

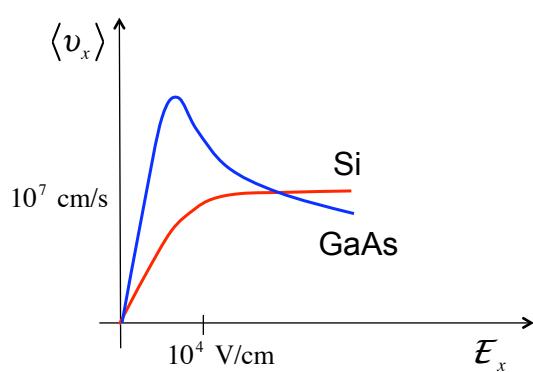
**ECE-656: Fall 2009**

## **Lecture 32: High-Field Transport**

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### velocity vs. field characteristics



## outline

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- 1) Brief Introduction
- 2) Current Equation**
- 3) Qualitative features of high field transport
- 4) Saturated velocity
- 5) Electron temperature model
- 6) Survey of results
- 7) Quick Summary

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## current equation

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$$J_{nx} = nq\mu_n \mathcal{E}_x + 2\mu_n \frac{d(nu_{xx})}{dx}$$

This is an “exact” steady-state current equation, but....

$$\mu_n [f(\vec{r}, \vec{p}, t)] \quad u_{xx} [f(\vec{r}, \vec{p}, t)] = \left\langle \frac{1}{2} p_z v_z \right\rangle$$

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## current equation: bulk semiconductor

bulk semiconductor:  $\mathcal{E}_x, u_{xx} \approx \text{constant}$

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{2u_{xx}}{q}$$

$$J_{nx} = nq\mu_n \mathcal{E}_x + 2\mu_n \frac{d(nu_{xx})}{dx}$$

$$\text{near equilibrium: } u_{xx} \approx \frac{k_B T_L}{q}$$

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## field-dependent mobility

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx}$$

Goal: Find mobility and diffusion coefficient without solving BTE

In general, however:  $\mu_n[f(\vec{r}, \vec{p}, t)] \quad D_n[f(\vec{r}, \vec{p}, t)]$

In a bulk semiconductor,  $f$  is determined by  $\mathcal{E}$ , so there is a one-to-one mapping between  $\mathcal{E}$  and  $f$ .

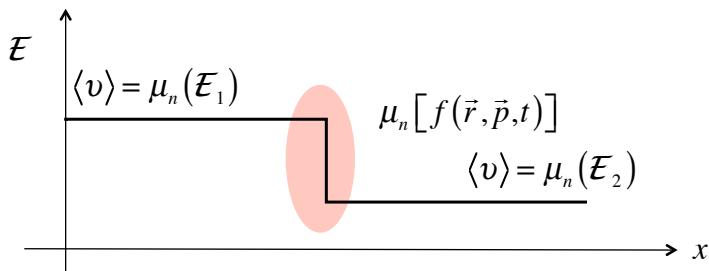
$\mu_n(\mathcal{E}) \quad D_n(\mathcal{E})$  Electric **field dependent** mobility and diffusion coefficient.

$$J_{nx} = nq\mu_n(\mathcal{E}) \mathcal{E}_x + qD_n(\mathcal{E}) \frac{dn}{dx}$$

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## field-dependent mobility



The concept of a field-dependent mobility applies only when the the electric field changes slowly with position.

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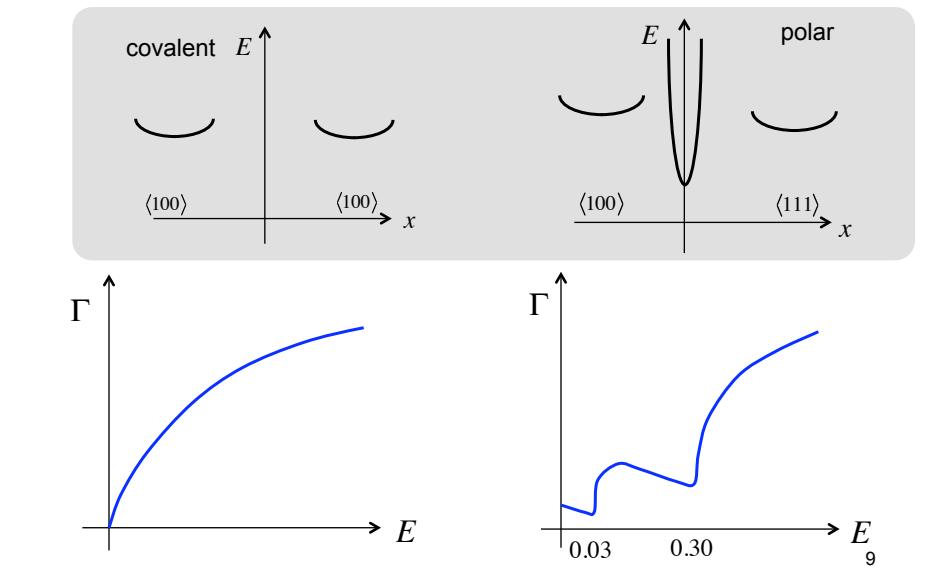
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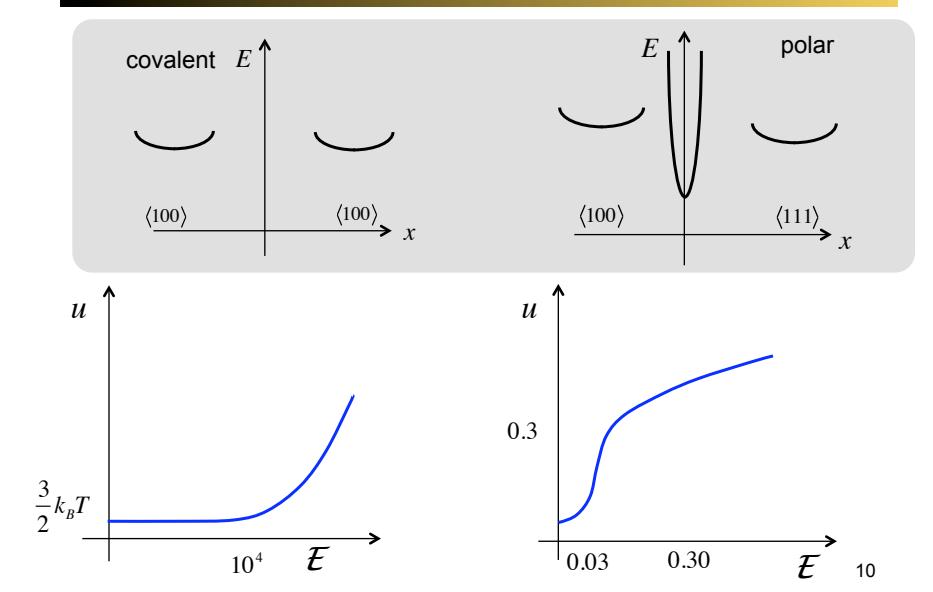
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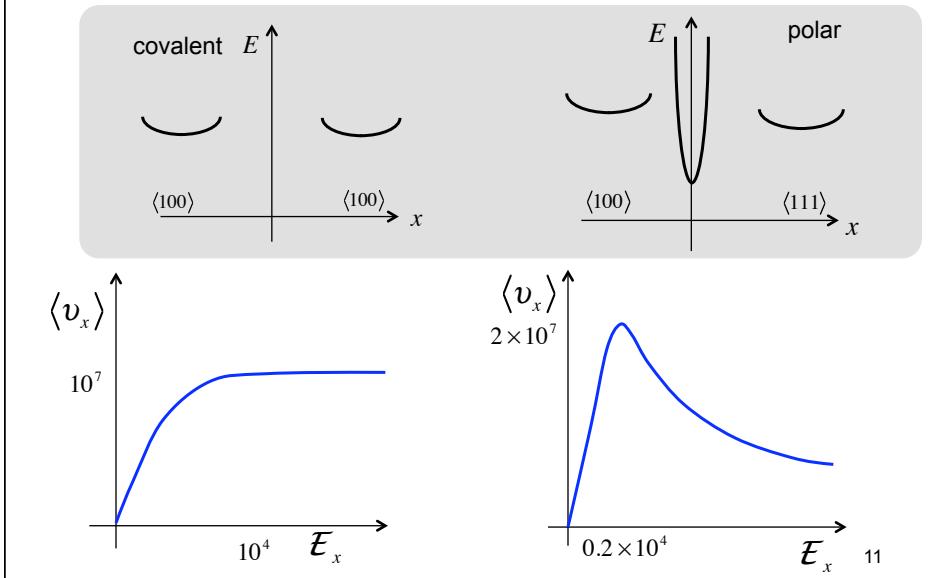
## covalent vs. polar semiconductors



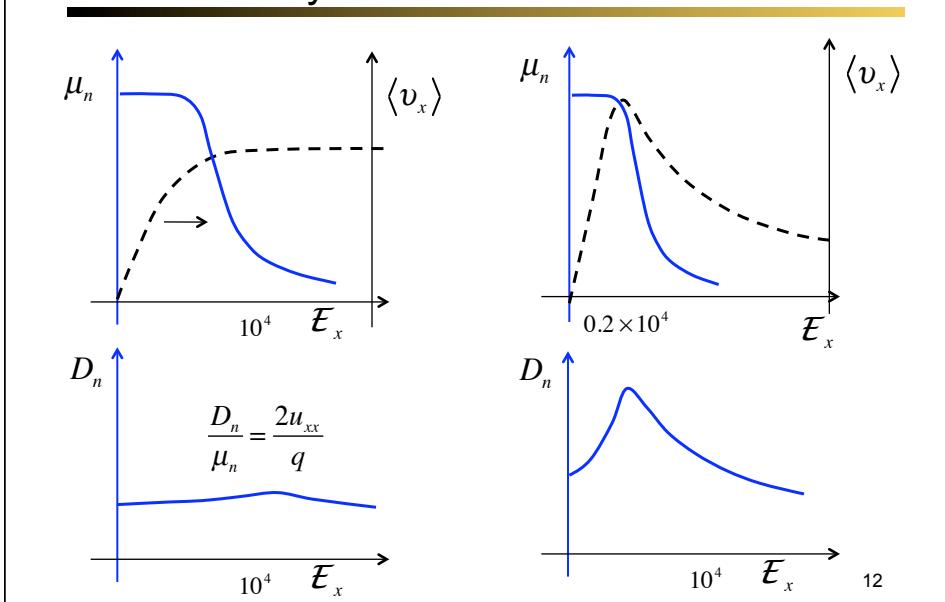
## average energy vs. electric field



## average velocity vs. electric field



## mobility and diffusion coefficient



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## can we calculate $v_{SAT}$ ?

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$$\mu_n = \frac{q\langle\tau_m\rangle}{m^*} \quad (\text{momentum balance})$$

$$J_{nx}\mathcal{E}_x = \frac{n(u - u_0)}{\langle\tau_E\rangle} \quad (\text{energy balance}) \quad nq\mu_n\mathcal{E}_x^2 = \frac{n(u - u_0)}{\langle\tau_E\rangle}$$

$$u = u_0 + \langle\tau_E\rangle q\mu_n\mathcal{E}_x^2 = u_0 + \frac{\langle\tau_E\rangle\langle\tau_m\rangle}{m^*} q^2\mathcal{E}_x^2 \approx \frac{\langle\tau_E\rangle\langle\tau_m\rangle}{m^*} q^2\mathcal{E}_x^2$$

$$\langle\tau_m\rangle \approx \langle\tau\rangle \quad (\text{ave. time between collisions})$$

$$\langle\tau_E\rangle \approx \frac{u}{\hbar\omega_0} \langle\tau\rangle = \frac{u}{\hbar\omega_0} \langle\tau_m\rangle$$

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can we calculate  $v_{SAT}$ ?

$$\mu_n = \frac{q\langle\tau_m\rangle}{m^*} \quad u \approx \frac{\langle\tau_E\rangle\langle\tau_m\rangle}{m^*} q^2 \mathcal{E}_x^2 \quad \langle\tau_E\rangle \approx \frac{u}{\hbar\omega_0} \langle\tau\rangle = \frac{u}{\hbar\omega_0} \langle\tau_m\rangle$$

$$u \approx \frac{\langle\tau_m\rangle^2 u}{\hbar\omega_0 m^*} q^2 \mathcal{E}_x^2$$

$$\langle\tau_m\rangle \approx \frac{\sqrt{\hbar\omega_0 m^*}}{q\mathcal{E}_x} \rightarrow \mu_n = \sqrt{\frac{\hbar\omega_0}{m^*}} \frac{1}{\mathcal{E}_x}$$

$$\langle v_x \rangle = \mu_n \mathcal{E}_x \rightarrow v_{SAT} = \sqrt{\frac{\hbar\omega_0}{m^*}}$$

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saturation velocity

$$v_{SAT} \approx \sqrt{\frac{\hbar\omega_0}{m^*}}$$

$$\text{Si: } \hbar\omega_0 = 0.063 \text{ eV} \quad v_{SAT} \approx 1.0 \times 10^7 \text{ cm/s}$$

$$\text{Ge: } \hbar\omega_0 = 0.037 \text{ eV} \quad v_{SAT} \approx 0.6 \times 10^7 \text{ cm/s}$$

$$\text{SiC: } \hbar\omega_0 = 0.12 \text{ eV} \quad v_{SAT} \approx 1.5 \times 10^7 \text{ cm/s}$$

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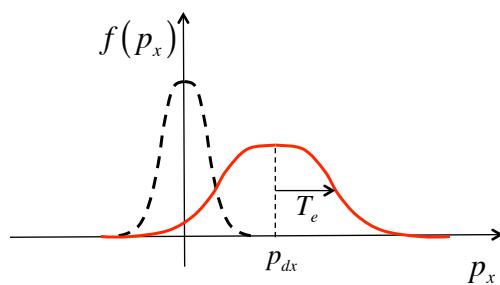
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## electron temperature approach

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- 1) Goal: to compute  $\langle v_x \rangle = v_{dx}(\mathcal{E})$

- 2) Assume:  $f(\vec{p}) = e^{-|\vec{p} - m^* \vec{v}_d|^2 / 2m^* k_B T_e}$



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## electron temperature approach

$$f(\vec{p}) = e^{-|\vec{p} - m^* \vec{v}_d|^2 / 2m^* k_B T_e}$$

2 unknowns:  $v_{dx}, T_e$  ...need 2 equations

1) Momentum balance:

$$J_{nx} = nq\mu_n \mathcal{E}_x \rightarrow v_{dx} = -\mu_n \mathcal{E}_x$$

2) Energy balance:

$$J_{nx} \mathcal{E}_x = nq\mu_n \mathcal{E}_x^2 = \frac{n(u - u_0)}{\langle \tau_E \rangle}$$

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## electron temperature approach

$$J_{nx} \mathcal{E}_x = nq\mu_n \mathcal{E}_x^2 = \frac{n(u - u_0)}{\langle \tau_E \rangle}$$

$$u = u_0 + q \langle \tau_E \rangle \mu_n \mathcal{E}_x^2 = u_0 + \frac{q^2 \langle \tau_E \rangle \langle \tau_m \rangle}{m^*} \mathcal{E}_x^2$$

$$u_0 \approx \frac{3}{2} k_B T_L$$

$$u \approx \frac{3}{2} k_B T_e \quad (\text{neglects drift energy})$$

$$\frac{T_e}{T_L} = 1 + \frac{2q^2 \langle \tau_E \rangle \langle \tau_m \rangle}{3k_B T_L m^*} \mathcal{E}_x^2$$

$$f(\vec{p}) = e^{-|\vec{p} - m^* \vec{v}_d|^2 / 2m^* k_B T_e}$$

$$v_{dx} = -\mu_n(T_e) \mathcal{E}_x$$

$$\frac{T_e}{T_L} = 1 + \frac{2q^2 \langle \tau_E \rangle \langle \tau_m \rangle}{3k_B T_L m^*} \mathcal{E}_x^2$$

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## aside: neglect of the drift energy

$$u = \frac{1}{2} m^* v_d^2 + \frac{3}{2} k_B T_e \approx \frac{3}{2} k_B T_e$$

$$\frac{m^* v_d^2 / 2}{3k_B T_e / 2} \ll 1 ?$$

$$v_d^2 = \mu_n^2 \mathcal{E}_x^2 = q^2 \langle \tau_m \rangle^2 \mathcal{E}_x^2 / m^{*2}$$

$$\frac{T_e}{T_L} = 1 + \frac{2q^2 \langle \tau_E \rangle \langle \tau_m \rangle}{3k_B T_L m^*} \mathcal{E}_x^2 \rightarrow \frac{3}{2} k_B T_e \approx \frac{q^2 \langle \tau_E \rangle \langle \tau_m \rangle}{m^*} \mathcal{E}_x^2$$

$$\frac{m^* v_d^2 / 2}{3k_B T_e / 2} = \frac{\langle \tau_m \rangle}{\langle \tau_E \rangle} \ll 1 \quad \text{typically well-satisfied}$$

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## electron temperature model

$$f(\vec{p}) = e^{-|\vec{p} - m^* \vec{v}_d|^2 / 2 m^* k_B T_e}$$

$$v_{dx} = -\mu_n(T_e) \mathcal{E}_x$$

$$\frac{T_e}{T_L} = 1 + \frac{2\mu_n \langle \tau_E \rangle}{3k_B T_L} \mathcal{E}_x^2$$

need to specify:

$$\mu_n(T_e) \text{ or } \langle \tau_m \rangle(T_e) \text{ and } \langle \tau_E \rangle(T_e)$$

“it can be shown”

$$\mu_n(T_e) = \mu_0 \sqrt{T_L/T_e} \quad (\text{ADP})$$

$$\mu_n(T_e) = \mu_0 (T_e/T_L)^{3/2} \quad (\text{II})$$

for ODP IV scattering in Si:

$$1/\langle \tau_E \rangle = \frac{2}{3} \frac{C}{k_B T_L} \sqrt{T_L/T_e}$$

$$C \approx 10^{-8} \text{ W}$$

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## the procedure

- 1) Identify the scattering mechanism that controls momentum relaxation

e.g. ADP scattering in Si  $\mu_n(T_e) = \mu_0 \sqrt{T_L/T_e}$

- 2) Identify the scattering mechanism that controls energy relaxation

e.g. IV scattering in Si  $1/\langle\tau_E\rangle = \frac{2}{3} \frac{C}{k_B T_L} \sqrt{T_L/T_e}$

- 3) Solve the energy balance equation for  $T_e$

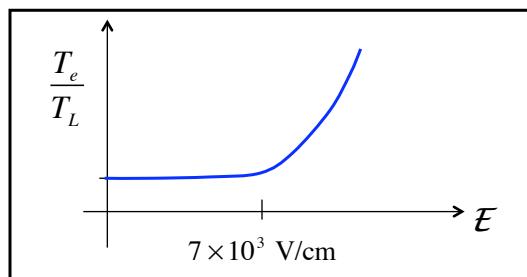
$$\frac{T_e}{T_L} = 1 + \frac{2\mu_n \langle\tau_E\rangle}{3k_B T_L} \mathcal{E}_x^2$$

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## result (for silicon)

$$\frac{T_e}{T_L} = 1 + \frac{q\mu_{n0}}{C} \mathcal{E}_x^2 = 1 + (\mathcal{E}/\mathcal{E}_c)^2 \quad \mathcal{E}_c \approx 7 \times 10^3 \text{ V/cm}$$



$$\mu_n(T_e) = \mu_0 \sqrt{T_L/T_e} = \frac{\mu_{n0}}{\sqrt{1 + (\mathcal{E}/\mathcal{E}_c)^2}}$$

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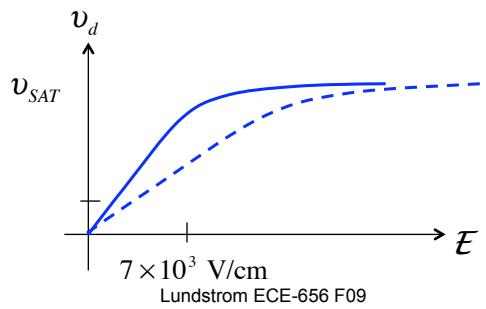
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## velocity vs. field characteristic

$$\mu_n(T_e) = \frac{\mu_{n0}}{\sqrt{1 + (\mathcal{E}/\mathcal{E}_c)^2}}$$

$$v_d = \mu_n(T_e)\mathcal{E} = \frac{\mu_{n0}\mathcal{E}}{\sqrt{1 + (\mathcal{E}/\mathcal{E}_c)^2}}$$

$$v_{SAT} = \mu_{n0}\mathcal{E}_c = 1 \times 10^7 \text{ cm/s}$$



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## high- field diffusion

$$D_n = \frac{k_B T_e}{q} \mu_n(T_e) = D_{n0} \sqrt{1 + (\mathcal{E}/\mathcal{E}_c)^2}$$

$$\text{but....in practice, } D_n(\mathcal{E}) \approx D_{n0} \quad \left( \frac{D_n}{\mu_n} = \frac{2u_{xx}}{q} \right)$$

a failure of the electron temperature model!

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## <111> Silicon: low-field

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$$\mathcal{E}_z = -100 \text{ V/cm}$$

$$\langle v_z \rangle = 8.1 \times 10^4 \text{ cm/s}$$

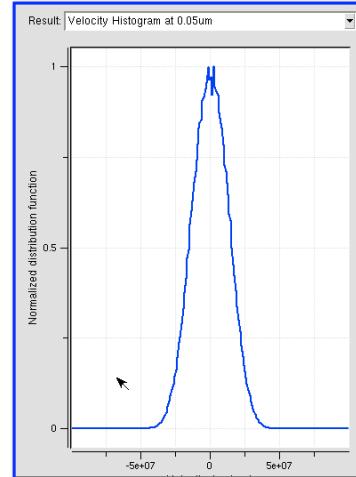
$$\mu_n(\mathcal{E}_z) = 810 \text{ cm}^2/\text{V-s}$$

$$u = 0.04 \text{ eV} \quad (1.5k_B T / q = 0.39 \text{ eV})$$

$$u_{zz} = 0.0135 \text{ eV} \quad (u_{zz} / u = 0.34)$$

$$u_{drift} \sim 10^{-7} \text{ eV} \quad (u_{drift} / u \sim 10^{-5})$$

$$n(x,y,z)/n = 0.33 / 0.335 / 0.335$$



(simulations performed with DEMONs on [www.nanoHUB.org](http://www.nanoHUB.org))

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## <111> Silicon: high-field

$$\mathcal{E}_z = -10^5 \text{ V/cm}$$

$$\langle v_z \rangle = 1.04 \times 10^7 \text{ cm/s}$$

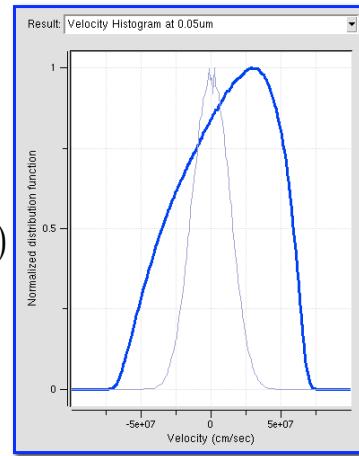
$$\mu_n(\mathcal{E}_z) = 104 \text{ cm}^2/\text{V-s}$$

$$u = 0.364 \text{ eV} \quad (1.5k_B T / q = 0.039 \text{ eV})$$

$$u_{zz} = 0.145 \text{ eV} \quad (u_{zz} / u = 0.40)$$

$$u_{drift} = 0.008 \text{ eV} \quad (u_{drift} / u = 0.02)$$

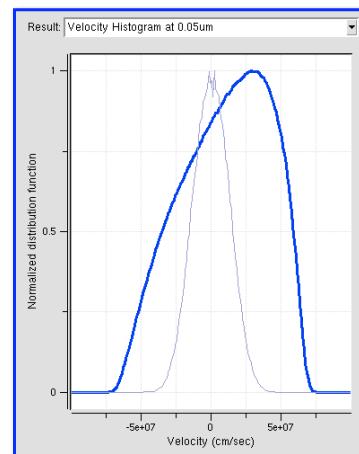
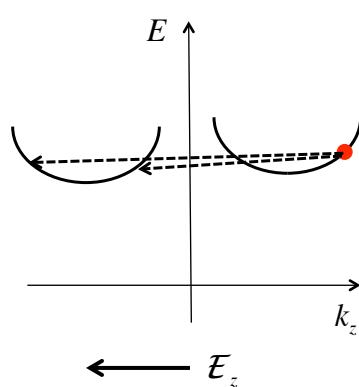
$$n(x,y,z)/n = 0.336 / 0.331 / 0.333$$



(simulations performed with DEMONs on [www.nanoHUB.org](http://www.nanoHUB.org))

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## <111> Silicon: high-field



(simulations performed with DEMONs on [www.nanoHUB.org](http://www.nanoHUB.org))

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## <100> Silicon: high-field

$$\mathcal{E}_z = -10^5 \text{ V/cm}$$

$$\langle v_z \rangle = 0.98 \times 10^7 \text{ cm/s}$$

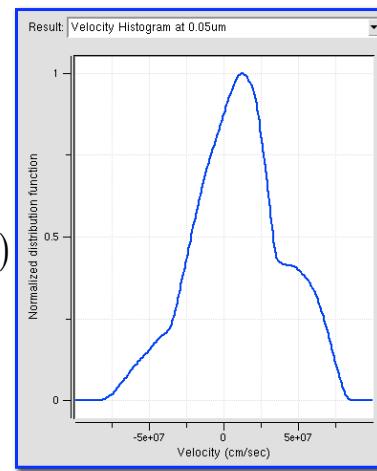
$$\mu_n(\mathcal{E}_z) = 98 \text{ cm}^2/\text{V-s}$$

$$u = 0.346 \text{ eV} \quad (1.5k_B T / q = 0.039 \text{ eV})$$

$$u_{zz} = 0.138 \text{ eV} \quad (u_{zz} / u = 0.40)$$

$$u_{drift} = 0.007 \text{ eV} \quad (u_{drift} / u = 0.02)$$

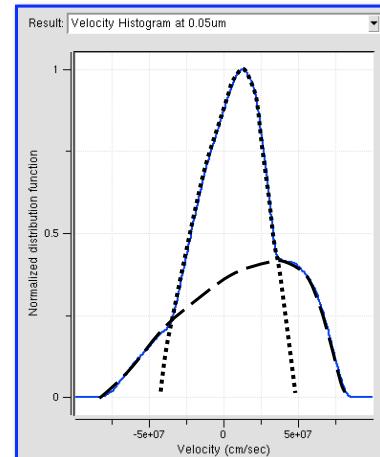
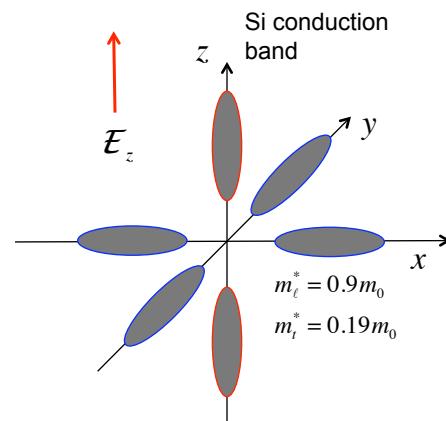
$$n(x,y,z)/n = 0.306 / 0.309 / 0.385$$



(simulations performed with DEMONs on [www.nanoHUB.org](http://www.nanoHUB.org))

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## <100> Silicon: high-field



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## summary

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- 1) High-field transport leads to field-dependent mobilities and diffusion coefficients (when the field varies slowly in space and time).
- 2) The electron temperature approach provides a qualitative (and sometimes quantitative) way to view high-field (**hot carrier**) transport.
- 3) Rapidly varying electric fields lead to “off-equilibrium”, “non-local” or “non-stationary” transport effects that cannot be described with (local) field-dependent field dependent transport parameters.

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## questions

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