velocity vs. field characteristics

\( \langle v_x \rangle \)

\( 10^7 \text{ cm/s} \)

\( 10^4 \text{ V/cm} \)

\( E_x \)

Si

GaAs
**outline**

1) Brief Introduction  
2) **Current Equation**  
3) Qualitative features of high field transport  
4) Saturated velocity  
5) Electron temperature model  
6) Survey of results  
7) Quick Summary

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**current equation**

\[ J_{nx} = nq\mu_nE_x + 2\mu_n \frac{d(nu_{sx})}{dx} \]

This is an “exact” steady-state current equation, but….  

\[ \mu_n[f(\vec{r},\vec{p},t)] \]  
\[ u_{sx}[f(\vec{r},\vec{p},t)] = \left\langle \frac{1}{2} p_z v_z \right\rangle \]
current equation: bulk semiconductor

bulk semiconductor: \( E_x, u_{xx} \approx \text{constant} \)

\[
J_n = n q \mu_n E_x + q D_n \frac{dn}{dx}
\]

\[
D_n = \frac{2u_{xx}}{\mu_n q}
\]

near equilibrium: \( u_{xx} \approx \frac{k_B T_i}{q} \)

field-dependent mobility

\[
J_n = n q \mu_n E_x + q D_n \frac{dn}{dx}
\]

Goal: Find mobility and diffusion coefficient without solving BTE

In general, however:

\[
\mu_n \left[ f(\vec{r}, \vec{p}, t) \right] \quad D_n \left[ f(\vec{r}, \vec{p}, t) \right]
\]

In a bulk semiconductor, \( f \) is determined by \( E \), so there is a one-to-one mapping between \( E \) and \( f \).

\[
\mu_n(E) \quad D_n(E)
\]

Electric field dependent mobility and diffusion coefficient.
The concept of a field-dependent mobility applies only when the electric field changes slowly with position.

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covalent vs. polar semiconductors

average energy vs. electric field
average velocity vs. electric field

- Covalent (100) direction: \( \langle v_x \rangle \) vs. \( E_x \)
  - \( \langle v_x \rangle \) approximately \( 10^7 \) as \( E_x \) increases

- Polar (111) direction: \( \langle v_x \rangle \) vs. \( E_x \)
  - \( \langle v_x \rangle \) approximately \( 2 \times 10^7 \) as \( E_x \) increases

mobility and diffusion coefficient

- Mobility \( \mu_n \) vs. \( E_x \)
  - As \( E_x \) increases, \( \mu_n \) decreases

- Diffusion coefficient \( D_n \) vs. \( E_x \)
  - \( D_n \) is approximately \( \frac{2u_{sx}}{q} \) as \( E_x \) increases

\[ \frac{D_n}{\mu_n} = \frac{2u_{sx}}{q} \]
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can we calculate \( \nu_{\text{SAT}} \)?

\[
\mu_n = \frac{q \langle \tau_m \rangle}{m^*} \text{ (momentum balance)}
\]

\[
J_m E_x = \frac{n(u - u_0)}{\langle \tau_k \rangle} \text{ (energy balance)}
\]

\[
nq \mu_n E_x^2 = \frac{n(u - u_0)}{\langle \tau_E \rangle}
\]

\[
u = u_0 + \tau_k q \mu_n E_x^2 = u_0 + \frac{\langle \tau_k \rangle \langle \tau_m \rangle}{m^*} q^2 E_x^2 \approx \frac{\langle \tau_E \rangle \langle \tau_m \rangle}{m^*} q^2 E_x^2
\]

\[
\langle \tau_m \rangle \approx \langle \tau \rangle \text{ (ave. time between collisions)}
\]

\[
\langle \tau_E \rangle \approx \frac{u}{\hbar \omega_0} \langle \tau \rangle = \frac{u}{\hbar \omega_0} \langle \tau_m \rangle
\]
can we calculate $\nu_{\text{SAT}}$?

$$\mu_n = \frac{q \langle \tau_m \rangle}{m^*}$$

$$u = \frac{\langle \tau_E \rangle \langle \tau_m \rangle}{m^*} q^2 \mathcal{E}_x^2$$

$$\langle \tau_m \rangle \approx \frac{u}{\hbar \omega_0} \langle \tau \rangle = \frac{u}{\hbar \omega_0} \langle \tau_m \rangle$$

$$\langle \tau \rangle = \frac{\langle \tau_m \rangle^2}{\hbar \omega_0 m^* q^2 \mathcal{E}_x^2}$$

$$\langle \nu_x \rangle = \mu_n \mathcal{E}_x \rightarrow \nu_{\text{SAT}} = \sqrt{\frac{\hbar \omega_0}{m^*}}$$

### Saturation Velocity

<table>
<thead>
<tr>
<th>Material</th>
<th>$\hbar \omega_0$</th>
<th>$\nu_{\text{SAT}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>0.063 eV</td>
<td>$1.0 \times 10^7$ cm/s</td>
</tr>
<tr>
<td>Ge</td>
<td>0.037 eV</td>
<td>$0.6 \times 10^7$ cm/s</td>
</tr>
<tr>
<td>SiC</td>
<td>0.12 eV</td>
<td>$1.5 \times 10^7$ cm/s</td>
</tr>
</tbody>
</table>
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electron temperature approach

1) Goal: to compute $\langle v_x \rangle = v_{dx} (E)$

2) Assume: $f(\vec{p}) = e^{-[p-m^*v_x]^2/2m^*k_BT_e}$
electron temperature approach

\[ f(\tilde{p}) = e^{-\frac{1}{2}m^*\tilde{v}_p \tilde{v}_p^T / m^*k_BT_e} \]

2 unknowns: \( \nu_{dx}, T_e \) ...need 2 equations

1) Momentum balance:
   \[ J_{nx} = nq \mu_n \mathcal{E}_x \rightarrow \nu_{dx} = -\mu_n \mathcal{E}_x \]

2) Energy balance:
   \[ J_{nx} \mathcal{E}_x = nq \mu_n \mathcal{E}_x^2 = \frac{n(u - u_0)}{\tau_E} \]

\[ u = u_0 + q \langle \tau_E \rangle \mu_n \mathcal{E}_x = u_0 + \frac{q^2 \langle \tau_E \rangle \langle \tau_m \rangle \mathcal{E}_x^2}{m^*} \]

\[ u_0 \approx \frac{3}{2} k_B T_L \]

\[ u = \frac{3}{2} k_B T_e \quad \text{(neglects drift energy)} \]

\[ \frac{T_e}{T_L} = 1 + \frac{2q^2 \langle \tau_E \rangle \langle \tau_m \rangle \mathcal{E}_x^2}{3k_B T_L m^*} \]

\[ \nu_{dx} = -\mu_n (T_e) \mathcal{E}_x \]

\[ T_e = 1 + \frac{2q^2 \langle \tau_E \rangle \langle \tau_m \rangle \mathcal{E}_x^2}{3k_B T_L m^*} \]
aside: neglect of the drift energy

\[ u = \frac{1}{2} m^* \vec{v}_d^2 + \frac{3}{2} k_B T_e = \frac{3}{2} k_B T_e \]

\[ \frac{m^* \vec{v}_d^2 / 2}{3k_B T_e / 2} \ll 1? \]

\[ \vec{v}_d^2 = \mu_n^2 E_x^2 = \frac{q^2 \langle \tau_m \rangle^2 E_x^2}{m^*} \]

\[ \frac{T_e}{T_L} = 1 + \frac{2q^2 \langle \tau_E \rangle \langle \tau_m \rangle E_x^2}{3k_B T_L m^*} \approx \frac{3}{2} \frac{k_B T_e}{m^*} \]

\[ \frac{m^* \vec{v}_d^2 / 2}{3k_B T_e / 2} \frac{\langle \tau_m \rangle}{\langle \tau_E \rangle} \ll 1 \]

typically well-satisfied

---

electron temperature model

\[ f(\vec{p}) = e^{-\frac{\vec{p} \cdot \vec{v}_d}{2m^*k_B T_e}} \]

\[ \vec{v}_{dx} = -\mu_n (T_e) E_x \]

\[ \frac{T_e}{T_L} = 1 + \frac{2\mu_n \langle \tau_E \rangle}{3k_B T_L} E_x^2 \]

need to specify:

\[ \mu_n (T_e) \text{ or } \langle \tau_m \rangle (T_e) \text{ and } \langle \tau_E \rangle (T_e) \]

“it can be shown”

\[ \mu_n (T_e) = \mu_0 \sqrt{T_L / T_e} \text{ (ADP)} \]

\[ \mu_n (T_e) = \mu_0 (T_e / T_L)^{3/2} \text{ (II)} \]

for ODP IV scattering in Si:

\[ \frac{1}{\langle \tau_E \rangle} = \frac{2}{3} \frac{C}{k_B T_L} \sqrt{T_L / T_e} \]

\[ C \approx 10^{-8} \text{ W} \]
the procedure

1) Identify the scattering mechanism that controls momentum relaxation
   
   e.g. ADP scattering in Si \[ \mu_n(T_e) = \mu_0 \sqrt{T_L/T_e} \]

2) Identify the scattering mechanism that controls energy relaxation
   
   e.g. IV scattering in Si \[ \frac{1}{\tau_E} = \frac{2}{3} C \frac{E}{k_B T_L} \sqrt{T_L/T_e} \]

3) Solve the energy balance equation for \( T_e \)
   
   \[ \frac{T_e}{T_L} = 1 + \frac{2\mu_n(\tau_E)}{3k_B T_L} E_x^2 \]

result (for silicon)

\[ \frac{T_e}{T_L} = 1 + \frac{q\mu_n(0)}{C} E_x^2 = 1 + \left( \frac{E}{E_C} \right)^2 \quad E_C \approx 7 \times 10^3 \text{ V/cm} \]

\[ \frac{T_e}{T_L} = 1 + \frac{q\mu_n(0)}{C} E_x^2 = 1 + \left( \frac{E}{E_C} \right)^2 \quad E_C \approx 7 \times 10^3 \text{ V/cm} \]

![Graph showing \( \frac{T_e}{T_L} \) vs. \( E \)]

\[ \mu_n(T_e) = \mu_0 \sqrt{T_L/T_e} = \frac{\mu_n(0)}{\sqrt{1 + \left( \frac{E}{E_C} \right)^2}} \]
velocity vs. field characteristic

\[ \mu_n(T_e) = \frac{\mu_{n0}}{\sqrt{1 + (E/E_C)^2}} \]

\[ v_d = \mu_n(T_e)E = \frac{\mu_{n0}E}{\sqrt{1 + (E/E_C)^2}} \]

\[ v_{SAT} = \mu_{n0}E_c = 1 \times 10^7 \text{ cm/s} \]

high- field diffusion

\[ D_n = \frac{k_B T_e}{q} \mu_n(T_e) = D_{n0} \sqrt{1 + (E/E_C)^2} \]

but….in practice, \[ D_n(E) \approx D_{n0} \left( \frac{D_n}{\mu_n} = \frac{2u_{so}}{q} \right) \]

a failure of the electron temperature model!
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<111> Silicon: low-field

\[ \bar{E}_z = -100 \text{ V/cm} \]

\[ \langle v_z \rangle = 8.1 \times 10^4 \text{ cm/s} \]

\[ \mu_n(\bar{E}_z) = 810 \text{ cm}^2/\text{V-s} \]

\[ u = 0.04 \text{ eV} \quad (1.5k_B T / q = 0.39 \text{ eV}) \]

\[ u_z = 0.0135 \text{ eV} \quad (u_z / u = 0.34) \]

\[ u_{\text{drift}} \sim 10^{-7} \text{ eV} \quad (u_{\text{drift}} / u \sim 10^{-5}) \]

\[ n(x,y,z)/n = 0.33 / 0.335 / 0.335 \]

(simulations performed with DEMONs on www.nanoHUB.org)
\( E_z = -10^5 \text{ V/cm} \)
\( \langle v_z \rangle = 1.04 \times 10^7 \text{ cm/s} \)
\( \mu_n (E_z) = 104 \text{ cm}^2/\text{V-s} \)
\( u = 0.364 \text{ eV} \quad (1.5 k_B T / q = 0.039 \text{ eV}) \)
\( u_{cc} = 0.145 \text{ eV} \quad \left( u_{cc} / u = 0.40 \right) \)
\( u_{drift} = 0.008 \text{ eV} \quad \left( u_{drift} / u = 0.02 \right) \)
\( n(x,y,z)/n = 0.336 / 0.331 / 0.333 \)

(simulations performed with DEMONs on www.nanoHUB.org)
\( E_z = -10^5 \) V/cm
\(<v_z> = 0.98 \times 10^7\) cm/s
\( \mu_n(E_z) = 98\) cm\(^2\)/V-s
\( u = 0.346\) eV \( (1.5k_B T / q = 0.039\) eV\)
\( u_{zz} = 0.138\) eV \( (u_{zz} / u = 0.40)\)
\( u_{\text{drift}} = 0.007\) eV \( (u_{\text{drift}} / u = 0.02)\)
\( n(x,y,z)/n = 0.306 / 0.309 / 0.385\)

(simulations performed with DEMONs on www.nanoHUB.org)
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### summary

1) High-field transport leads to field-dependent mobilities and diffusion coefficients (when the field varies slowly in space and time).

2) The electron temperature approach provides a qualitative (and sometimes quantitative) way to view high-field (hot carrier) transport.

3) Rapidly varying electric fields lead to “off-equilibrium”, “non-local” or “non-stationary” transport effects that cannot be described with (local) field-dependent field dependent transport parameters.
questions

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