outline

1) Review of velocity overshoot
2) Steady-state, spatial transients
3) Heterojunction launching ramps
4) Repeated velocity overshoot
5) Questions?
rapidly varying electric fields

\[ \nu_x(t) = -\mu_n E_z \]

actual \[ \nu(t) \]

Lundstrom ECE-656 F09
Monte Carlo simulation

Fig. 8.9 Evolution of the distribution function during a velocity overshoot transient. The average drift velocity and energy are shown in (a), and the evolution of the corresponding distribution function is shown in (b). The results were obtained by Monte Carlo simulation of electron transport in silicon by E. Constant [8.10].

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Rees effect (GaAs)

Fig. 8.10 (a) Applied electric field vs. time. (b) Ave. drift velocity vs. time. (c) Ave. electron energy vs. time. (Monte Carlo simulations from E. Constant [8.10].

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velocity overshoot

\[ p_{dx} = \langle p_x \rangle \quad \text{Let’s find an equation for the ave. } x\text{-directed momentum.} \]

\[ \frac{dp_{dx}}{dt} = -qE_x - \frac{p_{dx}}{\langle \tau_m \rangle} \]

\[ u = \langle E(p) \rangle \quad \text{Let’s find an equation for the ave. } x\text{-directed momentum.} \]

\[ \frac{du}{dt} = -qu_{dx}E_x - \frac{(u - u_0)}{\langle \tau_E \rangle} \]

\[ u_{dx}(t) = -\mu_n E_x \left( 1 - e^{-t/\langle \tau_m \rangle} \right) \]

\[ u(t) = u_0 + q\langle \tau_E \rangle \mu_n E_x^2 \left( 1 - e^{-t/\langle \tau_E \rangle} \right) \]

But, we have ignored diffusion (an ensemble effect).

Lundstrom ECE-656 F09
momentum and energy balance

\[
\frac{dp_{dx}}{dt} = -q\mathcal{E}_x - \frac{p_{dx}}{\langle \tau_m \rangle}
\]

\[
\frac{du}{dt} = -q\nu_{dx} \mathcal{E}_x - \frac{(u - u_0)}{\langle \tau_E \rangle}
\]

\[
\frac{dP_x}{dt} = -\frac{d(2W_{xx})}{dx} - qn\mathcal{E}_x - \frac{P_x}{\langle \tau_m \rangle}
\]

\[
P_x = n\langle p_x \rangle = np_{dx}
\]

\[
\frac{dW}{dt} = -\frac{d(F_W)}{dx} + J_x \mathcal{E}_x - \frac{W - W_0}{\langle \tau_E \rangle}
\]

\[
W = n\langle E(p) \rangle = nu
\]

What effect do these spatial gradients have?
**Question:** If we change the horizontal axis to distance, what does the steady-state velocity vs. position characteristic look like?
Steady-state current is constant: \[ J_{nx} = n(x)q \left\langle v_x(x) \right\rangle \]
carrier velocity vs. position

\[ \nu_{dz}(t) \]

\[ n(x) \quad \nu_{dx}(x) \]
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a familiar example

\[ \Delta n(x) \approx \Delta n(0) \left( 1 - \frac{x}{W_B} \right) \]

\[ J_n \approx q D_n \frac{\Delta n(0)}{W_B} \]

\[ \langle \nu(x) \rangle \propto \frac{1}{1 - \frac{x}{W_B}} \]

\[ J_{nx} = n(x) q \langle \nu_x(x) \rangle \]
example Monte Carlo simulations

DEMONs

1D, steady-state Monte Carlo simulation for Si and GaAs

Piecewise constant electric field profiles.

http://nanohub.org/resources/1934
low-high field structure

periodic boundary conditions

\[ J_{nx} = n(x)q\langle v_x(x) \rangle \]
low-high field structure

\[ \langle \nu(x) \rangle \]

\[ u(x) \]

periodic boundary conditions

0.039 eV
velocity histograms

$\mathcal{E} = -10 \text{ V/cm}$

$\mathcal{E} = -10^5 \text{ V/cm}$

[Diagram showing velocity histograms with electric field values]
energy histograms

\[ n(E) = f(E) D(E) \]

\[ \mathcal{E} = -10^5 \text{ V/cm} \]

\[ n(E) \propto e^{-E/k_BT} \sqrt{E} \]
off-equilibrium nanoscale MOSFETs

Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992
low-high-low field structure

periodic boundary conditions

velocity undershoot

\[ u(x) > u_0 \]
temporal vs. spatial transients

Fig. 8.13  (a) Applied electric field in time and space.  (b) Average velocity versus position for a pulse applied in space (solid line) and time (dashed line).  (c) Steady-state carrier density (solid line) and energy (dashed line.) The results were obtained by Monte Carlo simulation of electron transport in GaAs by E. Constant [8.10].

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\[ z = \int_{0}^{t} \nu(t') \, dt' \]
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Question: How do we experimentally tell whether base transport is ballistic or diffusive?

Answer: Look at the base current.

\[ I_B \propto \frac{t_t}{\tau_n} \]

ballistic: \( t_t = \frac{W_B}{v_{ball}} \)
diffusive: \( t_t = \frac{W_B^2}{2D_n} \)

base transit time scaling

~80% traverse a 300Å base ballistically or quasi-ballistically

base transit time scaling

\[ n_0 \nu_{inj} \rightarrow (1 - \Gamma) n_0 \nu_{inj} \rightarrow n_{mw}(D_n/W) \]

\[ n_0 \nu_{inj} = (1 - \Gamma) n_0 \nu_{inj} + n_{mw}(D_n/W) \]

\[ \frac{n_0}{n_{mw}} = \Gamma \frac{\nu_{inj}}{(D_n/W)} \]

Dodd and Lundstrom, *APL*, 61, 27, 1992
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