ECE-656: Fall 2009

Lecture 35:
Ballistic Transport

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first: let’s finish up L34

1) Review of velocity overshoot
2) Steady-state, spatial transients
3) Heterojunction launching ramps
4) Repeated velocity overshoot
5) Questions?
temporal vs. spatial transients

Fig. 8.13  (a) Applied electric field in time and space. (b) Average velocity versus position for a pulse applied in space (solid line) and time (dashed line). (c) Steady-state carrier density (solid line) and energy (dashed line.) The results were obtained by Monte Carlo simulation of electron transport in GaAs by E. Constant [8.10].

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\[ z = \int_{0}^{t} \nu(t') \, dt' \]
can VO be maintained over large distances?

Fig. 8.14  (a) A series of electric field impulses in time or space. (b) Expected average velocity versus time profile. (c) Expected steady-state velocity vs. position profile.

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outline

1) Schottky barriers
2) Transport across a thin base
3) High-field collectors
a familiar problem with strong gradients

\begin{align*}
J_n &= nq\mu_n \mathcal{E} + k_B T \mu_n \frac{dn}{dx} \\
\mu_n(\mathcal{E}) &= \frac{\mu_n^0}{\sqrt{1 + (\mathcal{E}/\mathcal{E}_C)^2}}
\end{align*}

Should we use a field-dependent mobility?
pn junction: energy balance

\[ J_n \mathcal{E} \approx \frac{3}{2} nk_B \left( \frac{T_e - T_L}{\langle \tau_E \rangle} \right) \]  
(ignores diffusion)

\[ \frac{T_e}{T_L} = 1 + \left( \frac{\mathcal{E}}{\mathcal{E}_C} \right)^2 \quad \mathcal{E}_C^2 = \frac{3nk_B T_L \mathcal{E}}{\langle \tau_E \rangle J_n} \]

1) equilibrium:

\[ J_n = 0 \quad \mathcal{E}_C \to \infty \]

\[ T_e \to T_L \quad \mu_n \to \mu_n^o \]
$\frac{T_e}{T_L} = 1 + \left( \frac{\mathcal{E}}{\mathcal{E}_C} \right)^2 \quad \mathcal{E}_C^2 = \frac{3nk_BT_L\mathcal{E}/2}{\langle \tau_E \rangle J_n}$

2) forward bias:

- $J_n < 0$  \quad $\mathcal{E}_C^2 < 0$
- $T_e < T_L$  \quad $\mu_n > \mu_n^0$

3) reverse bias:

- $J_n > 0$  \quad $\mathcal{E}_C^2 > 0$
- $T_e > T_L$  \quad $\mu_n < \mu_n^0$

FB: $J_n g\mathcal{E} < 0$  cooling

RB: $J_n g\mathcal{E} > 0$  heating
Schottky barriers: diffusion theory

solve:

\[ J_n = n q \mu_n \mathcal{E} + k_B T \mu_n \frac{dn}{dx} \]

\[ n(0) = n_0 \quad n(-W) = N_D \]

result:

\[ J(V_A) = q N_D \mu_n \mathcal{E}(0) e^{-q\phi_B n/k_B T} \left( e^{qV_A/k_B T} - 1 \right) \]

\[ \mu_n \leq \mu_n^o \]
Schottky barriers: TE theory

assume:

ballistic transport

result:

\[ J(V_A) = q \frac{N_D}{2} \nu_T e^{-q \phi_{Bn}/k_B T} \left( e^{qV_A/k_B T} - 1 \right) \]

DD fails when: \( \mu_n \mathcal{E}(0) > \frac{\nu_T}{2} \)

but why, what went wrong?
Schottky barriers: aside

TE result:

\[ J(V_A) = q \frac{N_D}{2} \nu T e^{-q\phi_{Bn}/k_BT} \left( e^{qV_A/k_BT} - 1 \right) \]

This result can be easily obtained by solving the ballistic BTE. See Lecture 13.
outline

1) Schottky barriers

2) Transport across a thin base

3) High-field collectors
diffusion across a thin base

\[ F = -D_n \frac{d\Delta n}{dx} \]

\[ F = D_n \frac{\Delta n(0)}{W_B} = \Delta n(0) \nu_{diff} \]

\[ \nu_{diff} = \frac{D_n}{W_B} \ll \nu_T \]

\[ D_n = \frac{\nu_T \lambda_0}{2} \]

\[ W_B \gg \lambda_0 \]

Fick’s Law describes diffusion across bases that are many mfps long.
why does DD fail?

\[ J_n = nq \mu_n E + 2 \mu_n d \left( nu_{xx} \right) / dx \]

\[ u_{xx} = \frac{k_B T}{2} + \frac{1}{2} m^* \nu_{dx}^2 \]

\[ J_{nx} = -qn \nu_{dx} \]

\[ J_{nx}^2 + \frac{n^2 q^2}{2 \mu_n m^* (dn/ dx)} J_{nx} - \frac{n^2 q^3 D_n}{2 \mu_n m^*} = 0 \]

\[ \begin{align*}
   i) \quad & \frac{dn}{dx} \rightarrow 0 \quad J_{nx} \rightarrow qD_n \frac{dn}{dx} \\
   ii) \quad & \frac{dn}{dx} \rightarrow \infty \quad J_{nx} \rightarrow qn \sqrt{\frac{k_B T}{2 m^*}} = qn \nu_{th} \\
   & D_n \frac{1}{n} \frac{dn}{dx} < \nu_{diff} < \nu_{th}
\end{align*} \]
diffusion velocity vs. conc. gradient

\[ J_{nx} = qD_{eff} \frac{dn}{dx} = nq\left( D_{eff} \frac{1}{n} \frac{dn}{dx} \right) \]

\( D_{eff} \) must be reduced for high concentration gradients.
Schottky barriers

\[ \left| \frac{dn}{dx} \right| \text{ is very large.} \]

\[ D_{\text{eff}} < D^o \]

\[ \mu_{\text{eff}} = \frac{D_{\text{eff}}}{k_BT/q} < \mu^o \]

The mobility, \( \mu \), should be reduced in a forward-biased PN junction even though the carrier are not heated.

\[ \mu_n \mathcal{E}(0) \rightarrow \nu_{th} \]

DD fails when: \( \mu_n \mathcal{E}(0) > \frac{\nu_T}{2} \)
is Fick’s Law wrong?

\[ F = -D_n \frac{dn}{dx} = D_n \frac{n(0)}{W_B} = n(0) \nu_{\text{diff}} \]

\[ \nu_{\text{diff}} = \frac{D_n}{W_B} \]

We just showed that we can get around this problem by reducing \( D_n \), but....
thin base transport again

\[ \Delta n(0) \]

\[ \nu_{\text{diff}} = \frac{D_n}{W_B} \]

\[ F = n(W_B)\nu_T \]

\[ n(W_B) > 0 \]
Fick’s Law in a thin base

\[ F = D_n \frac{n(0) - n(W_B)}{W_B} \]

\[ F = n(W_B) \nu_T \]

\[ n(W_B) = \frac{n(0)}{1 + \nu_T/(D_n/W_B)} \]

\[ F = n(0) \frac{D_n}{W_B} \times \frac{1}{1 + \left(\frac{D_n}{W_B}\right) \nu_T} \]

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Fick’s Law

\[ F = n(0) \frac{D_n}{W_B} \times \frac{1}{1 + \left( \frac{D_n}{W_B} \right) \nu_T} \]

\[ \frac{D_n}{W_B} \ll \nu_T \quad F \rightarrow n(0) \frac{D_n}{W_B} \]

\[ \frac{D_n}{W_B} \gg \nu_T \quad F \rightarrow n(0) \nu_T \]

No need to reduce \( D \) because \( dn/dx \rightarrow 0 \) when \( W_B \ll \lambda \)

Fick’s Law always holds – no matter how small the base!
outline

1) Schottky barriers
2) Transport across a thin base
3) High-field collectors
how good is the collector?

\[ \Delta n(0) \]

\[ n(W_B) > 0 \]
backscattering from the collector

\[
T_B = \frac{\lambda_0}{\lambda_0 + W_B} \\
T_C = 1 - R_C \approx \frac{\lambda_0}{\lambda_0 + 1}
\]

See Sec. 8.8 of Lundstrom
questions

1) Schottky barriers
2) Transport across a thin base
3) High-field collectors
Question & Answer 1