ECE-656: Fall 2009

Lecture 36:
The Course in One Lecture

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Conservation Laws:

$$\nabla \cdot \vec{D} = \rho$$

$$\frac{\partial n}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_n}{q} \right) + (G_n - R_n)$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_p}{q} \right) + (G_p - R_p)$$

Constitutive Relations:

$$\vec{D} = \kappa \varepsilon_0 \vec{E} = -\kappa \varepsilon_0 \vec{\nabla} V$$

$$\rho = q \left( p - n + N_D^+ - N_A^- \right)$$

$$\vec{J}_n = nq \mu_n \vec{E} + qD_n \vec{\nabla} n$$

$$\vec{J}_p = pq \mu_p \vec{E} - qD_p \vec{\nabla} p$$

$$R = f(n, p)$$

eetc.
“the drift-diffusion equation”

\[ \bar{J}_p = -pq \mu_p \vec{\nabla}V - qD_p \vec{\nabla}p \]

1) How is the DD equation derived?

2) What determines the mobility and diffusion coefficient?

3) What physics does it miss?

4) How do we describe transport without the DD equation?
“bottom-up” vs. “top down”

\[ \vec{J}_p = -pq\mu_p \vec{\nabla}V - qD_p \vec{\nabla}p \]

Historically, the DD equation was developed to describe transport for small gradients in concentration and potential in structures that were large compared to the mean-free-path for scattering. As time went on, it was extended to describe smaller and smaller structures.

In ECE-656, we turned this around and started by examining transport in small structures, and then we worked up to larger structures.
current at the nanoscale

If we apply a bias between the two contacts, what current flows?

gate oxide
SiON ~ 1.1 nm
channel ~ 30 nm
L3: Landauer model

\[ I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE \]

1) A difference in Fermi functions cause current to flow

2) \( M(E) \): density of conducting channels

3) \( T(E) \): transmission \((0 < T < 1)\)

4) Important assumptions:
   - contacts are “ideal” (absorbing, in equilibrium)
   - inelastic scattering only in contacts
conductance

\[ I = \frac{2q}{h} \int T(E) M(E)(f_1 - f_2) \, dE \]

**When** \((V_2 - V_1)\) **is small,**
\[ f_1 \approx f_2 \approx f_0 \quad (f_1 - f_2) \approx -\frac{\partial f_0}{\partial E}(q\Delta V) \]

**G**\[ G = \frac{I}{\Delta V} = \frac{2q^2}{h} \int T(E) M(E)\left(-\frac{\partial f_0}{\partial E}\right) \, dE \] \[ \frac{2q^2}{h} = \frac{1}{12.7 \text{k}\Omega} \]
L5-7: nanoresistors

\[ G = \frac{1}{R} = \frac{2q^2}{h} \int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE \]

\[ \frac{2q^2}{h} = \frac{1}{12.7 \, k\Omega} \]

For \( T = 0K \) or for strongly degenerate systems, \( (-\partial f_0 / \partial E) \approx \delta(E_F) \)

\[ G = \frac{2q^2}{h} T(E_F)M(E_F) \]

For ballistic conductors, \( T = 1 \).

\( M(E) \) is the number of conducting channels at energy, \( E \).
L4: Density of states / Density of modes

Carrier densities are determined by the density of states.

Current flow is determined by the density of modes.

To determine $M(E)$:

1D: simply count the subbands

2D: $M(E) \sim$ width of the resistor, $W$

3D: $M(E) \sim$ cross-sectional area, $A$
DOS depends on bandstructure and dimensionality

\[
D_{1D}(E) = \frac{L}{\pi \hbar} \sqrt{\frac{2m^*}{E - \epsilon_1}} \Theta(E - \epsilon_1)
\]

\[
D_{2D}(E) = A \frac{m^*}{\pi \hbar^2} \Theta(E - \epsilon_1)
\]

\[
D_{3D}(E) = \Omega \frac{m^* \sqrt{2m^*(E - E_C)}}{2\pi^2 \hbar^3}
\]

\[
(E(k) = E_C + \hbar^2 k^2 / 2m^*)
\]
DOM depends on bandstructure and dimensionality

\[ M_{1D}(E) = \Theta(E - \varepsilon_1) \]

\[ M_{2D}(E) = \frac{W \sqrt{2m^*(E - \varepsilon_1)}}{\pi \hbar} \]

\[ M_{3D}(E) = A \frac{m^*}{2\pi \hbar^2} (E - E_C) \]

\[ (E(k) = E_C + \hbar^2 k^2 / 2m^*) \]
DOM and bandstructure

For a simple $E(k)$, and given dimensionality, L4 shows how we can work out $M(E)$.

For a numerical table of $E(k)$, the prescription for determining $M(E)$ is described in L17.
$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$

$\lambda$ is the “mean-free-path for backscattering”

1) Ballistic: $\lambda \gg L$, $T \approx 1$

$G = \frac{2q^2}{h} M(E_F)$

2) Diffusive: $\lambda \ll L$, $T \ll 1$

$G = \frac{2q^2}{h} M(E_F) \frac{\lambda(E_F)}{L}$

$G = \frac{2q^2}{h} T(E_F) M(E_F)$

Explains why current $\sim 1/L$ (1D), $W/L$ (2D), and $A/L$ (3D)
mobility and diffusion coefficient

\[ G = \frac{2q^2}{\hbar} \int T(E)M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE = \frac{2q^2}{\hbar} \left\langle T(E_F)M(E_F) \right\rangle \]

Conventionally: \( G = nq\mu_n \)

For \( T = 0K \):

\[ \mu_n = \frac{q\tau(E_F)}{m^*} \]

For non-degenerate conditions:

\[ \mu_n = \frac{D_n}{k_BT/q} \]

\[ D_n = \frac{\nu T \langle \lambda \rangle}{2} \]

\[ \nu_T = \sqrt{\frac{2k_BT}{\pi m^*}} \]
three ways to write the diffusive conductance

For $T = 0K$: 

$$G = \frac{2q^2}{h} T(E_F)M(E_F) = \frac{2q^2}{h} \frac{\lambda(E_F)}{L} M(E_F)$$

For 2D, diffusive, $T = 0K$: 

$$G_{2D} = \sigma S \frac{W}{L}$$

1) \( \sigma S = \frac{2q^2}{h} \lambda(E_F)M(E_F)/W \)

2) \( \sigma S = n_S q \mu_n \quad (\mu_n = q \tau(E_F)/m^*) \)

3) \( \sigma S = q^2 D_{2D}(E_F)D_n(E_F) \quad (D_n(E_F) = v^2(E_F)\tau(E_F)/2) \)
L10: Landauer and the DD equation

\[ J_n = -nq \mu_n \frac{dV}{dx} + qD_n \frac{dn}{dx} \]

\[ I_n = - \left\{ \frac{2q^2}{h} \int T(E)M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \right\} \Delta V \quad T(E) = \frac{\lambda(E)}{\lambda(E) + L} \approx \frac{\lambda(E)}{L} \]

\[ J_n = \frac{I_n}{A} = - \left\{ \frac{2q^2}{h} \int T(E) \frac{M(E)}{A} \left( -\frac{\partial f_0}{\partial E} \right) dE \right\} \frac{\Delta V}{L} \quad \Delta F_n = -q\Delta V \]

\[ J_n = \sigma_n \frac{d \left( \frac{F_n}{q} \right)}{dx} \bigg|_T \quad \sigma_n = \frac{2q^2}{h} \int \frac{\lambda(E)M(E)}{A} \left( -\frac{\partial f_0}{\partial E} \right) dE \]
The DD equation

\[ J_n = \sigma_n \frac{d(F_n/q)}{dx} \bigg|_T \]
\[ \sigma_n = \frac{2q^2}{h} \int \frac{\lambda(E) M(E)}{A} \left( -\frac{\partial f_0}{\partial E} \right) dE \]

\[ n(x) = N_C e^{(E_n(x) - E_C(x))/k_B T} \]  
(Boltzmann statistics)

\[ J_n = nq\mu_n \mathcal{E}_x + k_B T \mu_n \frac{dn}{dx} \]

The Landauer approach also gives us a derivation of the DD equation and an understanding of its underlying assumptions.
L8: driving forces for current flow

\[ I = \frac{2q}{h} \int T(E)M(E)(f_1 - f_2) \, dE \]

Anything that causes a difference in Fermi functions leads to current flow.

1) Differences in Fermi level (caused by differences in voltage)

2) Differences in temperature.

For small difference (linear transport):

\[ (f_1 - f_2) \approx \left( -\frac{\partial f_0}{\partial E} \right) q \Delta V - \left( -\frac{\partial f_0}{\partial E} \right) \frac{(E - E_F)}{T} \Delta T \]
 electrons absorb thermal energy, \( E - E_{F1} \)

cold

contact 1

Net power dissipated: \( P_D = IV \)

hot

electrons dissipate energy, \( E - E_{F2} \)

bottom of CB

electrons enter contact 1 at the Fermi energy, \( E_{F1} \)

electrons leave contact 2 at the Fermi energy, \( E_{F2} \)

electrons

cold hot

\( E_{F2} = E_{F1} - qV \)
coupled current equations

\[ I = G \Delta V - [SG] \Delta T \]

(electrons carry charge)

\[ I_Q = T[SG] \Delta V - K_0 \Delta T \]

(electrons carry thermal energy)

**alternative form:**

\[ \Delta V = RI - S \Delta T \]

\[ I_Q = -\pi I - K_e \Delta T \]

- **R**: resistance
- **S**: Seebeck coefficient
- **\(\pi\)**: Peltier coefficient
- **\(K_e\)**: thermal conductance

\[ G = \frac{1}{R} = \left(\frac{2q^2}{h}\right)I_0 \]

\[ S = \left[ \frac{SG}{G} \right] = \left( \frac{-k_B}{q} \right) \frac{I_1}{I_0} \]

\[ \pi = TS \]

\[ K_e = \left( \frac{2k_B^2 T}{h} \right) \left[ I_2 - \frac{I_1^2}{I_0} \right] \]

\[ I_j = \int_{-\infty}^{+\infty} \left( \frac{E - E_F}{k_B T_L} \right)^j T(E)M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]
DD and thermoelectric effects

Just as the Landauer approach in the diffusive limit leads to a DD equation when $T$ is constant, we can do the same with temperature gradients.

\[ J_n = \sigma_n \frac{d(F_n/q)}{dx} + S_T \frac{dT}{dx} \]

\[ \mathcal{E}_x = \rho J_x + S \frac{dT}{dx} \quad S = S_T / \sigma_n \]

\[ J^Q_x = \pi J_x - \kappa_e \frac{dT}{dx} \quad \pi = TS \]

\[ S = \left( \frac{k_B}{-q} \right) \left[ \delta + \frac{E_C - E_F}{k_B T} \right] \]
The equilibrium distribution function

For low-bias transport, the distribution function is very nearly the equilibrium distribution.

\[ f_0 = \frac{1}{1 + e^{(E - E_F)/k_BT}} \]

or

\[ f_0 \approx e^{-(E - E_F)/k_BT} \ll 1 \]

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The non-equilibrium distribution function

To **find** the distribution function under bias, we should solve the Boltzmann Transport Equation (BTE).

After solving the BTE, we can find any quantity of interest:

\[
n_\phi (\vec{r}, t) = \sum_{\vec{p}} \phi(\vec{p}) f(\vec{r}, \vec{p}, t)
\]
L12: The Boltzmann Transport Equation

**semiclassical transport**

\[
\frac{d\left(\frac{\hbar}{k}\right)}{dt} = \vec{F}_e(\vec{r})
\]

\[
\vec{v}_g(t) = \frac{1}{\hbar} \nabla_k E\left[\vec{k}(t)\right]
\]

\[
\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{v}_g(t')dt'
\]

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_r f + \vec{F}_e \cdot \nabla_p f = \hat{C}f
\]

\[
\hat{C}f(\vec{r}, \vec{p}, t) = \sum_{p'} S(\vec{p}', \vec{p}) f(\vec{p}') \left[1 - f(\vec{p})\right]
\]

\[
- \sum_{p'} S(\vec{p}, \vec{p}') f(\vec{p}) \left[1 - f(\vec{p}')\right]
\]
solution to the BTE

\[
\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_r f + \vec{F}_e \cdot \nabla_p f = \hat{C}f
\]

1) Equilibrium: \( \hat{C}f (\vec{r}, \vec{p}, t) = 0 \)

*Fermi level and temperature are constant*

\[
\hat{C}f (\vec{r}, \vec{p}, t) = 0
\]

2) Ballistic:

*Each state is populated according to an equilibrium Fermi function.*

\[
n(x) = \int D_1(x_1, E) f_0(E_{F1}) + D_2(x_1, E) f_0(E_{F2}) dE
\]
filling states in a ballistic device

\[ E_F^2 = E_F^1 - qV_A \]
relaxation time approximation

\[ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_r f + \mathbf{F}_e \cdot \nabla_p f = \hat{C} f = -\left( \frac{f(\mathbf{p}) - f_0(\mathbf{p})}{\tau_f} \right) \]

The RTA can be justified for:

1) near-equilibrium conditions

2) isotropic or elastic scattering

\[ \tau_f = \tau_m \]
relaxation time approximation

\[
\frac{\partial f}{\partial t} + \vec{v} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \hat{C}f = -\left( \frac{f(\vec{p}) - f_0(\vec{p})}{\tau_f} \right)
\]

\[
f(\vec{p}) = f_0(\vec{p}) + q \tau_f \mathcal{E}_x \frac{\partial f_0}{\partial p_x}
\]

\[
J_n = nq \mu_n \mathcal{E}_x + qD_n \frac{dn}{dx}
\]

\[
D_n = \langle \nu_x^2 \tau_f \rangle
\]

\[
\mu_n = \frac{q\langle \tau_f \rangle}{m^*}
\]

\[
\langle X \rangle \equiv \frac{\sum_k X f_0(E)}{\sum_k f_0(E)}
\]

\[
\langle \langle \tau_f \rangle \rangle \equiv \frac{\langle E \tau_f(E) \rangle}{\langle E \rangle}
\]

\[
dp_x = -q \tau_0 \mathcal{E}_x
\]
BTE vs. Landauer

**BTE:**
- requires an $E(k)$ for the semi-classical treatment
- ”hard” to apply boundary conditions
- works best in the diffusive regime
- $B$-fields readily incorporated
- anisotropic transport readily treated
- can be mathematically complex

**Landauer:**
- does not require an $E(k)$
- readily treats small devices with idealize boundary conditions
- works from the diffusive to ballistic regime
- physically transparent
L17: BTE vs. Landauer (mathematics)

\[ G = \frac{2q^2}{h} I_0 \]

\[ I_0 = \int \Sigma(E) \left( -\frac{\partial f_s}{\partial E} \right) dE \]

\[ \Sigma(E) = \frac{h}{L^2} \sum_{\vec{k}} \nu_x^2 \tau_f \delta(E - E_k) \]

\[ T(E) = \frac{\lambda(E)}{L} \quad \lambda(E) = 2 \frac{\langle \nu_x^2 \tau_f \rangle}{\langle |\nu_x| \rangle} \]

\[ M(E) = \frac{h}{2L} \sum_k |\nu_x| \delta(E - E_k) \]

\[ \langle \bullet \rangle = \frac{\sum_{\vec{k}} \langle \bullet \rangle \delta(E - E_k)}{\sum_{\vec{k}} \delta(E - E_k)} \]
B-fields

\[ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_r f + \mathbf{F}_e \cdot \nabla_p f = \mathbf{Cf} = - \left( \frac{f(\mathbf{p}) - f_0(\mathbf{p})}{\tau_f} \right) \]

\[ \mathbf{F}_e = -q\mathbf{E} - q\mathbf{v} \times \mathbf{B} \]

\[ J_i = \sigma_{ij}(\mathbf{B}) \mathbf{E}_j \]

\[ \sigma_{ij}(\mathbf{B}) = \sigma_0 \begin{bmatrix} 1 & -\mu_H B_z \\ +\mu_H B_z & 1 \end{bmatrix} \]  \hspace{1cm} (2D)

\[ \mathbf{J} = \sigma_0 \mathbf{E} - \sigma_0 \mu_H \mathbf{E} \times \mathbf{B} \]

(Hall effect)
L18: Strong B-fields

\[ \sigma_{ij}(B_z) = \frac{n q \mu_n}{1 + \mu_n B_z} \begin{bmatrix} 1 & -\mu_n B_z \\ \mu_n B_z & 1 \end{bmatrix} \]

1) A magnetic field affects both the diagonal and off-diagonal components of the magneto-conductivity tensor.

2) Small magnetic field means: \( \mu_n B_z \ll 1 \) \( \omega_c \tau \ll 1 \) \( \omega_c = \frac{q B_z}{m^*} \) (parabolic bands)

3) Landau levels develop. \( E_n = \left( n + \frac{1}{2} \right) \hbar \omega_c \)
scattering

1) Landauer: \( T(E) = \frac{\lambda(E)}{\lambda(E) + L} \)

1) How is this equation derived?
2) How is mfp for backscattering related to the scattering time

\( \lambda(E) \propto \nu(E) \tau(E) \)

2) BTE: \( \hat{C}f = \sum_{\vec{p}', \vec{p}} S(\vec{p}', \vec{p}) f(\vec{p}') [1 - f(\vec{p})] - \sum_{\vec{p}} S(\vec{p}, \vec{p}') f(\vec{p}) [1 - f(\vec{p}')] \)

How is the transition rate, \( S(p, p') \) computed?

3) General:

How do we simply describe the physical effects of scattering
L19: characteristic times

\[
\frac{1}{\tau(\bar{p})} = \sum_{\bar{p}',\uparrow} S(\bar{p}, \bar{p}')
\]

\[
\frac{1}{\tau_m(\bar{p})} = \sum_{\bar{p}',\uparrow} S(\bar{p}, \bar{p}') \frac{\Delta p_z}{p_z}
\]

\[
\frac{1}{\tau_E(\bar{p})} = \sum_{\bar{p}',\uparrow} S(\bar{p}, \bar{p}') \frac{\Delta E}{E_0}
\]

\[
\bar{p}(t = 0)
\]

\[
t = 0
\]

\[
t \approx \tau
\]

\[
t \approx \tau_m \geq \tau
\]

\[
t \approx \tau_E > \tau_m \geq \tau
\]
L21: Fermi’s Golden Rule

\[ S(p, p') = \frac{2\pi}{\hbar} |H_{p'p}|^2 \delta(E' - E - \Delta E) \]

\[ H_{p',p} = \int_{-\infty}^{+\infty} \psi_f^* U_S(\vec{r}) \psi_i d\vec{r} \]

\[ E' = E_0 + \Delta E \]

\[ \Delta E = 0 \text{ for a static } U_S \]

\[ \Delta E = \pm \hbar \omega \text{ for an oscillating } U_S \]

\[ \frac{1}{\tau(p)} = \sum_{p'\uparrow} S(p, p') \propto D_f(E) \]

For an electron with energy, \( E \), its scattering rate is proportional to the density of final states at energy, \( E \) (1D, 2D, 3D).
scattering in semiconductors

- ionized impurities
- neutral impurities
- dislocations
- surface roughness
- alloy

- electron-electron
- electron-plasmon
- electron-hole

- intravalley
  - ADP
  - ODP
  - POP
  - PZ

- intervalley
  - acoustic
  - optical
covalent vs. polar semiconductors

covalent

\[ E \]

\[ \langle 100 \rangle \]

\[ \Gamma \]

ADP + equivalent IV

\[ E \]

\[ 1.0 \]

polar

\[ E \]

\[ \langle 100 \rangle \]

\[ \langle 111 \rangle \]

\[ \Gamma \]

POP EMS ABS IV

\[ E \]

\[ 0.30 \]

\[ 0.03 \]

\[ 1.0 \]
L26: scattering in quantum wells

\[
\frac{1}{\tau_{n,n'}} = \frac{2\pi}{\hbar} U_{ac} \frac{D_{2D}(E)}{2} \left(\frac{2 + \delta_{n,n'}}{2}\right) = \Gamma_0 \left(\frac{2 + \delta_{n,n'}}{2}\right)
\]

\[
\frac{1}{\tau_{1,n'}}(E) = \begin{cases} 
7\Gamma_0/2 & \text{if } E = 1 \\
5\Gamma_0/2 & \text{if } E = 4 \\
3\Gamma_0/2 & \text{if } E = 9 
\end{cases}
\]

Note: energy is referenced to the bottom of the first subband.
The quantities of interest to device researchers are moments of the distribution function:

\[ n_\phi (\vec{r}, t) = \sum_{\vec{p}} \phi (\vec{p}) f (\vec{r}, \vec{p}, t) \]

These quantities satisfy a continuity equation:

\[ \frac{\partial n_\phi}{\partial t} = -\nabla \cdot \vec{F}_\phi + G_\phi - R_\phi \]

A clear prescription for generating a continuity (or balance) equation exists, but simplifying the resulting equations for use in practice is an art.
example: the drift-diffusion equation again

\[ \phi(\vec{p}) = (-q) \frac{p_i}{m^*} \quad n_\phi(\vec{r}, t) = J_{ni}(\vec{r}, t) \]

\[ \vec{J}_n + \langle \tau_m \rangle \frac{\partial \vec{J}_n(\vec{r}, t)}{\partial t} = nq \mu_n \vec{E} + 2\mu_n \nabla \cdot \vec{W} \]

\[ \mu_n = \frac{q\langle \tau_m \rangle}{m^*} \quad W_{ij} = n\left\langle \frac{p_i \nu_j}{2} \right\rangle \]

assume:

i) \( W \) is diagonal
ii) near-equilibrium conditions
ii) slow variations in time and space

DD equation
L29: four balance equations

\[
\frac{\partial n(\vec{r},t)}{\partial t} = -d\left[ \frac{J_{nx}}{(-q)} \right] dx
\]

0th moment of BTE

\[
\langle \tau_m \rangle \frac{\partial J_{nx}(\vec{r},t)}{\partial t} + J_{nx} = nq \mu \mathcal{E}_x + 2\mu \frac{dW_{xx}}{dx}
\]

1st moment of BTE

\[
\frac{\partial W(\vec{r},t)}{\partial t} = -\frac{dF_{Wx}}{dx} + J_{nx} \mathcal{E}_x - \frac{(W-W_0)}{\langle \tau_E \rangle}
\]

2nd moment of BTE

\[
\langle \tau_{Fw} \rangle \frac{\partial F_W(\vec{r},t)}{\partial t} + F_{Wx} = -3n \frac{q}{m^*} u \mathcal{E}_x - \langle \tau_{Fw} \rangle \frac{dX_{xx}}{dx}
\]

3rd moment of BTE
1) “free flight” for $t_C$ seconds.
2) update $E(t_C^-)$ and $r(t_C^-)$
3) identify collision
4) update $E(t_C^+)$ and $p(t_C^+)$
5) Set $t = 0$ and repeat

1) free flights: semi-classical equations of motion
2) scattering: quantum mechanical transition rate $S(p, p')$
L32: hot carrier transport

\[ J_{nx} E_x = \frac{n(u - u_0)}{\langle \tau_E \rangle} \]

\[ \nu_{dx} = -\mu_n E_x = -\frac{q\langle \tau_m \rangle}{m^*} E_x \]

\[ \langle \tau_m \rangle \downarrow \quad \text{as} \quad u \uparrow \]
covalent vs. polar semiconductors

![Covalent vs. Polar Semiconductors Diagram](image)
field-dependent DD equation

\[ J_{nx} = nq\mu_n E_x + qD_n \frac{dn}{dx} \quad \mu_n = q\langle \tau_m \rangle / m^* \quad D_n / \mu_n = 2u_{xx} / q \]

**Goal:** Find mobility and diffusion coefficient without solving BTE

In general, however:

\[ \mu_n \left[ f(\vec{r}, \vec{p}, t) \right] \quad D_n \left[ f(\vec{r}, \vec{p}, t) \right] \]

In a bulk semiconductor, \( f \) is determined by \( E \), so there is a one-to-one mapping between \( E \) and \( f \).

Electric **field dependent** mobility and diffusion coefficient.

\[ J_{nx} = nq\mu_n(E)E_x + qD_n(E)\frac{dn}{dx} \]
The concept of a field-dependent mobility applies only when the electric field changes slowly with position.
VO occurs in the presence of scattering when the energy relaxation time is longer than the momentum relaxation time.
non-local transport in nanoscale MOSFETs

Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992
ballistic vs. diffusive transport

(source) $E_{F1}$ $X$ $X$ $X$ $X$ $X$

(diffusive) $W$ $f_1(E)$ $f_2(E)$

(ballistic) $W$ $f_1(E)$ $f_2(E)$

(scattering from boundaries assumed to be negligible)
Local density-of-states

\[ D_{3D}(E) = \frac{1}{\Omega} \sum_k \delta(E - E_k) \]

Local density of states (L13):

\[ D_i(E, x) \]

\( i = 1 \) or \( 2 \) for contact 1 or 2

In a ballistic device (L3):

\[ N(x) = \int \left[ D_1(E, x) f_1(E) + D_2(E, x) f_2(E) \right] dE \]
quantum transport

\[ D_{3D}(E) = \frac{1}{\Omega} \sum_k \delta(E - E_k) \]

\[ \rightarrow \psi^* (\vec{r}) \psi (\vec{r}) \sum_k \delta(E - E_k) \]

source + drain-injected LDOS in a carbon nanotube MOSFET

See: “Physics of Nanoscale MOSFETs,” NCN Summer School, July 2008
http://nanohub.org/resources/5306
quantum transport

source + drain-injected LDOS in a carbon nanotube MOSFET
the drift-diffusion equation

\[ \vec{J}_p = -pq \mu_p \vec{\nabla} V - qD_p \vec{\nabla} p \]

1) Still describes transport in a very large number of cases

2) ECE-656 has taught you how to relate mobility and diffusion coefficient to material parameters and it suggests how we can engineer these parameters with strain and quantum confinement.

3) We have also learned about the assumptions underlying the DD equation – (basically slow variations in time and space and applications to devices that are many mfp’s long).

4) We also learned that some problems cannot be treated by the DD equation – ballistic transport, non-local semiclassical transport, and quantum transport.

5) Finally, we learned some new techniques and how they related to the DD equations (Landauer approach, BTE, and an introduction to quantum transport.)