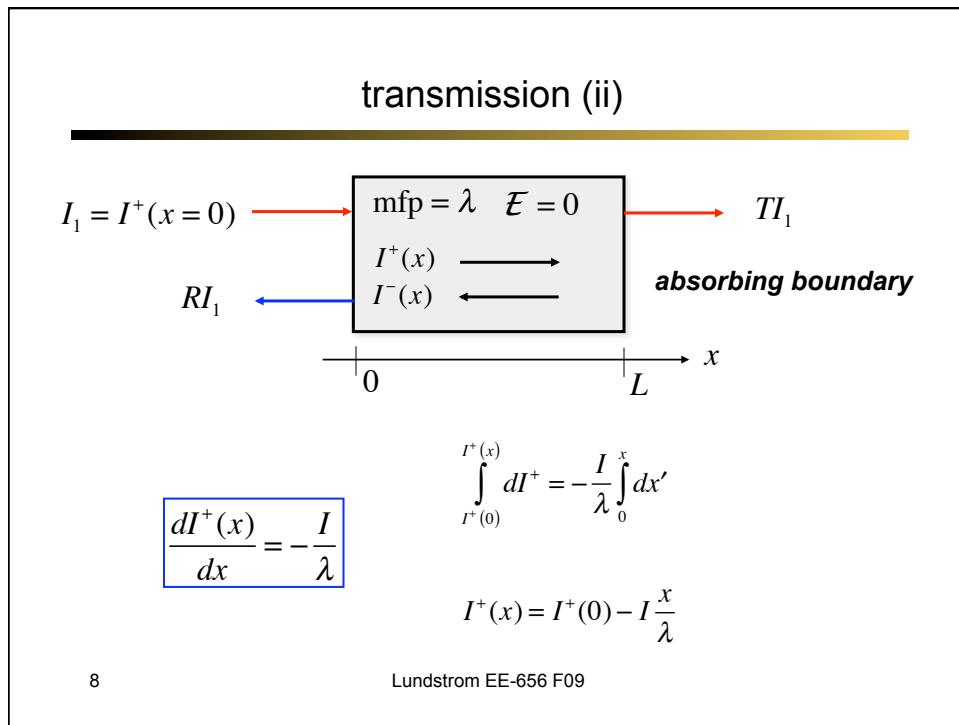
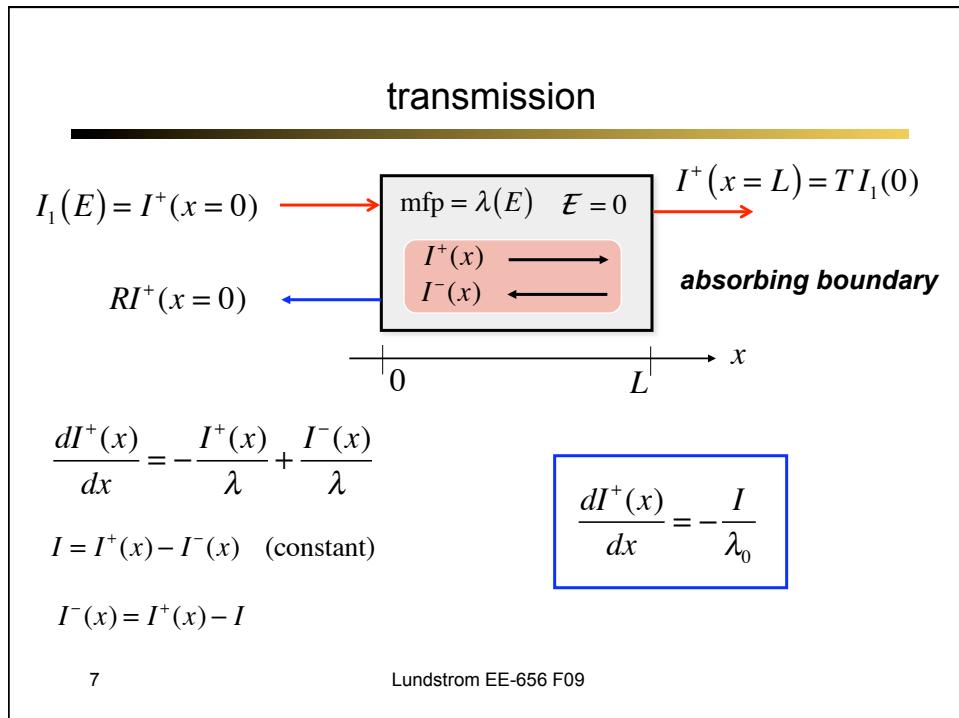
## problem specification



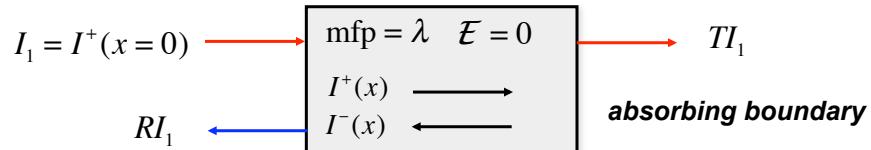
1) Inject from left only.

2) Ignore ‘vertical transport’ (elastic scattering or near-equilibrium), so  $T_{12}(E) = T_{21}(E) = T(E)$ .

Then relate  $T$  to the mean-free-path for backscattering within the slab.  
(No assumption about whether the slab length,  $L$ , is long or short compared to the mfp.)



### transmission (iii)



$$I^+(x) = I^+(0) - (I^+(x) - I^-(x)) \frac{x}{\lambda}$$

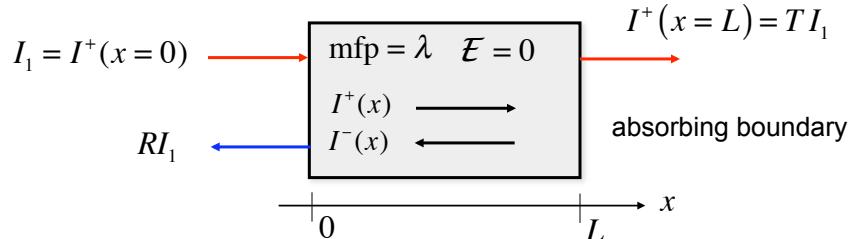
$$I^+(x) = I^+(0) - I \frac{x}{\lambda}$$

$$I^-(L) = 0$$

$$I^+(L) = I^+(0) - I^+(L) \frac{L}{\lambda}$$

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### transmission (iv)



$$I^+(L) = I^+(0) - I^+(L) \frac{L}{\lambda}$$

$$I^+(L) = \frac{I^+(0)}{1 + L/\lambda}$$

$$\frac{I^+(L)}{I^+(0)} = T = \frac{\lambda}{\lambda + L}$$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L} \quad T(E) + R(E) = 1$$

$$T \rightarrow 0 \quad L \gg \lambda$$

$$T \rightarrow 1 \quad L \ll \lambda$$

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## Lecture 17: mean-free-path

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$$\lambda(E) \equiv 2 \frac{\langle v_x^2 \tau_f \rangle}{\langle |v_x| \rangle}$$

$$\lambda(E) = 2v(E)\tau_f(E) \quad 1D$$

$$\lambda(E) = \frac{\pi}{2}v(E)\tau_f(E) \quad 3D$$

$$\lambda(E) = \frac{4}{3}v(E)\tau_f(E) \quad 3D$$

$$\langle |v_x| \rangle = \frac{\sum_k |v_x| \delta(E - E_k)}{\sum_k \delta(E - E_k)}$$

$$\langle v_x^2 \tau_f \rangle \equiv \frac{\sum_k v_x^2 \tau_f \delta(E - E_k)}{\sum_k \delta(E - E_k)}$$

This is an average over angle mfp for backscattering at a specific energy,  $E$ .

## Recall: Lecture 19: RTA

$$\hat{C}_f = -\frac{f_A(\vec{p})}{\tau_f} \quad \text{Can be rigorously justified near-equilibrium for elastic or isotropic scattering.}$$

$\tau_f = \tau_m$  Characteristic time for RTA is the momentum relaxation time.

$$\frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') [1 - (p'/p) \cos \alpha] \quad \text{The angle, } \alpha, \text{ is the polar angle between the incident and scattered electron (3D).}$$

$$\lambda(E) = 2 \frac{\langle v_x^2 \tau_m \rangle}{\langle |v_x| \rangle} \quad \text{Relates the mfp for backscattering to the microscopic transition rate when the RTA is valid. (Often produces reasonable results when RTA is not strictly justified.)}$$

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## energy-dependent mfp for backscattering

$$\lambda(E) = 2 \frac{\langle v_x^2 \tau_m \rangle}{\langle |v_x| \rangle}$$

$$\lambda(E) = 2v(E)\tau_m(E) \quad (1D)$$

$$\lambda(E) = (\pi/2)v(E)\tau_m(E) \quad (2D)$$

$$\lambda(E) = (4/3)v(E)\tau_m(E) \quad (3D)$$

$$\frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') [1 - (p'/p) \cos \alpha]$$

## average mfp for backscattering

$$G_{ch} = \left[ \frac{2q^2}{h} \int M'(E) \lambda(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \right] \frac{W}{L} \quad \text{Landauer in the diffusive limit}$$

$$M'(E) = g_v \frac{\sqrt{2m^*E}}{\pi\hbar} \quad \text{number of modes per unit width at energy, } E.$$

$$G_{ch} = \frac{2q^2}{h} \langle M' \rangle \langle \lambda \rangle \frac{W}{L}$$

$$\langle M' \rangle = \int M'(E) \left( -\frac{\partial f_0}{\partial E} \right) dE$$

$$\langle \lambda \rangle \approx \frac{\int M'(E) \lambda(E) \left( -\frac{\partial f_0}{\partial E} \right) dE}{\int M'(E) \left( -\frac{\partial f_0}{\partial E} \right) dE}$$

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## effective number of modes

$$\langle M' \rangle = \int M'(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \quad M'(E) = g_v \frac{\sqrt{2m^*E}}{\pi\hbar}$$

i) nondegenerate:  $\langle M' \rangle = \frac{n_s}{\sqrt{2D_{2D}k_B T}/g_v} \quad D_{2D} = g_v \frac{m^*}{\pi\hbar^2}$

ii)  $T = 0K$ :  $\langle M' \rangle = \sqrt{2g_v n_s / \pi}$

iii) General:  $\langle M' \rangle = g_v \frac{\sqrt{2\pi m^* k_B T}}{2\pi\hbar} \mathcal{F}_{-1/2}(\eta_F) \quad \eta_F = (E_F - E_C)/k_B T$

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## relating ave mfp to mobility

$$G_{ch} = \frac{2q^2}{h} \langle M' \rangle \langle \lambda \rangle \frac{W}{L} = n_s q \mu_n \frac{W}{L} \quad \langle \lambda \rangle = \frac{n_s \mu_n}{(2q/h) \langle M' \rangle}$$

i) nondegenerate:  $\langle \lambda \rangle = \frac{2(k_B T / q) \mu_{eff}}{v_T} \quad \left( D_n = \frac{v_T \langle \lambda \rangle}{2} \right)$

ii)  $T = 0K$ :  $\langle \lambda \rangle = \frac{k_F \mu_{eff}}{2(2q/h)}$   $k_F = \sqrt{\frac{2\pi n_s}{g_v}}$

iii) General:  $\langle \lambda \rangle = \frac{2(k_B T / q) \mu_{eff}}{v_T} \frac{\mathcal{F}_0(\eta_F)}{\mathcal{F}_{-1/2}(\eta_F)} \quad \eta_F = \ln(e^{n_s/N_{2D}} - 1)$

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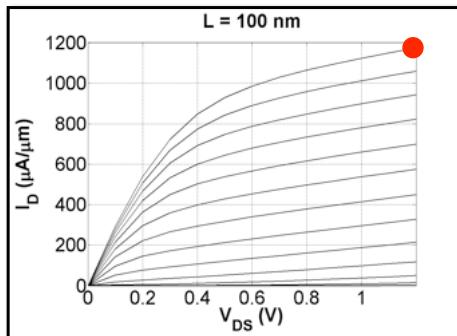
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## mfp in a Si N-MOSFET

2007 N-MOSFET

(unstrained Si technology)



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

What is the mfp at high gate voltage?

$$\mu_n(V_G = 1.2V) = 260 \text{ cm}^2/\text{V}\cdot\text{s}$$

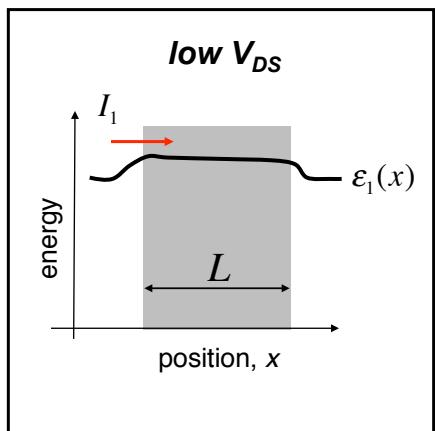
(measured for long channels)

$$n_s(V_G = 1.2V) = 7.9 \times 10^{12} \text{ cm}^{-2}$$

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C. Jeong, et al., *IEEE Trans. Electron Dev.*, 2009.

## example



$$\mu_n \approx 260 \text{ cm}^2/\text{V}\cdot\text{s}$$

$$\mu_n = \frac{v_T}{2(k_B T/q)} \lambda_0$$

$$\lambda_0 \approx 12 \text{ nm}$$

$$\langle \lambda \rangle = \frac{2(k_B T/q) \mu_{eff}}{v_T} \frac{\mathcal{F}_0(\eta_F)}{\mathcal{F}_{-1/2}(\eta_F)}$$

$$\lambda_0 \approx 14 \text{ nm}$$

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## questions

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