

ECE-656: Fall 2009

Lecture 33: Non-Local Transport

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outline

- 1) Review of high-field transport**
- 2) MC simulation of high-field transport
- 3) Velocity overshoot
- 4) Summary

balance equations

1) Momentum balance:

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx}$$

$$\mu_n = q\langle\tau_m\rangle/m^* \quad D_n/\mu_n = 2u_{xx}/q$$

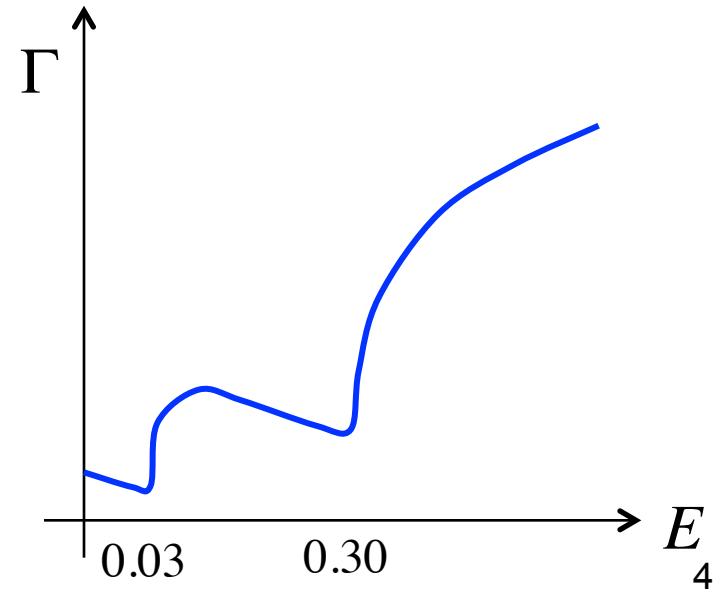
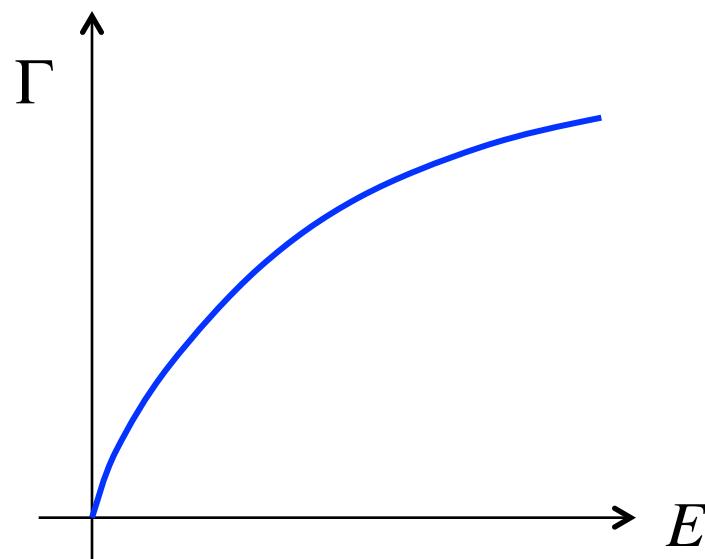
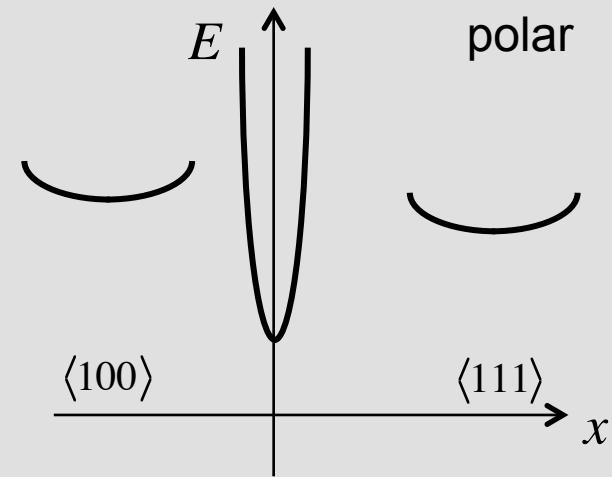
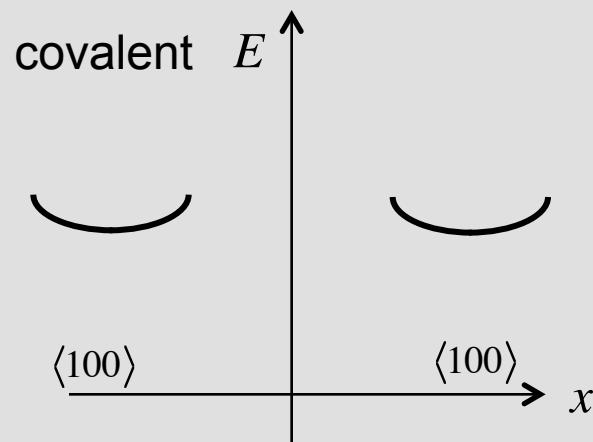
As the average energy increases, u_{xx} increases and $\langle\tau_m\rangle$ decreases

2) Energy balance: $\partial W(x,t)/\partial t = -dF_W/dx + J_{nx}\mathcal{E}_x - (W - W_0)/\langle\tau_E\rangle$

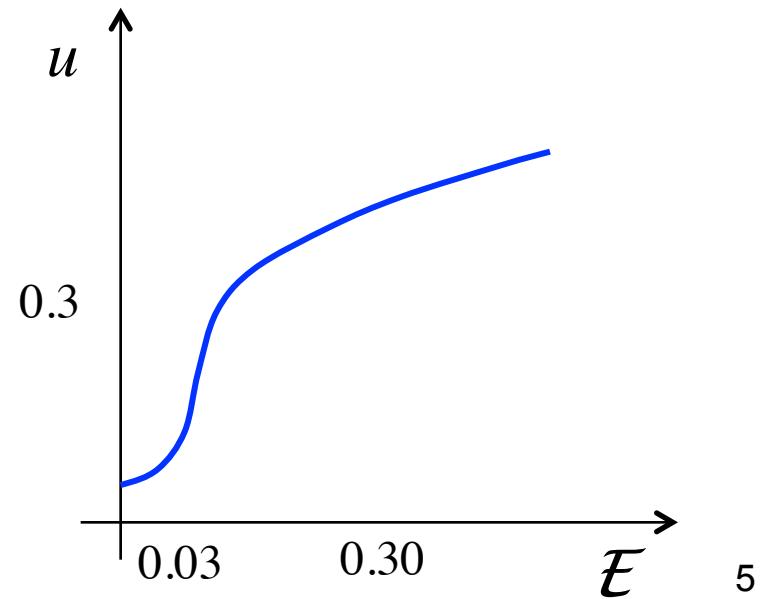
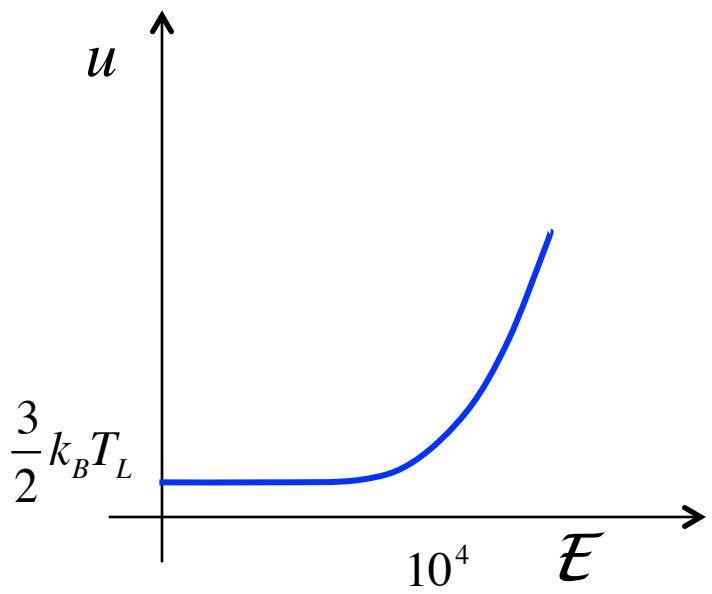
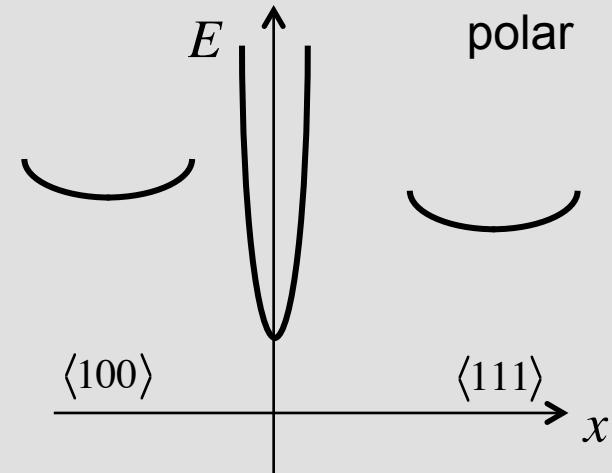
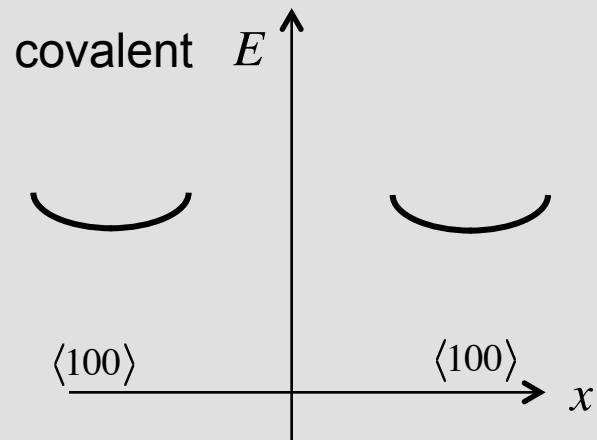
Steady-state, uniform:

$$J_{nx}\mathcal{E}_x = (W - W_0)/\langle\tau_E\rangle$$

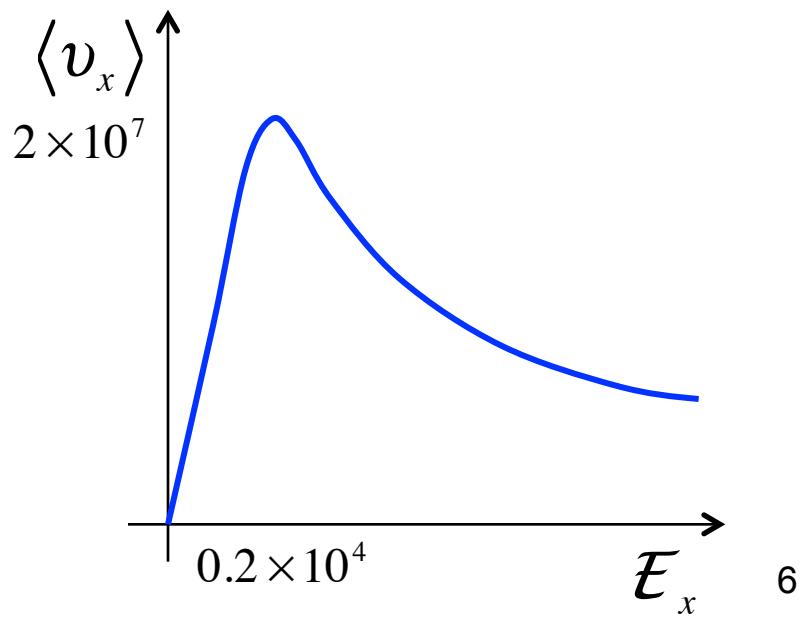
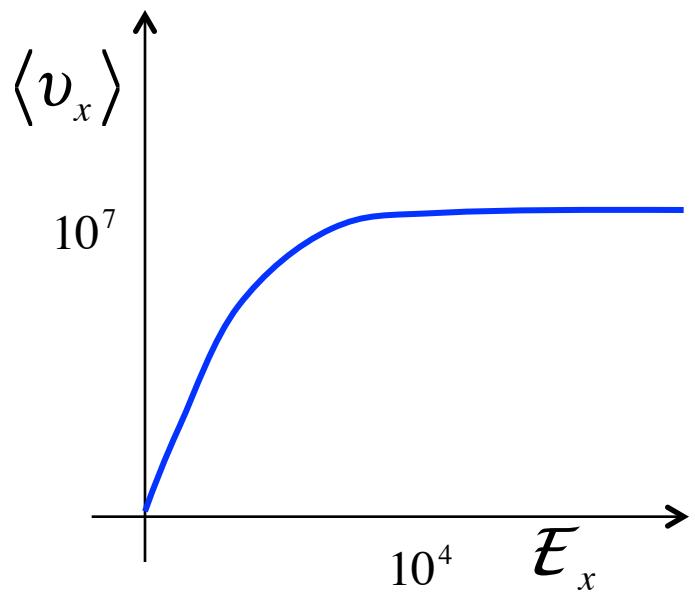
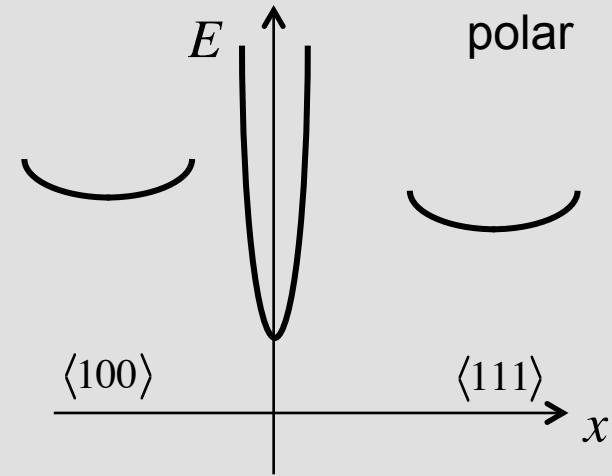
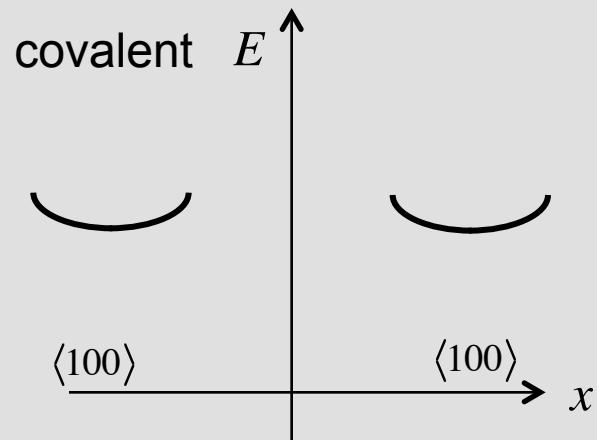
covalent vs. polar semiconductors



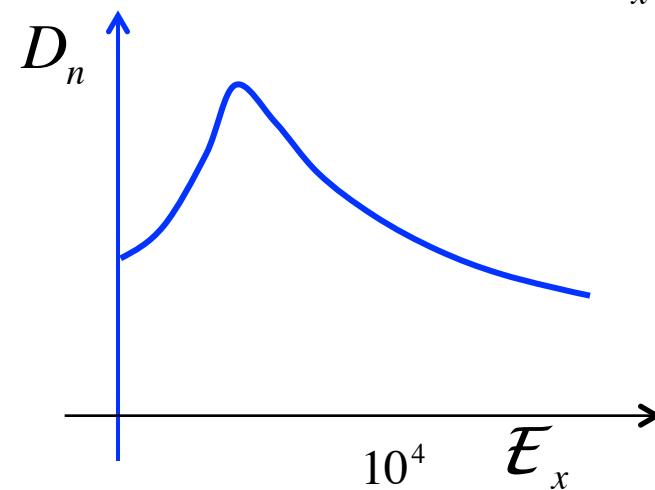
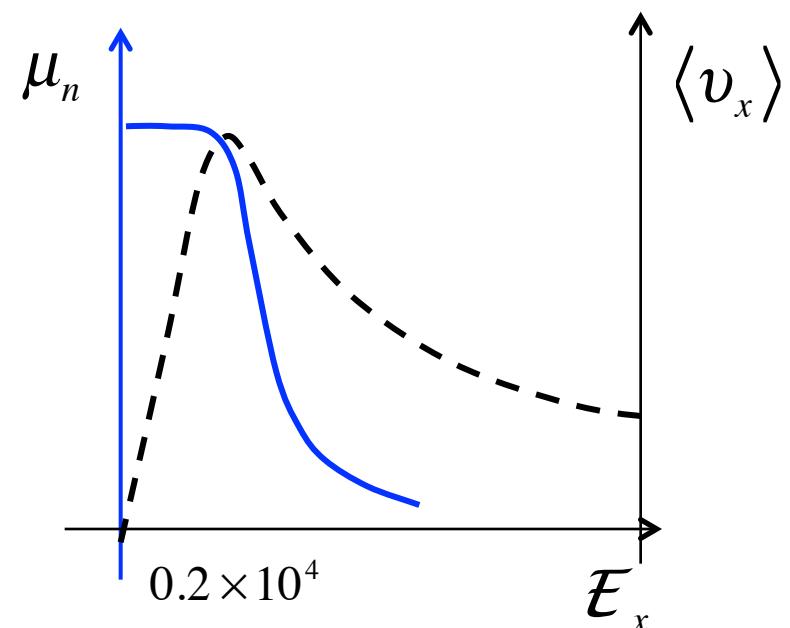
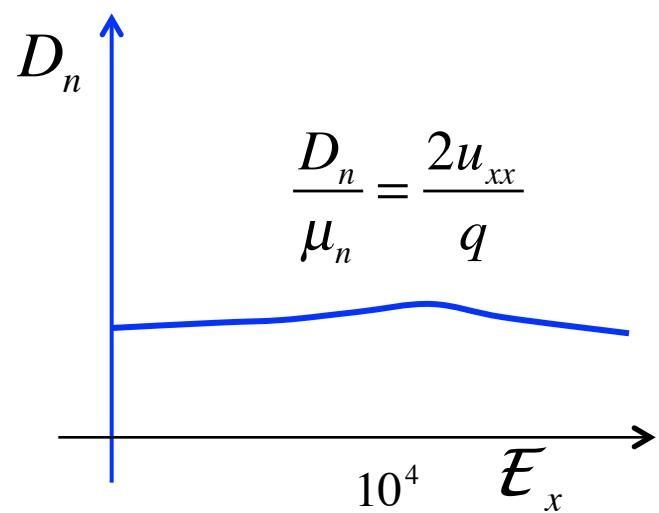
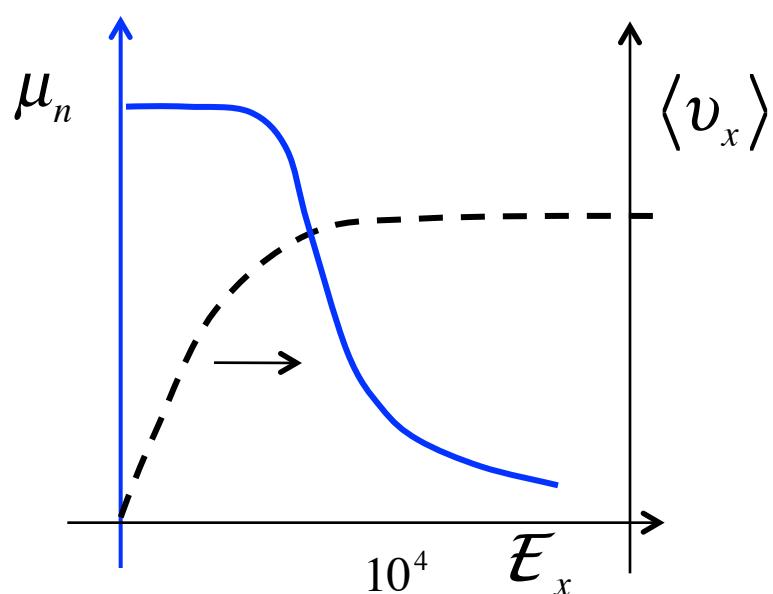
average energy vs. electric field



average velocity vs. electric field



mobility and diffusion coefficient



outline

- 1) Review of high-field transport
- 2) **MC simulation of high-field transport**
- 3) Velocity overshoot
- 4) Summary

<111> Silicon: low-field

$$\mathcal{E}_z = -100 \text{ V/cm}$$

$$\langle v_z \rangle = 8.1 \times 10^4 \text{ cm/s}$$

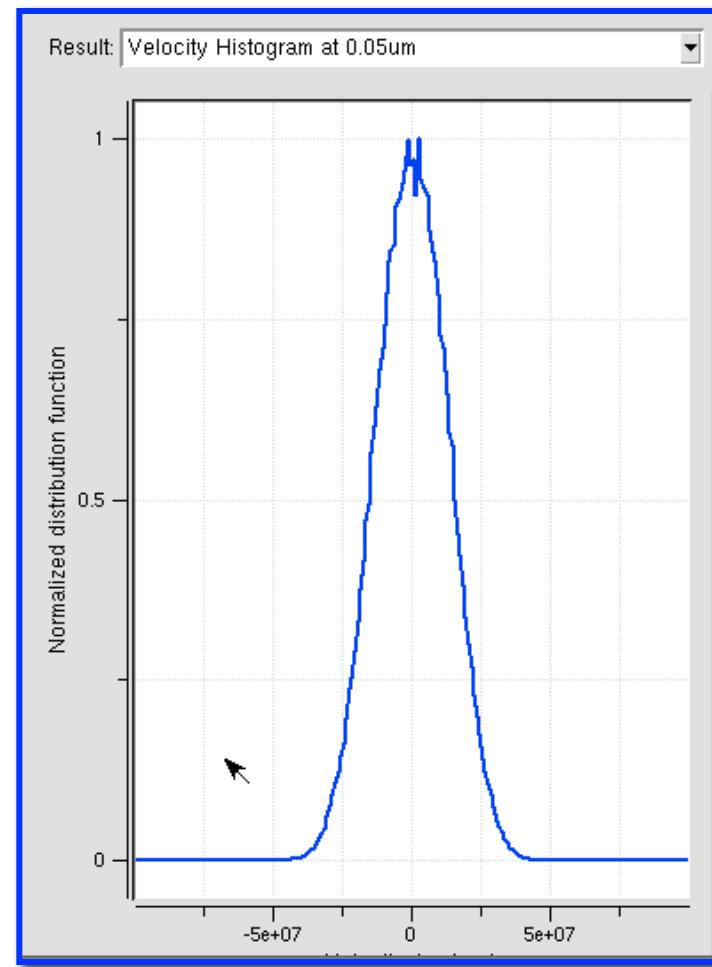
$$\mu_n(\mathcal{E}_z) = 810 \text{ cm}^2/\text{V-s}$$

$$u = 0.04 \text{ eV} \quad (1.5k_B T / q = 0.39 \text{ eV})$$

$$u_{zz} = 0.0135 \text{ eV} \quad (u_{zz} / u = 0.34)$$

$$u_{drift} \sim 10^{-7} \text{ eV} \quad (u_{drift} / u \sim 10^{-5})$$

$$n(x,y,z)/n = 0.33 / 0.335 / 0.335$$



(simulations performed with DEMONs on www.nanoHUB.org)

<111> Silicon: high-field

$$\mathcal{E}_z = -10^5 \text{ V/cm}$$

$$\langle v_z \rangle = 1.04 \times 10^7 \text{ cm/s}$$

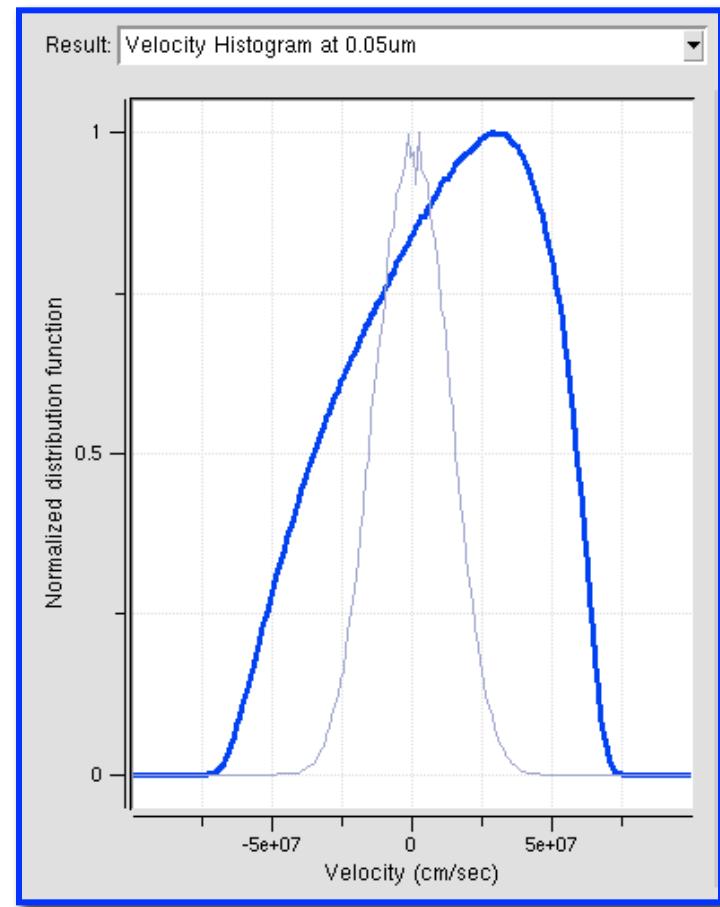
$$\mu_n(\mathcal{E}_z) = 104 \text{ cm}^2/\text{V-s}$$

$$u = 0.364 \text{ eV} \quad (1.5k_B T / q = 0.039 \text{ eV})$$

$$u_{zz} = 0.145 \text{ eV} \quad (u_{zz} / u = 0.40)$$

$$u_{drift} = 0.008 \text{ eV} \quad (u_{drift} / u = 0.02)$$

$$n(x,y,z)/n = 0.336 / 0.331 / 0.333$$



(simulations performed with DEMONs on www.nanoHUB.org)

momentum relaxation time

$$\mathcal{E}_z = -10^5 \text{ V/cm} \quad \langle v_z \rangle = 1.04 \times 10^7 \text{ cm/s} \quad \mu_n(\mathcal{E}_z) = 104 \text{ cm}^2/\text{V-s}$$

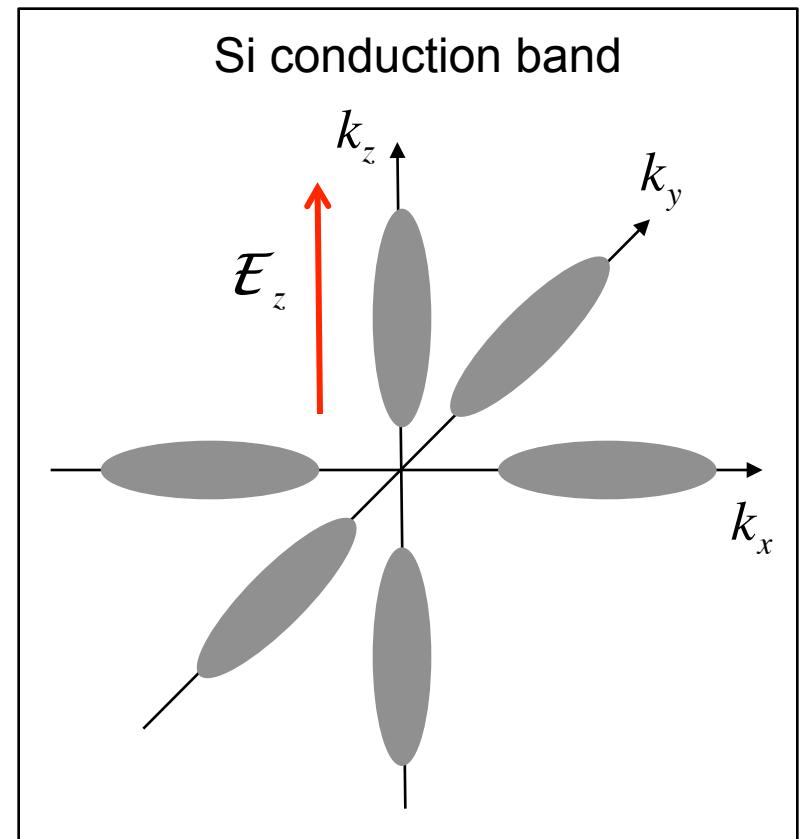
$$\mu_n = \frac{q\langle\tau_m\rangle}{m^*} \quad m^* = ?$$

$$\frac{1}{m_c^*} = \frac{1}{3m_\ell^*} + \frac{2}{3m_t^*} \rightarrow m_c^* = 0.26m_0$$

(ignores conduction band non-parabolicity)

$$\langle\tau_m\rangle = 0.02 \text{ ps} \quad (\mathcal{E}_z = 10^5 \text{ V/cm})$$

$$\langle\tau_m\rangle = 0.12 \text{ ps} \quad (\mathcal{E}_z = 10^2 \text{ V/cm})$$



energy relaxation time

$$\mathcal{E}_z = -10^5 \text{ V/cm} \quad u = 0.364 \text{ eV} \quad u_0 = 0.039 \text{ eV} \quad \langle v_z \rangle = 1.04 \times 10^7 \text{ cm/s}$$

$$J_{nx} \mathcal{E}_x = (W - W_0) / \langle \tau_E \rangle \rightarrow nq \langle v_z \rangle \mathcal{E}_x = n(u - u_0) / \langle \tau_E \rangle$$

$$\langle \tau_E \rangle = \frac{(u - u_0)/q}{\langle v_z \rangle \mathcal{E}_x} = 0.3 \text{ ps} \quad (\langle \tau_m \rangle = 0.02 \text{ ps})$$

$$\langle \tau_E \rangle \gg \langle \tau_m \rangle$$

electron temperature

$$\mathcal{E}_z = -10^5 \text{ V/cm}$$

$$u = 0.364 \text{ eV}$$

$$u_{zz} = 0.145 \text{ eV}$$

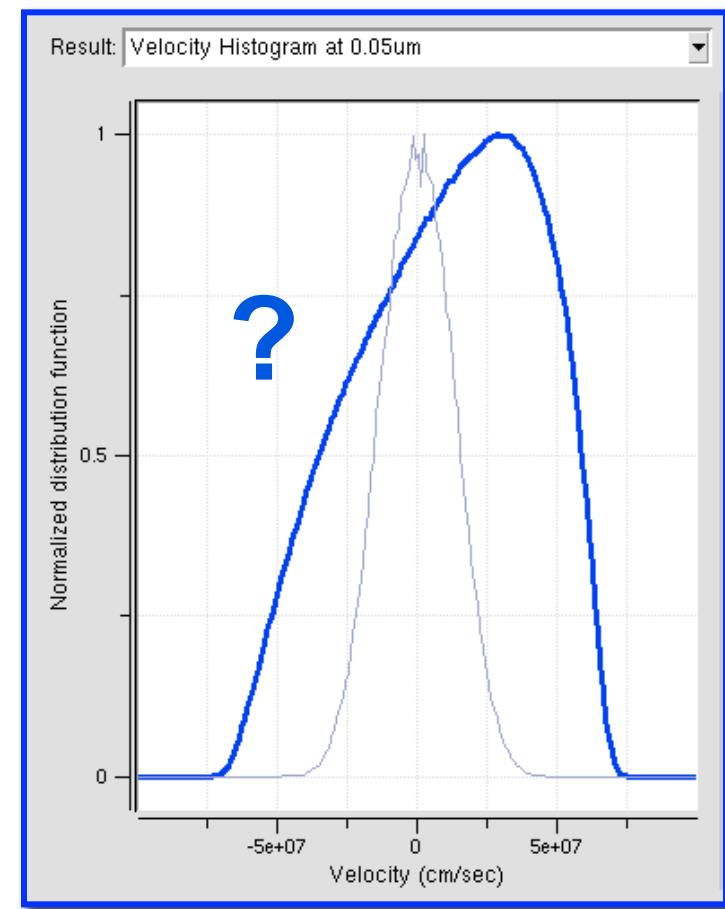
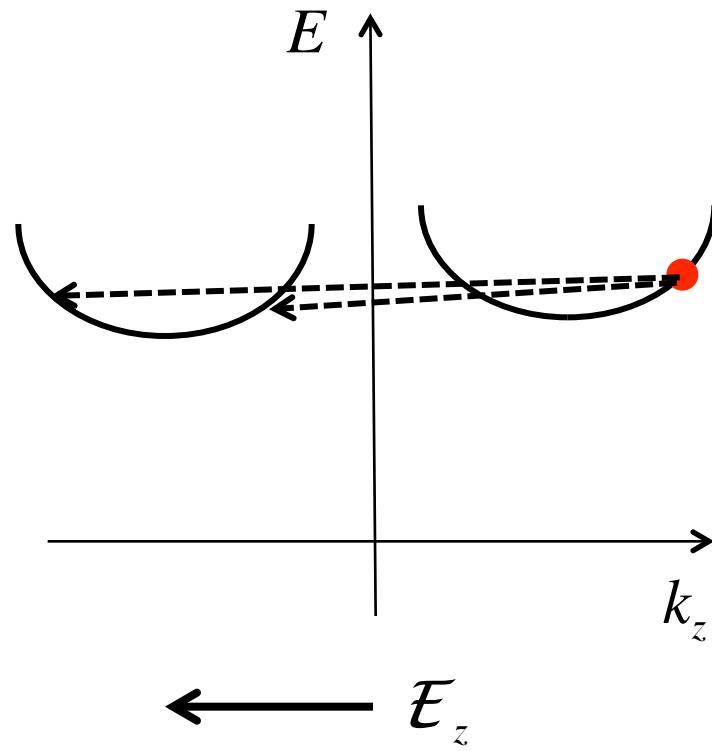
$$u = \frac{1}{2} m^* v_d^2 + \frac{3}{2} k_B T_e \approx \frac{3}{2} k_B T_e \rightarrow T_e \approx 2800 \text{ K}$$

$$\vec{T}_e = \begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix} = \frac{1}{2} m^* \begin{bmatrix} \langle c_x^2 \rangle & \langle c_x c_y \rangle & \langle c_x c_z \rangle \\ \langle c_y c_x \rangle & \langle c_y^2 \rangle & \langle c_y c_z \rangle \\ \langle c_z c_x \rangle & \langle c_z c_y \rangle & \langle c_z^2 \rangle \end{bmatrix} \approx \begin{bmatrix} T_{xx} & 0 & 0 \\ 0 & T_{yy} & 0 \\ 0 & 0 & T_{zz} \end{bmatrix}$$

$$\frac{1}{2} k_B T_{zz} = u_{zz} \rightarrow T_{zz} \approx 3400 \text{ K}$$

$$\frac{1}{2} k_B T_{xx} = \frac{1}{2} k_B T_{yy} = \frac{u - u_{zz}}{2} \rightarrow T_{xx} = T_{yy} \approx 2540 \text{ K}$$

<111> Silicon: high-field



(simulations performed with DEMONs on www.nanoHUB.org)

<100> Silicon: high-field

$$\mathcal{E}_z = -10^5 \text{ V/cm}$$

$$\langle v_z \rangle = 0.98 \times 10^7 \text{ cm/s}$$

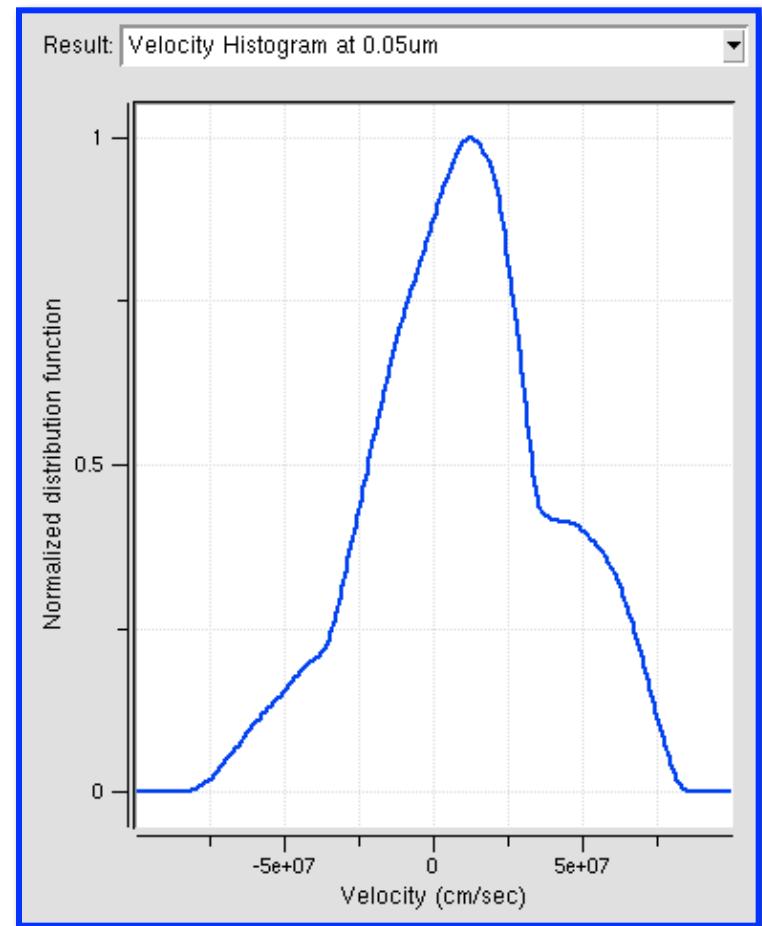
$$\mu_n(\mathcal{E}_z) = 98 \text{ cm}^2/\text{V-s}$$

$$u = 0.346 \text{ eV} \quad (1.5k_B T / q = 0.039 \text{ eV})$$

$$u_{zz} = 0.138 \text{ eV} \quad (u_{zz} / u = 0.40)$$

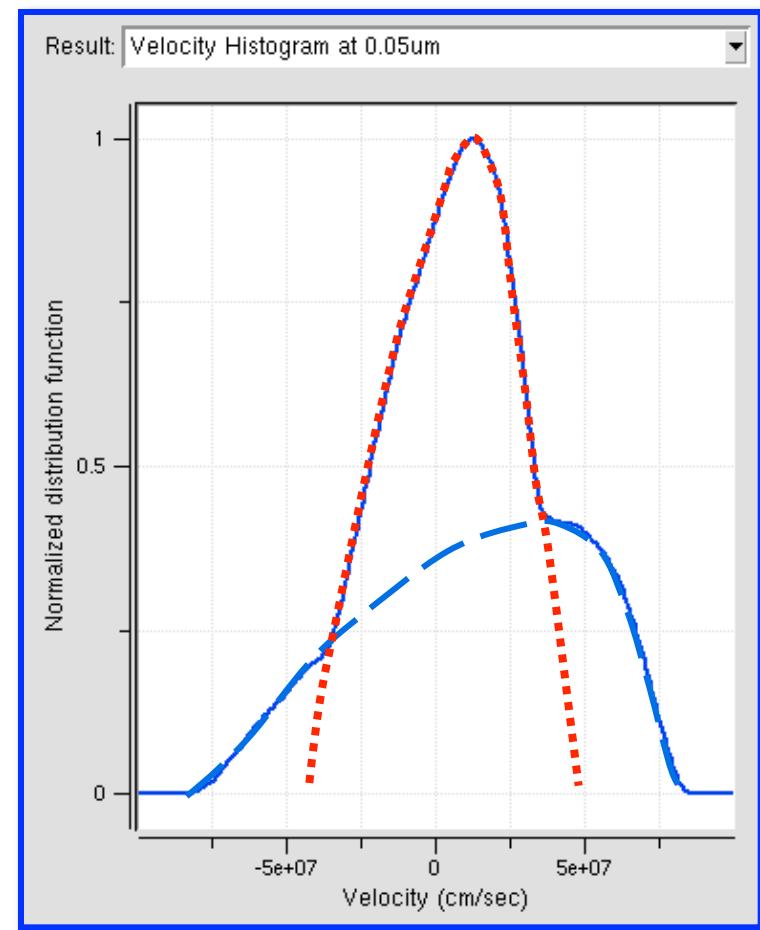
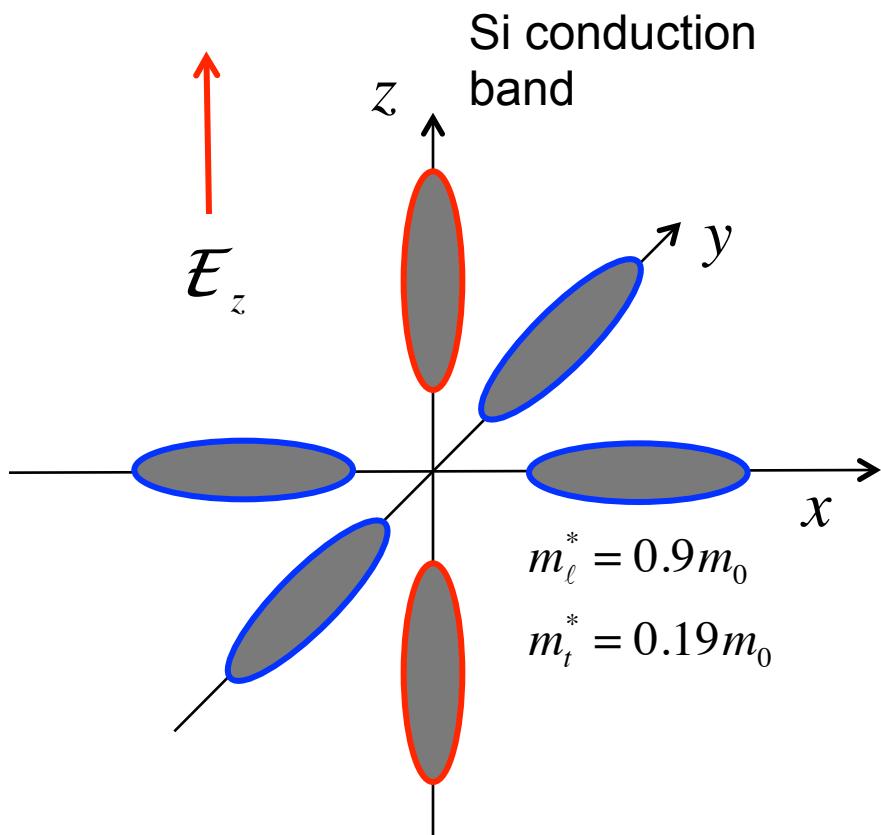
$$u_{drift} = 0.007 \text{ eV} \quad (u_{drift} / u = 0.02)$$

$$n(x,y,z)/n = 0.306 / 0.309 / 0.385$$



(simulations performed with DEMONs on www.nanoHUB.org)

<100> Silicon: high-field



(simulations performed with DEMONs on www.nanoHUB.org)

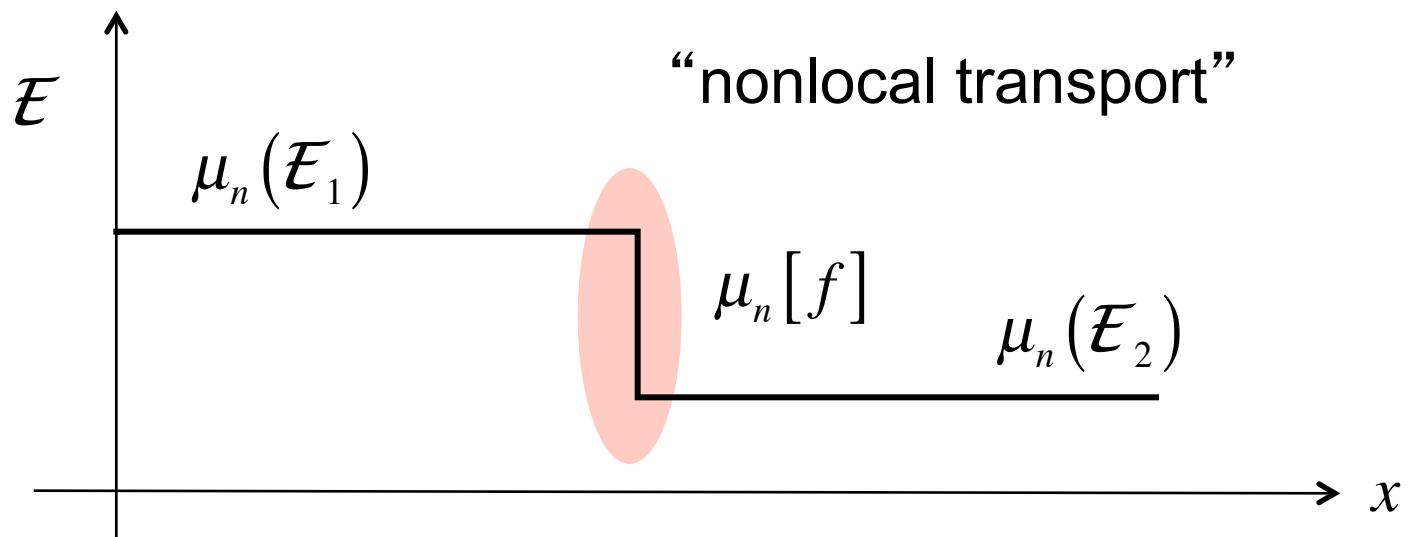
summary

- 1) High-field transport leads to field-dependent mobilities and diffusion coefficients (when the field varies slowly in space and time).
- 2) The electron temperature approach provides a qualitative (and sometimes quantitative) way to view high-field (**hot carrier**) transport.
- 3) In practice, the carrier distribution can be highly non-Maxwellian.

outline

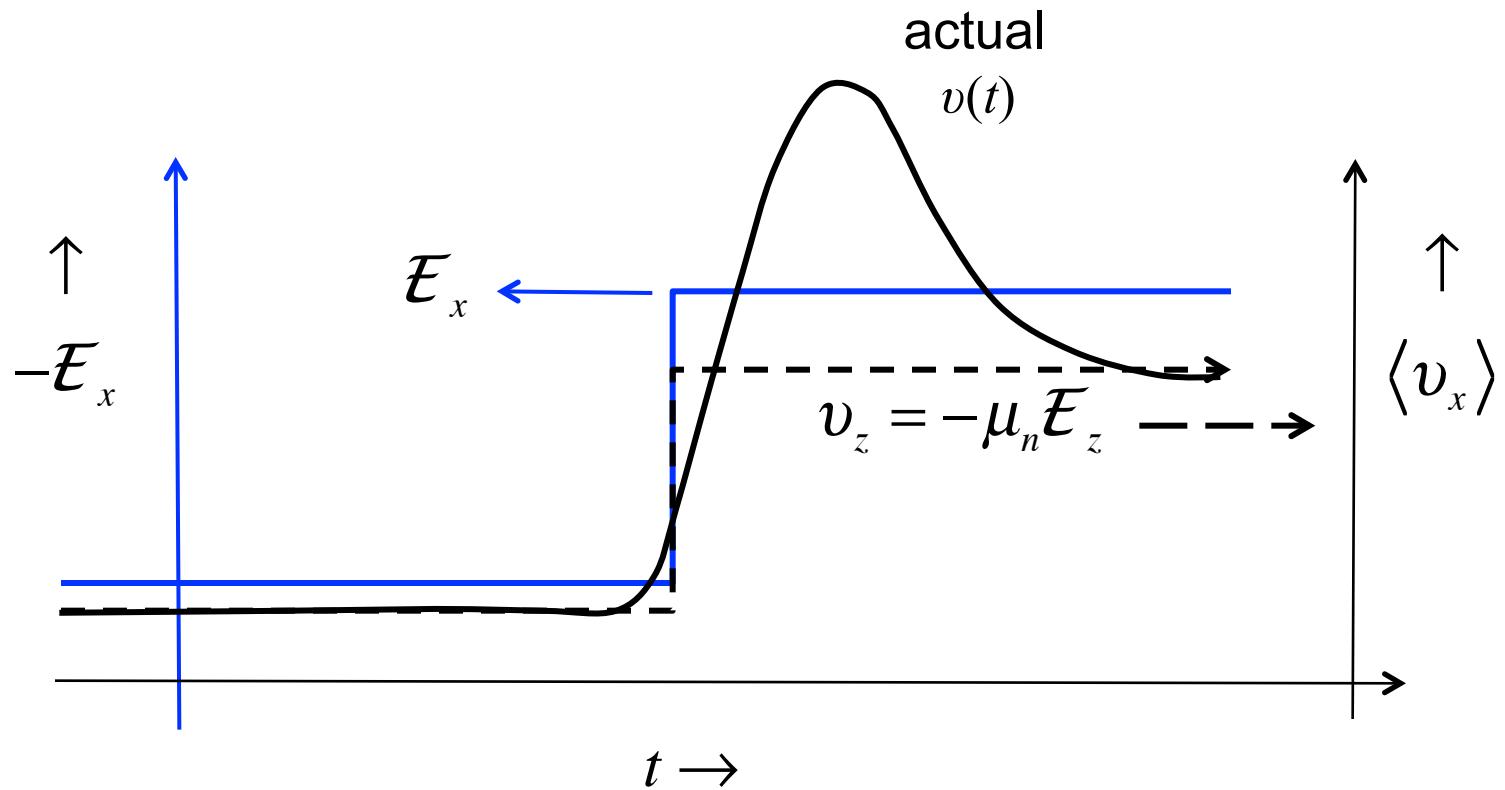
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off-equilibrium transport



The concept of a field-dependent mobility applies only when the electric field changes slowly with position.

rapidly varying electric fields



velocity overshoot

$p_{dx} = \langle p_x \rangle$ Let's find an equation for the ave. x -directed momentum.

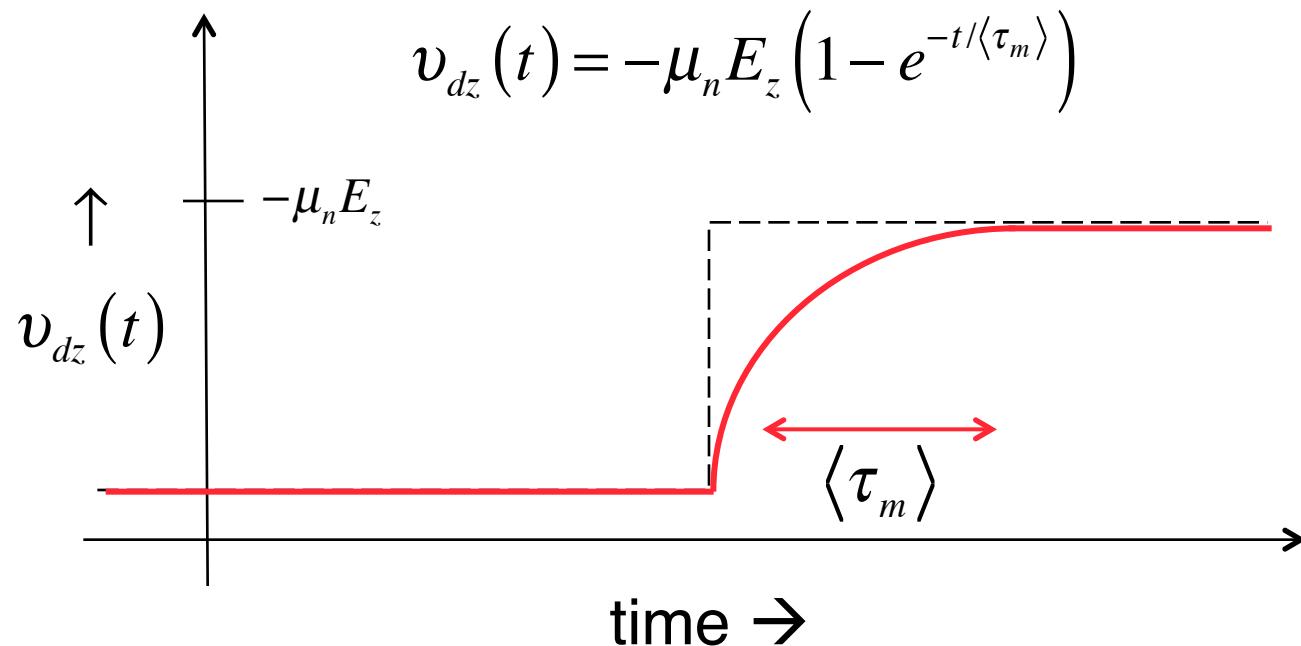
$$\frac{dp_{dx}}{dt} = -q\mathcal{E}_x - \frac{p_{dx}}{\langle \tau_m \rangle} \quad (\text{Ignores diffusion})$$

$$v_{dx} = -\mu_n \mathcal{E}_x \quad (\text{steady state solution})$$

$$v_{dx} = A e^{-t/\langle \tau_m \rangle} \quad (\text{homogeneous solution})$$

$$v_{dx}(t) = -\mu_n \mathcal{E}_x \left(1 - e^{-t/\langle \tau_m \rangle} \right)$$

velocity overshoot



But, μ_n is not constant $\mu_n(T_e)$

velocity overshoot

$$u = \left\langle \frac{1}{2} m^* v^2 \right\rangle \quad \frac{du}{dt} = -qv_{dx} \mathcal{E}_x - \frac{(u - u_0)}{\langle \tau_E \rangle}$$

1) steady state solution:

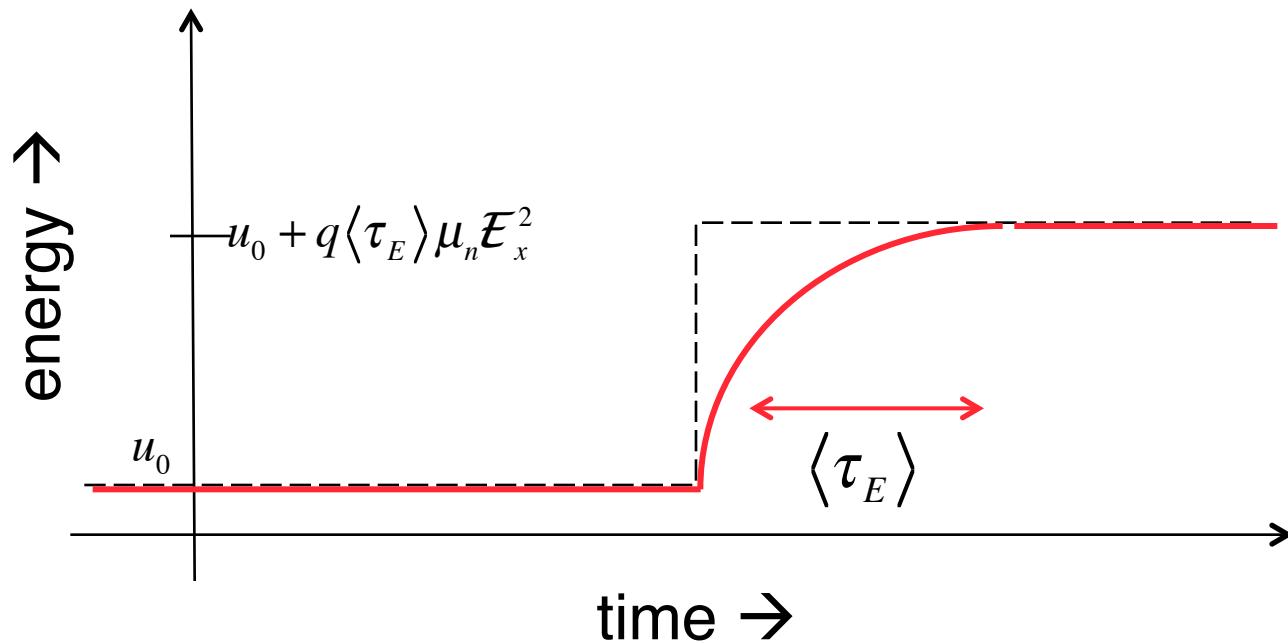
$$u_{ss} = u_0 - q \langle \tau_E \rangle v_{dx} \mathcal{E}_x = u_0 + q \langle \tau_E \rangle \mu_n \mathcal{E}_x^2$$

2) homogeneous solution:

$$u_H = C + A e^{-t/\langle \tau_E \rangle}$$

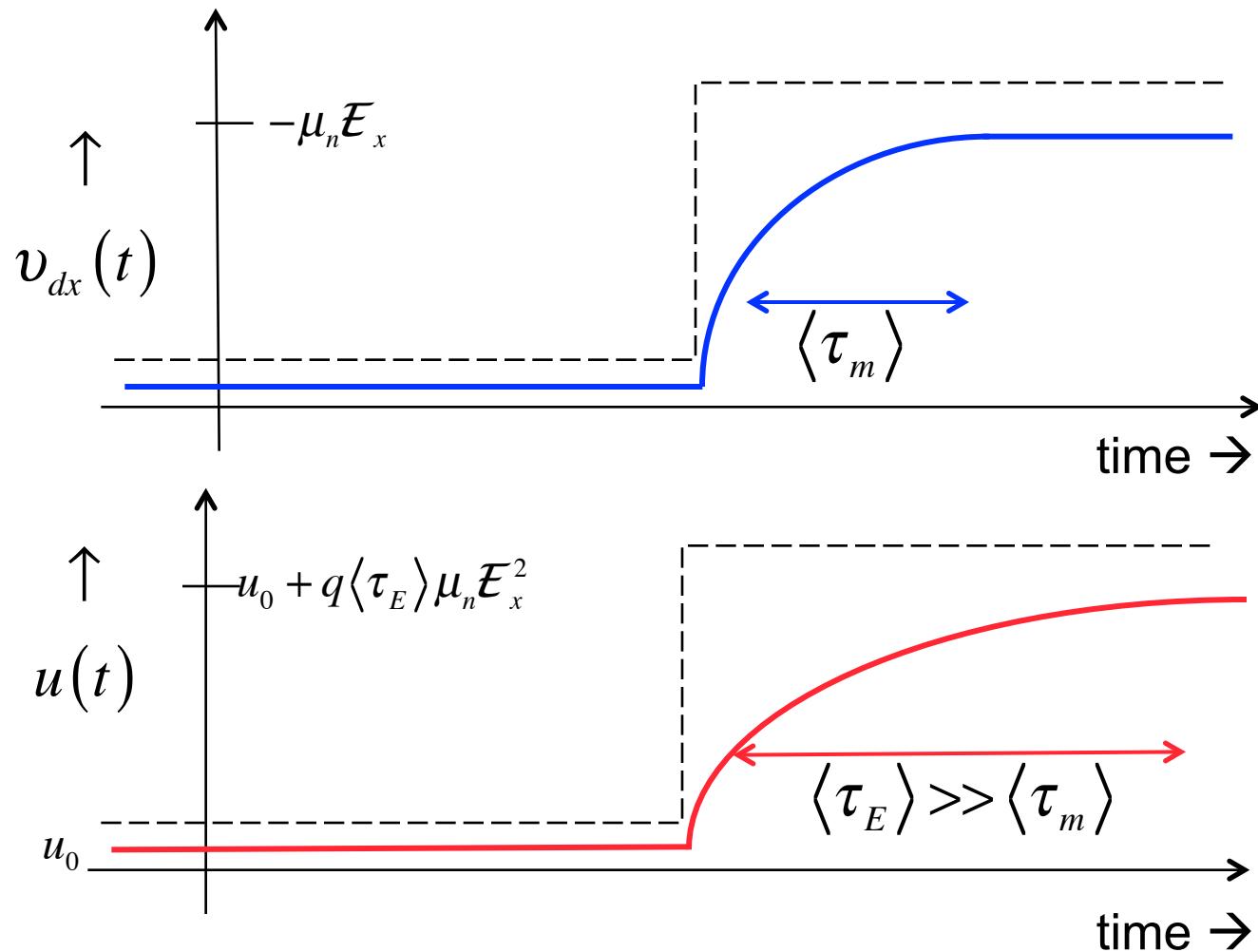
$$u(t) = u_0 + q \langle \tau_E \rangle \mu_n \mathcal{E}_x^2 \left(1 - e^{-t/\langle \tau_E \rangle} \right)$$

velocity overshoot

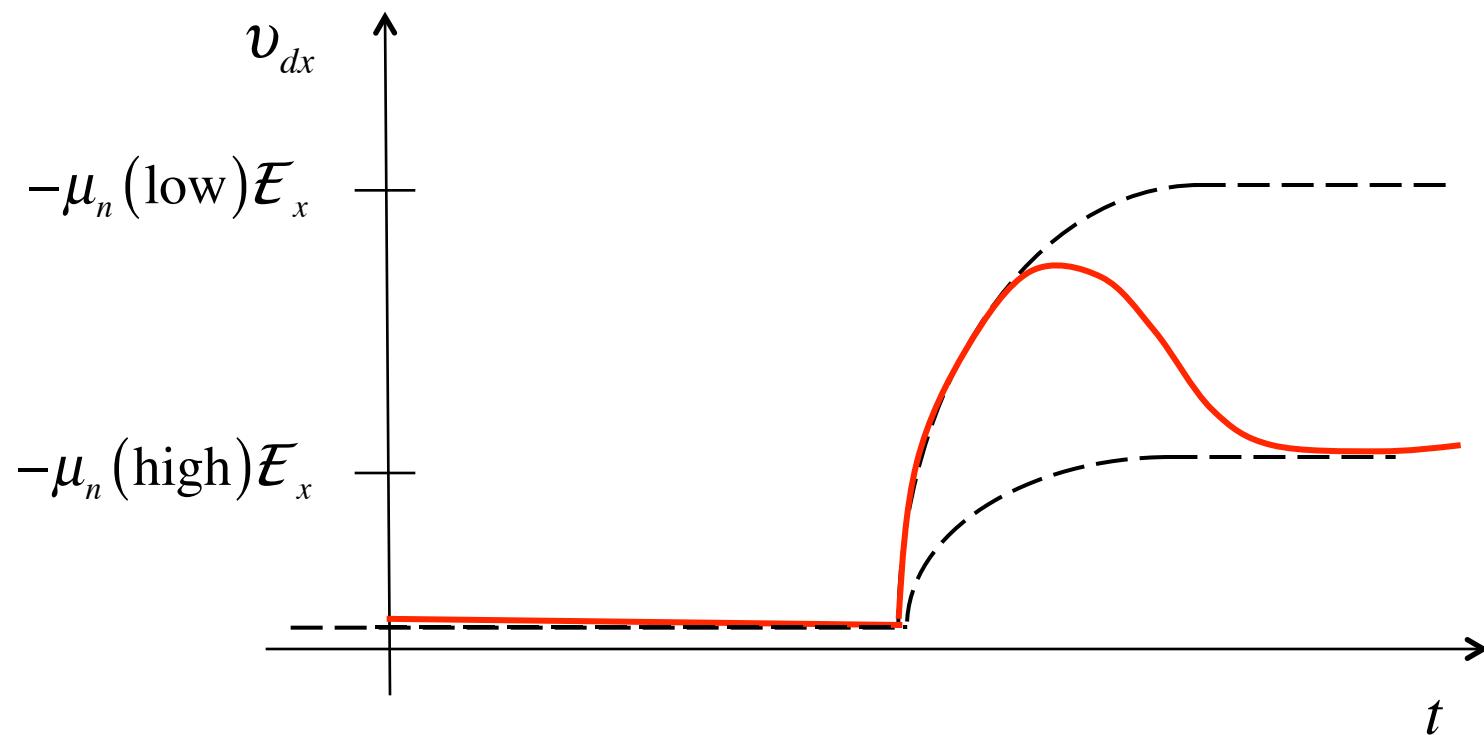


But, $\langle\tau_m\rangle$ and $\langle\tau_E\rangle$ are time dependent!

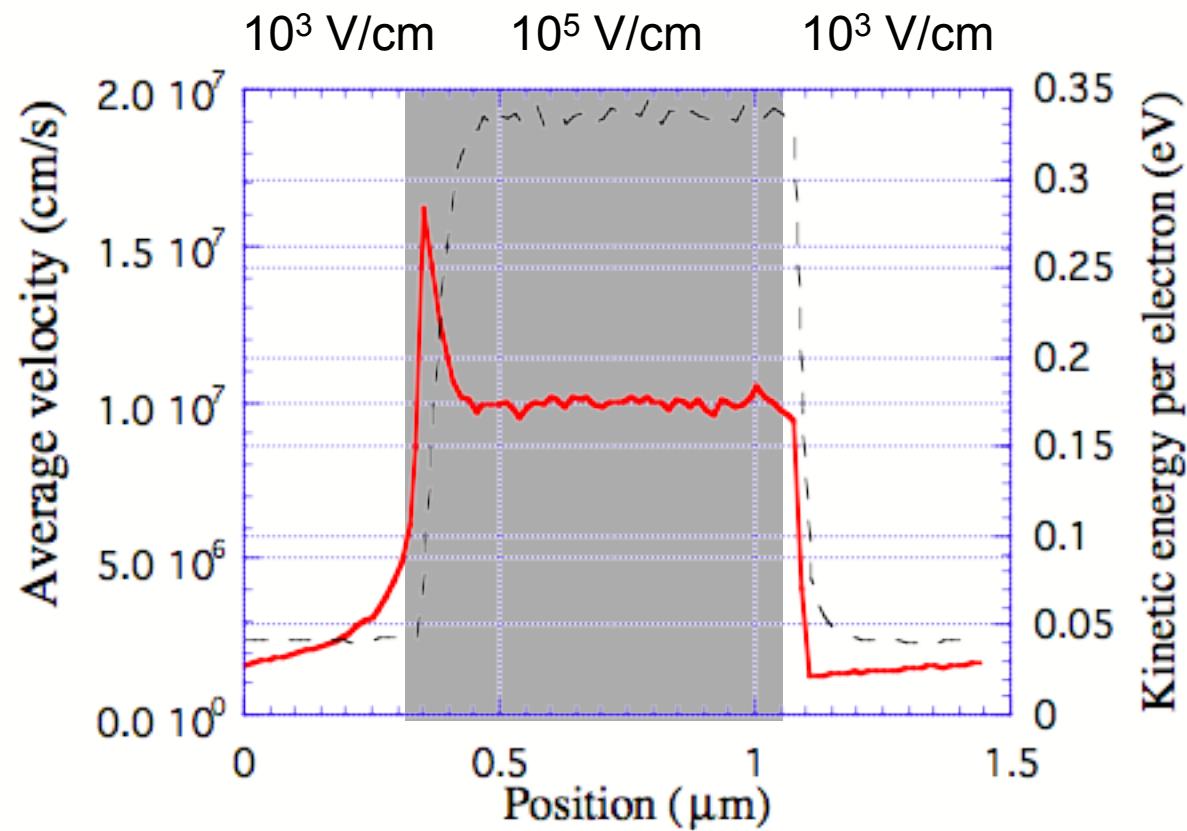
velocity overshoot



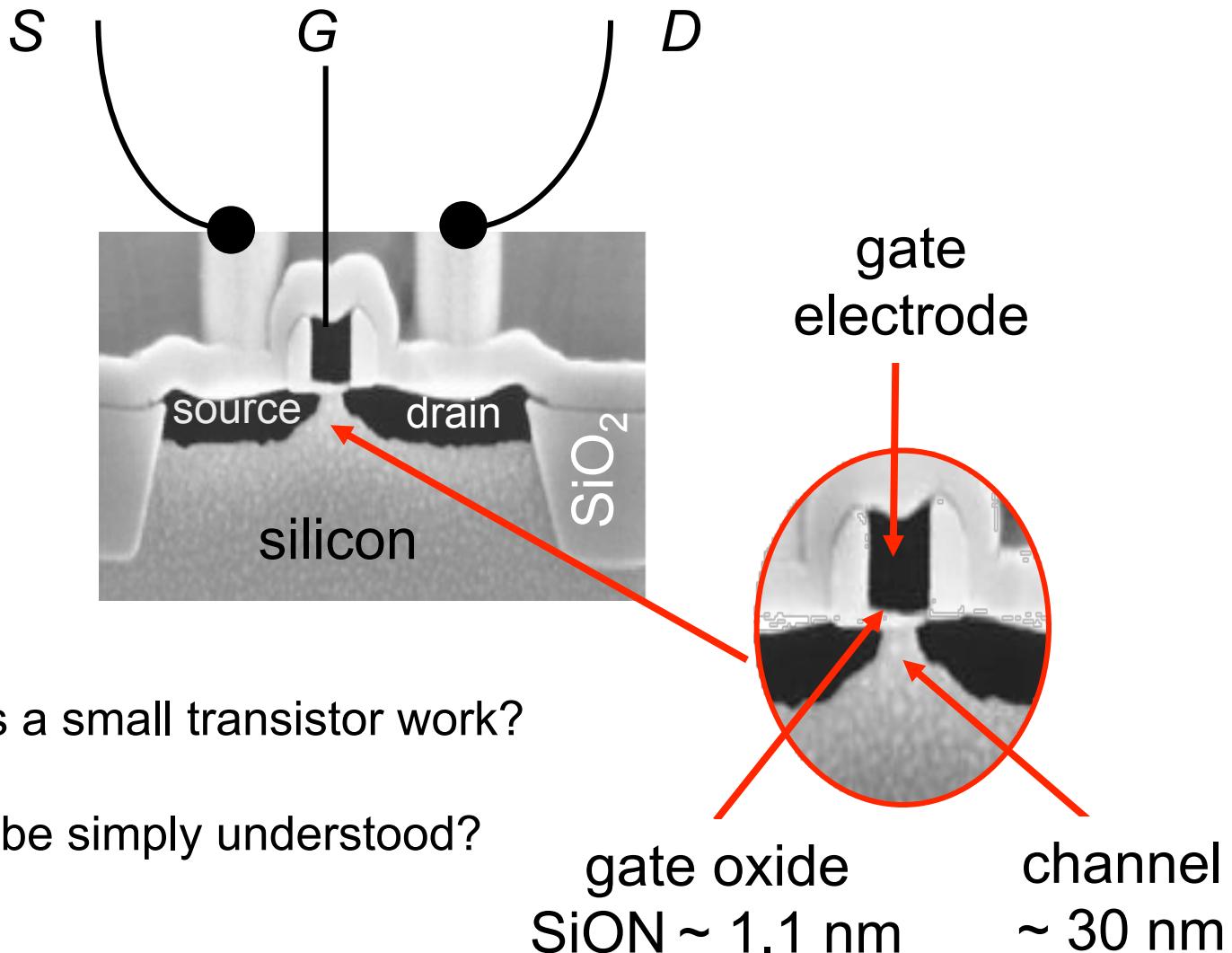
velocity overshoot



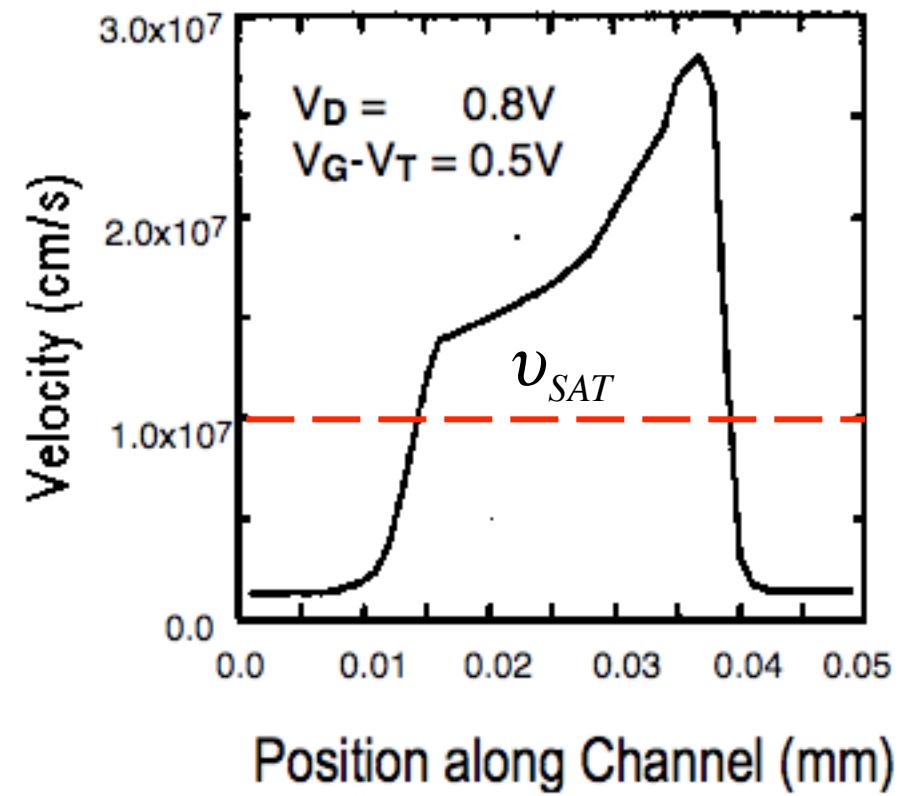
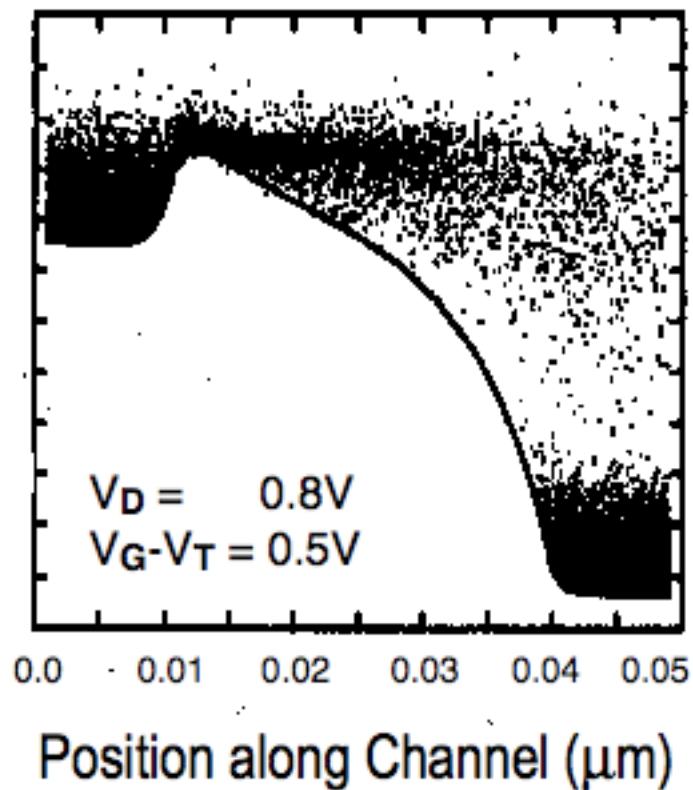
velocity overshoot in silicon



nanoscale MOSFETs 2009



off-equilibrium nanoscale MOSFETs



Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

outline

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summary

- 1) High-field transport leads to field-dependent mobilities and diffusion coefficients (when the field varies slowly in space and time)
- 2) The electron temperature approach provides a qualitative (and sometimes quantitative) way to view high-field (**hot carrier**) transport.
- 3) Rapidly varying electric fields lead to “off-equilibrium”, “non-local” or “non-stationary” transport effects such as velocity overshoot.
- 4) These effects can be described qualitatively with balance equations and quantitatively by Monte Carlo simulation.

questions

- 1) Review of high-field transport
- 2) MC simulation of high-field transport
- 3) Velocity overshoot
- 4) Summary

