ECE-656: Fall 2009

Lecture 36: The Course in One Lecture

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"the semiconductor equations"

Conservation Laws:

$$\nabla \bullet \vec{D} = \rho$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\vec{J}_n / -q \right) + \left(G_n - R_n \right)$$

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\vec{J}_p / \; q \right) + \left(G_p - R_p \right)$$

Constitutive Relations:

$$\vec{D} = \kappa \varepsilon_0 \vec{\mathcal{E}} = -\kappa \varepsilon_0 \vec{\nabla} V$$

$$\rho = q\left(p - n + N_D^+ - N_A^- \right)$$

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n$$

$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

$$R = f(n, p)$$

etc.

“the drift-diffusion equation”

$$\vec{J}_p = -pq\mu_p \vec{\nabla} V - qD_p \vec{\nabla} p$$

1) How is the DD equation derived?

2) What determines the mobility and diffusion coefficient?

3) What physics does it miss?

4) How do we describe transport without the DD equation?

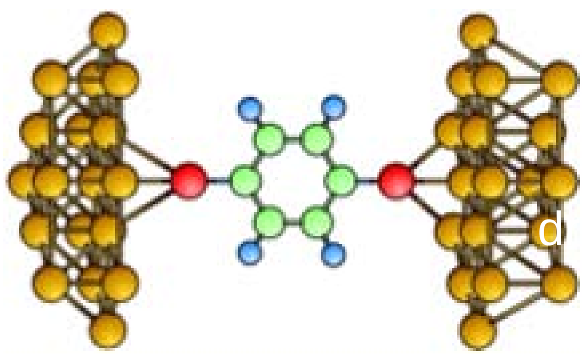
“bottom-up” vs. “top down”

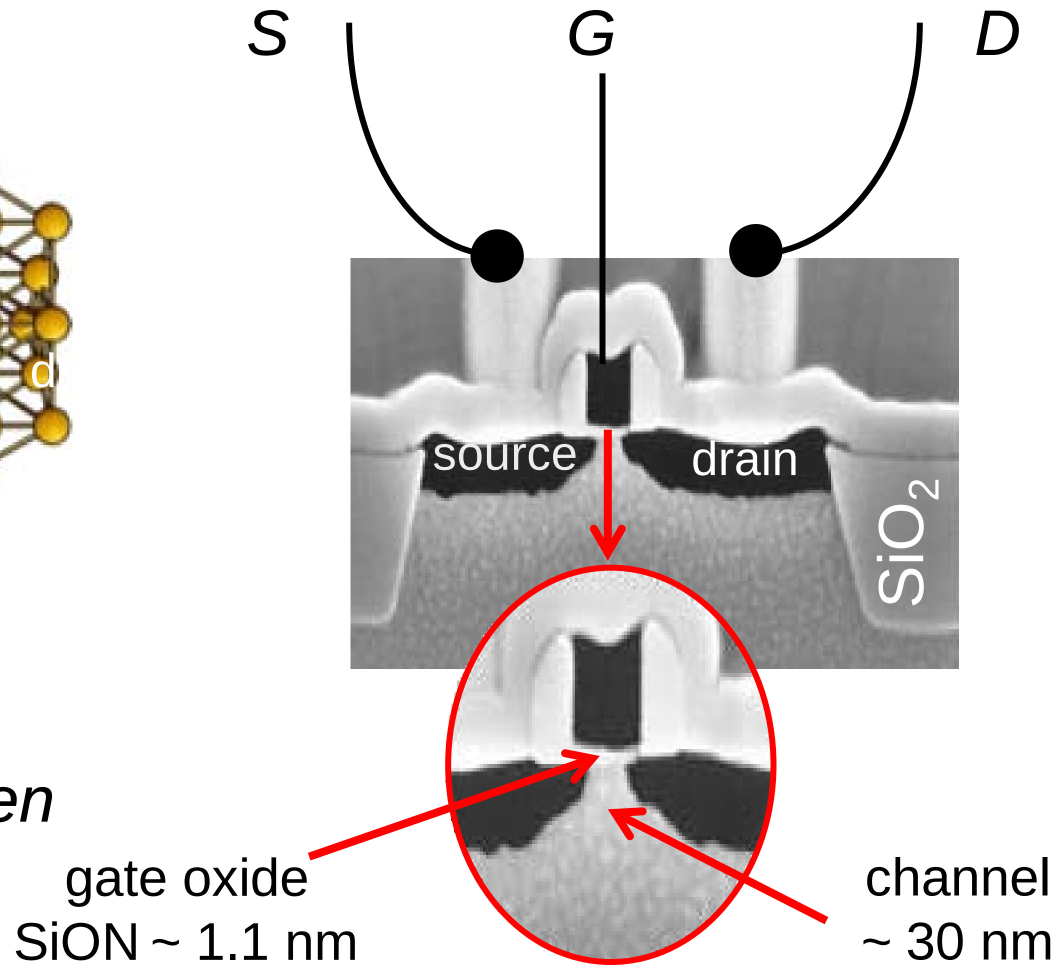
$$\vec{J}_p = -pq\mu_p \vec{\nabla} V - qD_p \vec{\nabla} p$$

Historically, the DD equation was developed to describe transport for small gradients in concentration and potential in structures that were large compared to the mean-free-path for scattering. As time went on, it was extended to describe smaller and smaller structures.

In ECE-656, we turned this around and started by examining transport in small structures, and then we worked up to larger structures.

current at the nanoscale





If we apply a bias between the two contacts, what current flows?

L3: Landauer model

$$I = \frac{2q}{h}\int T(E)M(E)\left(f_1 - f_2\right)dE$$

1) A difference in Femi functions cause current to flow

2) *M*(*E*) density of conducting channels

3) *T*(*E*): transmission (0 < *T* < 1)

4) Important assumptions:

-contacts are “ideal” (absorbing, in equilibrium)

-inelastic scattering only in contacts

conductance

$$I = \frac{2q}{h}\int T(E)M(E)\left(f_1 - f_2\right)dE$$

I > 0 when $V_2 > V_1$ because $f_1 > f_2$: $E_{F2} = E_{F1} - q\left(V_2 - V_1\right)$

When $(V_2 - V_1)$ is small, $f_1 \approx f_2 \approx f_0$ $\left(f_1 - f_2\right) \approx -\frac{\partial f_0}{\partial E}(q\Delta V)$

$$G = \frac{I}{\Delta V} = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad \frac{2q^2}{h} = \frac{1}{12.7\,k\Omega}$$

L5-7: nanoresistors

$$G = \frac{1}{R} = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad \frac{2q^2}{h} = \frac{1}{12.7\,k\Omega}$$

For T = 0K or for strongly degenerate systems, $\left(-\partial f_0/\partial E\right) \approx \delta\left(E_F\right)$

$$G = \frac{2q^2}{h}T\left(E_F\right)M\left(E_F\right)$$

For ballistic conductors, T = 1.

$M(E)$ is the number of conducting channels at energy, E.

L4: Density of states / Density of modes

Carrier densities are determined by the density of states.

Current flow is determined by the density of modes.

To determine $M(E)$:

1D: simply count the subbands

2D: $M(E)$ ~ width of the resistor, W

3D: $M(E)$ ~ cross-sectional area, A

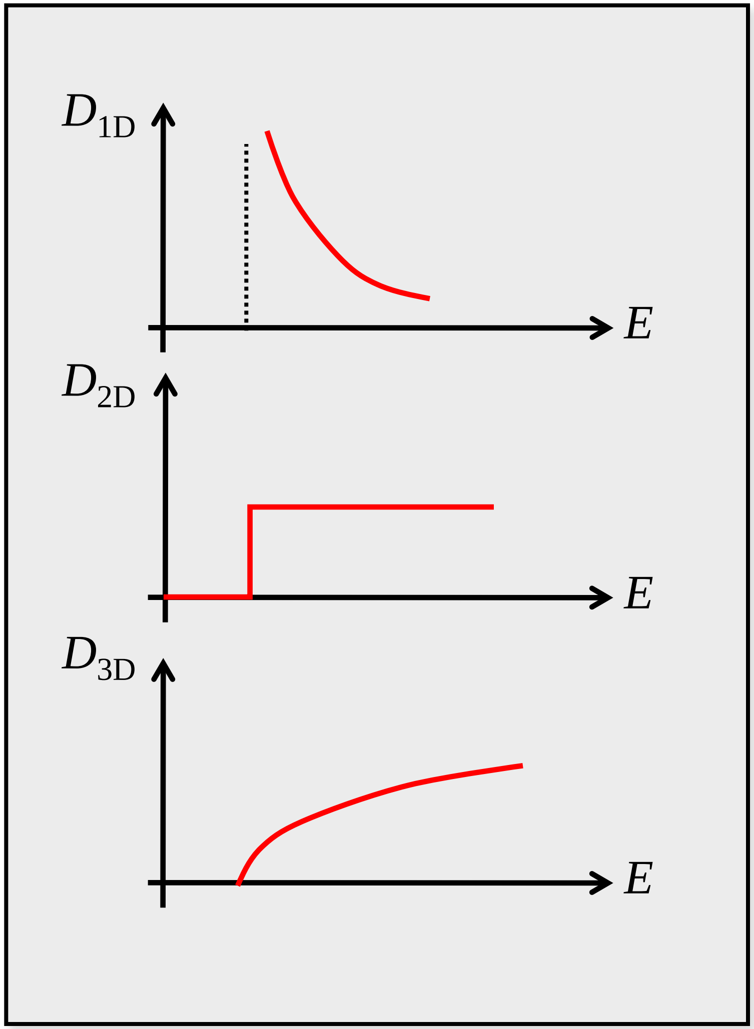
DOS depends on bandstructure and dimensionality

$$D_{1D}(E) = \frac{L}{\pi\hbar}\sqrt{\frac{2m^*}{(E-\varepsilon_1)}}\,\Theta(E-\varepsilon_1)$$

$$D_{2D}(E) = A\frac{m^*}{\pi\hbar^2}\Theta(E-\varepsilon_1)$$

$$D_{3D}(E) = \Omega\frac{m^*\sqrt{2m^*(E-E_C)}}{2\pi^2\hbar^3}$$

$$\left(E(k) = E_C + \hbar^2k^2/2m^*\right)$$



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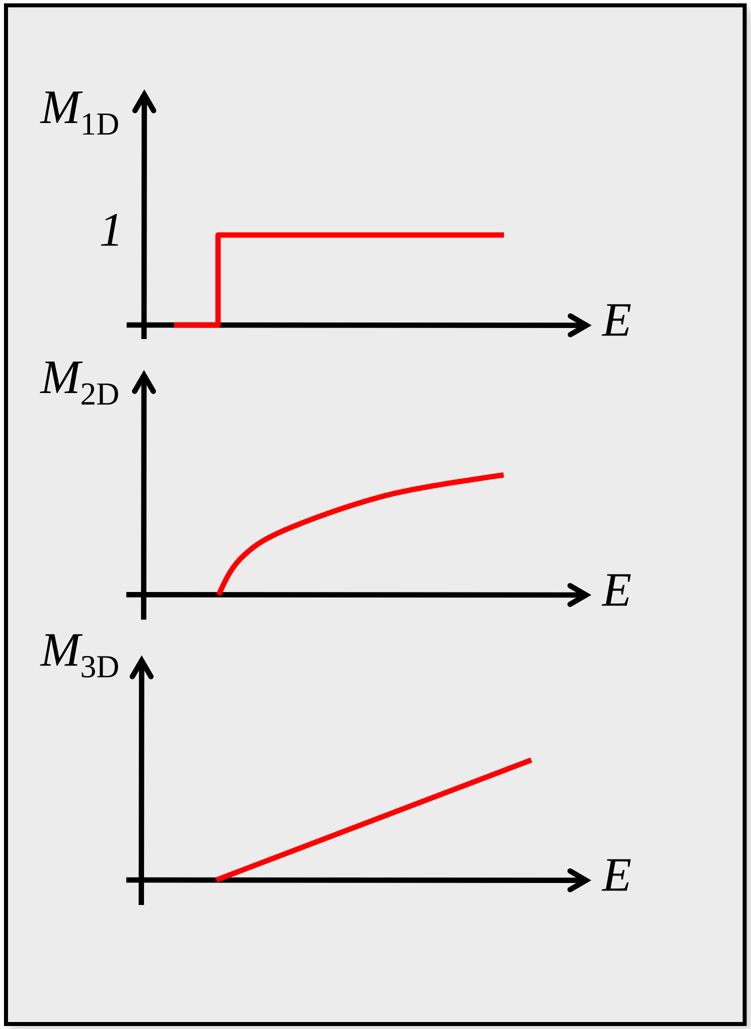
DOM depends on bandstructure and dimensionality

$$M_{1D}(E) = \Theta(E - \varepsilon_1)$$

$$M_{2D}(E) = W \frac{\sqrt{2m^*(E - \varepsilon_1)}}{\pi\hbar}$$

$$M_{3D}(E) = A \frac{m^*}{2\pi\hbar^2}(E - E_C)$$

$$\left(E(k) = E_C + \hbar^2 k^2 / 2m^*\right)$$



DOM and bandstructure

For a simple $E(k)$, and given dimensionality, L4 shows how we can work out M(E).

For a numerical table of $E(k)$, the prescription for determining $M(E)$ is described in L17.

ballistic vs. diffusive

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$ λ is the "mean-free-path for backscattering"

1) Ballistic : $\lambda >> L, \; T \approx 1$ $$G = \frac{2q^2}{h} M(E_F)$$

2) Diffusive : $\lambda << L, \; T << 1$ $$G = \frac{2q^2}{h} M(E_F) \frac{\lambda(E_F)}{L}$$

$$G = \frac{2q^2}{h} T(E_F) M(E_F)$$

Explains why current ~ 1/*L* (1D), *W*/*L* (2D), and *A*/*L* (3D)

mobility and diffusion coefficient

$$G = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE = \frac{2q^2}{h}\left\langle T(E_F)M(E_F)\right\rangle$$

Conventionally: $G = nq\mu_n$

For T = 0K:

$$\mu_n = \frac{q\tau(E_F)}{m^*}$$

For *non-degenerate conditions:*

$$\mu_n = \frac{D_n}{k_B T/q}$$

$$D_n = \frac{\upsilon_T\langle\lambda\rangle}{2} \qquad \upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$

three ways to write the diffusive conductance

For T = 0K: $$G = \frac{2q^2}{h} T(E_F) M(E_F) = \frac{2q^2}{h} \frac{\lambda(E_F)}{L} M(E_F)$$

For 2D, diffusive, T = 0K: $$G_{2D} = \sigma_S \frac{W}{L}$$

1) $$\sigma_S = \frac{2q^2}{h} \lambda(E_F) M(E_F) / W$$

2) $$\sigma_S = n_S q \mu_n \qquad \left(\mu_n = q\tau(E_F)/m^*\right)$$

3) $$\sigma_S = q^2 D_{2D}(E_F) D_n(E_F) \qquad \left(D_n(E_F) = \upsilon^2(E_F)\tau(E_F)/2\right)$$

L10: Landauer and the DD equation

$$J_n = -nq\mu_n \frac{dV}{dx} + qD_n \frac{dn}{dx}$$

$$I_n = -\left\{ \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right\} \Delta V \qquad T(E) = \frac{\lambda(E)}{\lambda(E) + L} \approx \frac{\lambda(E)}{L}$$

$$J_n = \frac{I_n}{A} = -\left\{ \frac{2q^2}{h} \int T(E) \frac{M(E)}{A} \left(-\frac{\partial f_0}{\partial E} \right) dE \right\} \frac{\Delta V}{L} \qquad \Delta F_n = -q\Delta V$$

$$J_n = \sigma_n \left. \frac{d\left(F_n/q\right)}{dx} \right|_T \qquad \sigma_n = \frac{2q^2}{h} \int \lambda(E) \frac{M(E)}{A} \left(-\frac{\partial f_0}{\partial E} \right) dE$$

the DD equation

$$J_n = \sigma_n \left. \frac{d\left(F_n/q\right)}{dx} \right|_T \qquad \sigma_n = \frac{2q^2}{h} \int \lambda(E) \frac{M(E)}{A} \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$n(x) = N_C\, e^{\left(F_n(x) - E_C(x)\right)/k_B T}$$

(Boltzmann statistics)

$$J_n = nq\mu_n \mathcal{E}_x + k_B T \mu_n \frac{dn}{dx}$$

The Landauer approach also gives us a derivation of the DD equation and an understanding of its underlying assumptions.

L8: driving forces for current flow

$$I = \frac{2q}{h}\int T(E)M(E)\left(f_1 - f_2\right)dE$$

Anything that causes a difference in Fermi functions leads to current flow.

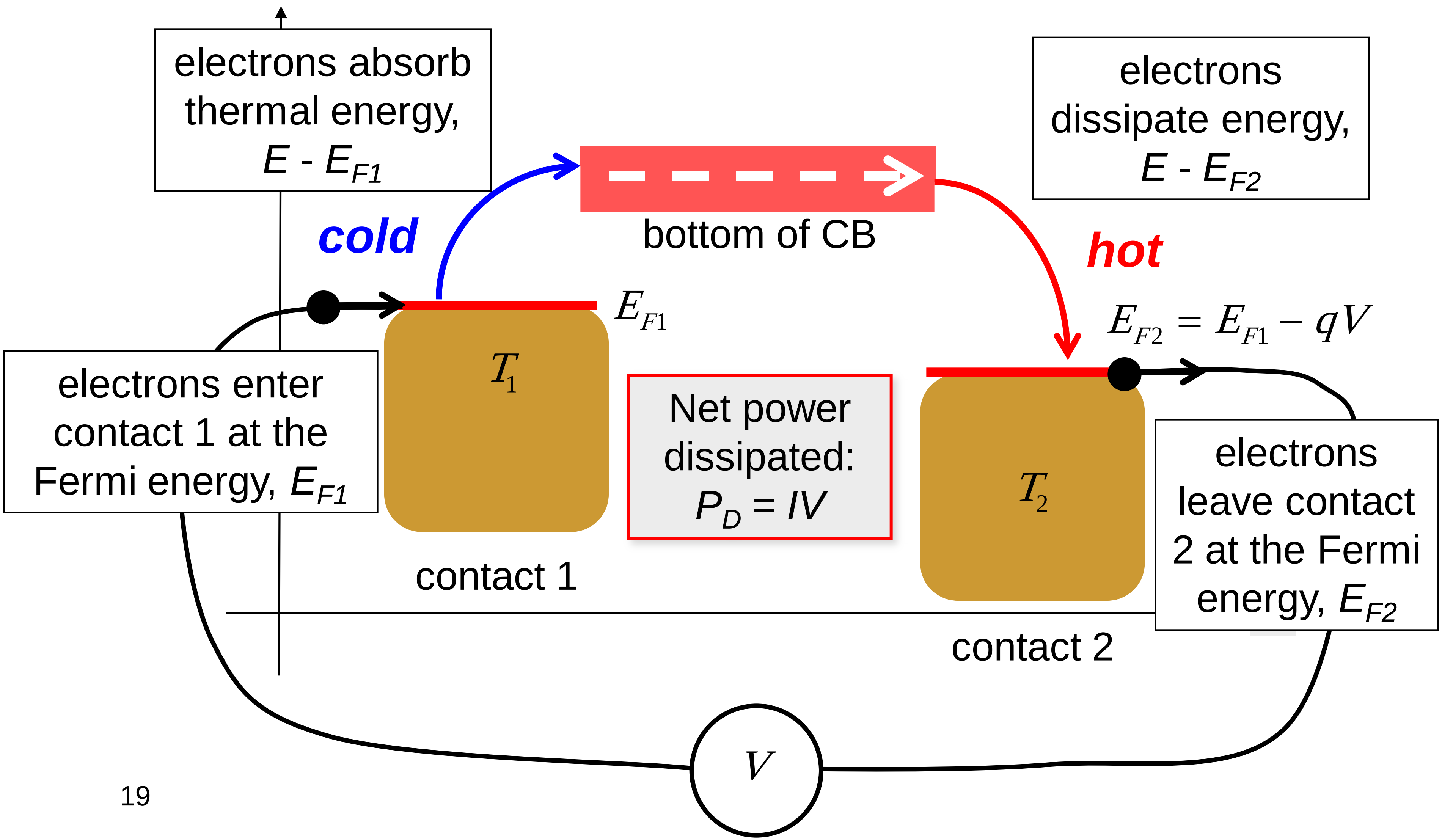
1) Differences in ***Fermi level*** (caused by differences in voltage)

2) Differences in ***temperature.***

For small difference (linear transport):

$$\left(f_1 - f_2\right) \approx \left(-\frac{\partial f_0}{\partial E}\right) q\Delta V - \left(-\frac{\partial f_0}{\partial E}\right)\frac{\left(E - E_F\right)}{T}\Delta T$$

physics of the Peltier effect



coupled current equations

$$I = G\Delta V - [SG]\Delta T$$

(electrons carry charge)

$$I_Q = T[SG]\Delta V - K_0 \Delta T$$

(electrons carry thermal energy)

alternative form:

$$\Delta V = RI - S\Delta T$$

$$I_Q = -\pi I - K_e \Delta T$$

R: resistance
S: Seebeck coefficient
π: Peltier coefficient
K_e: thermal conductance

$$G = 1/R = \left(2q^2/h\right) I_0$$

$$S = -\frac{[SG]}{G} = \left(-\frac{k_B}{q}\right)\frac{I_1}{I_0}$$

$$\pi = TS$$

$$K_e = \left(\frac{2k_B^2 T}{h}\right)\left[I_2 - \frac{I_1^2}{I_0}\right]$$

$$I_j = \int_{-\infty}^{+\infty}\left(\frac{E - E_F}{k_B T_L}\right)^j T(E) M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

DD and thermoelectric effects

Just as the Landauer approach in the diffusive limit leads to a DD equation when *T* is constant, we can do the same with temperature gradients.

$$J_n = \sigma_n \frac{d\left(F_n/q\right)}{dx} + S_T \frac{dT}{dx}$$

$$\mathcal{E}_x = \rho J_x + S\frac{dT}{dx} \qquad S = S_T/\sigma_n$$

$$J_x^Q = \pi J_x - \kappa_e \frac{dT}{dx} \qquad \pi = TS$$

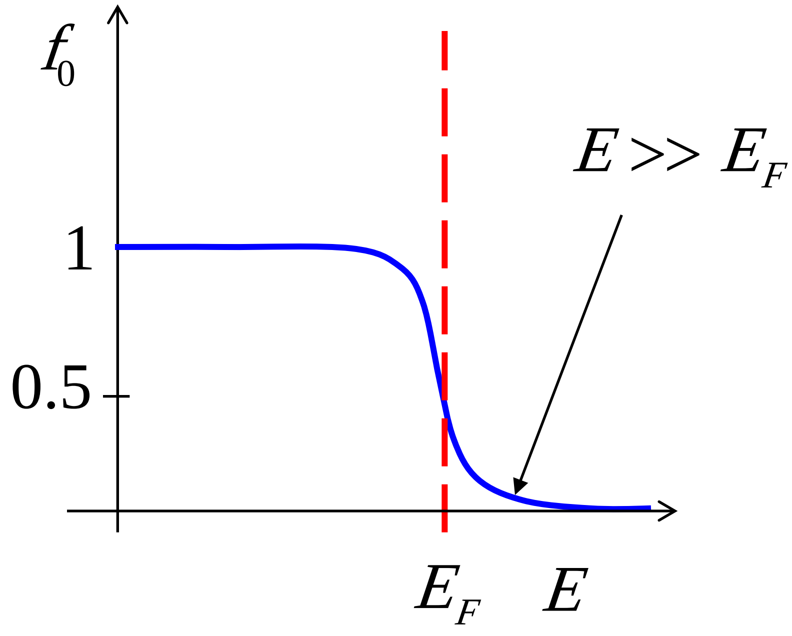
$$S = \left(\frac{k_B}{-q}\right)\left[\delta + \frac{E_C - E_F}{k_B T}\right]$$

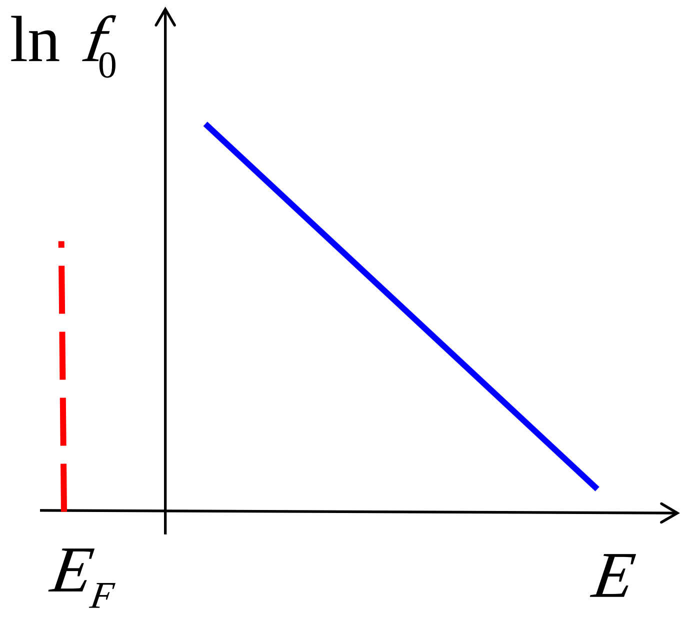
The equilibrium distribution function

For low-bias transport, the distribution function is very nearly the equilibrium distribution.

$$f_0 = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

$$f_0 \approx e^{-(E - E_F)/k_B T} << 1$$





(nondegenerate)

The non-equilibrium distribution function

To ***find*** the distribution function under bias, we should solve the Boltzmann Transport Equation (BTE).

After solving the BTE, we can find any quantity of interest:

$$n_\phi(\vec{r},t) = \sum_{\vec{p}} \phi(\vec{p}) f(\vec{r},\vec{p},t)$$

L12: The Boltzmann Transport Equation

semiclassical transport

$$\frac{d\left(\hbar\vec{k}\right)}{dt} = \vec{F}_e(\vec{r})$$

$$\vec{\upsilon}_g(t) = \frac{1}{\hbar}\nabla_k E\left[\vec{k}\left(t\right)\right]$$

$$\vec{r}\left(t\right) = \vec{r}\left(0\right) + \int_0^t \vec{\upsilon}_g(t')dt'$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \hat{C}f$$

$$\hat{C}f\left(\vec{r},\vec{p},t\right) = \sum_{p'} S\left(\vec{p}',\vec{p}\right) f\left(\vec{p}'\right)\left[1 - f\left(\vec{p}\right)\right]$$
$$- \sum_{p'} S\left(\vec{p},\vec{p}'\right) f\left(\vec{p}\right)\left[1 - f\left(\vec{p}'\right)\right]$$

solution to the BTE

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \hat{C} f$$

1) Equilibrium: $\hat{C} f \left(\vec{r}, \vec{p}, t\right) = 0$

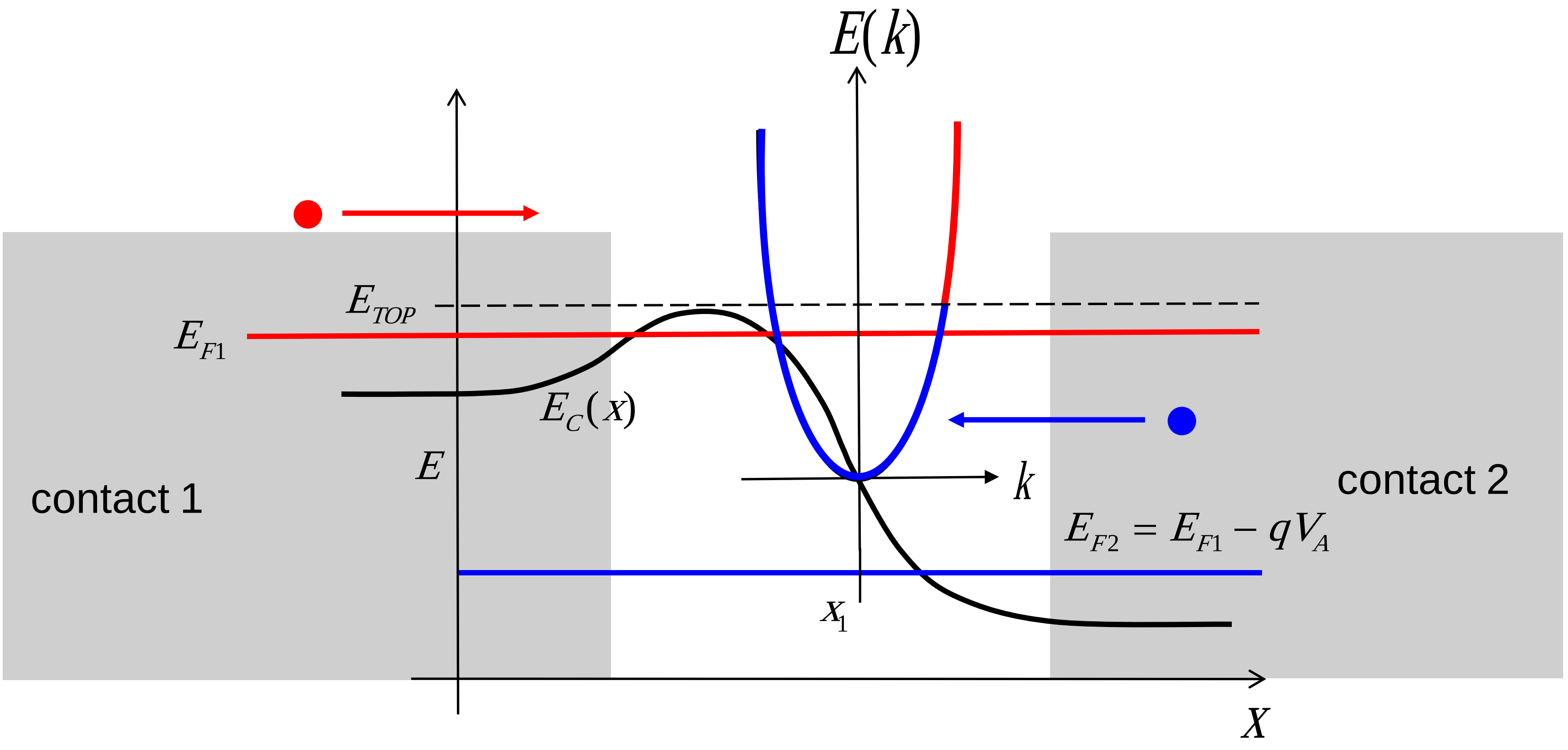
Fermi level and temperature are constant

2) Ballistic: $\hat{C} f \left(\vec{r}, \vec{p}, t\right) = 0$

Each state is populated according to an equilibrium Fermi function.

$$n(x) = \int D_1(x_1, E) f_0(E_{F1}) + D_2(x_1, E) f_0(E_{F2}) dE$$

filling states in a ballistic device



relaxation time approximation

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \hat{C}f = -\left(\frac{f(\vec{p}) - f_0(\vec{p})}{\tau_f} \right)$$

The RTA can be justified for:

1) near-equilibrium conditions

2) isotropic **or** elastic scattering

$$\tau_f = \tau_m$$

relaxation time approximation

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \hat{C} f = -\left(\frac{f(\vec{p}) - f_0(\vec{p})}{\tau_f} \right)$$

$$f(\vec{p}) = f_0(\vec{p}) + q \tau_f \mathcal{E}_x \frac{\partial f_0}{\partial p_x}$$

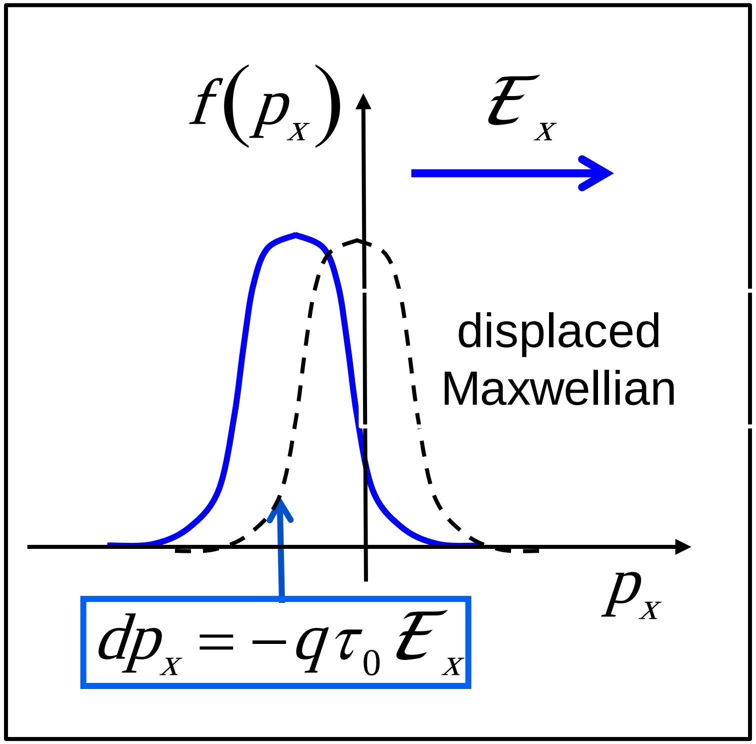
$$J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx}$$

$$D_n = \langle \upsilon_x^2 \tau_f \rangle$$

$$\mu_n = \frac{q \langle\langle \tau_f \rangle\rangle}{m^*}$$

$$\langle X \rangle \equiv \frac{\sum_k X f_0(E)}{\sum_k f_0(E)}$$

$$\langle\langle \tau_f \rangle\rangle \equiv \frac{\langle E \tau_f(E) \rangle}{\langle E \rangle}$$



BTE vs. Landauer

BTE:

-requires an $E(k)$ for the semi-classical treatment
-"hard" to apply boundary conditions
-works best in the diffusive regime
-B-fields readily incorporated
-anisotropic transport readily treated
-can be mathematically complex

Landauer:

-does not require an $E(k)$
-readily treats small devices with idealize boundary conditions
-works from the diffusive to ballistic regime
-physically transparent

L17: BTE vs. Landauer (mathematics)

$$G = \frac{2q^2}{h} I_0$$

$$I_0 = \int \Sigma(E)\left(-\frac{\partial f_S}{\partial E}\right) dE$$

$$\Sigma(E) = \frac{h}{L^2} \sum_{\vec{k}} \upsilon_x^2 \tau_f \delta(E - E_k)$$

$$G = \frac{2q^2}{h} I_0$$

$$I_0 = \int T(E) M(E)\left(-\frac{\partial f_S}{\partial E}\right) dE$$

$$\Sigma(E) = T(E) M(E)$$

$$T(E) = \frac{\lambda(E)}{L}$$

$$\lambda(E) \equiv 2\frac{\langle \upsilon_x^2 \tau_f \rangle}{\langle |\upsilon_x| \rangle}$$

$$\langle \bullet \rangle = \frac{\sum_{\vec{k}} (\bullet) \delta(E - E_k)}{\sum_{\vec{k}} \delta(E - E_k)}$$

$$M(E) = \frac{h}{2L} \sum_{\vec{k}} |\upsilon_x| \delta(E - E_k)$$

B-fields

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \hat{C} f = -\left(\frac{f(\vec{p}) - f_0(\vec{p})}{\tau_f} \right)$$

$$\vec{F}_e = -q\vec{\mathcal{E}} - q\vec{\upsilon} \times \vec{B}$$

$$J_i = \sigma_{ij}\left(\vec{B}\right)\mathcal{E}_j$$

$$\vec{J} = \sigma_0 \vec{\mathcal{E}} - \sigma_0 \mu_H \vec{\mathcal{E}} \times \vec{B}$$

$$\sigma_{ij}\left(\vec{B}\right) = \sigma_0 \begin{bmatrix} 1 & -\mu_H B_z \\ +\mu_H B_z & 1 \end{bmatrix}$$

(2D)

(Hall effect)

L18: Strong B-fields

$$\sigma_{ij}(B_z) = \frac{nq\mu_n}{1+\mu_n B_z}\begin{bmatrix} 1 & -\mu_n B_z \\ \mu_n B_z & 1 \end{bmatrix}$$

1) A magnetic field affects both the diagonal and off-diagonal components of the magneto-conductivity tensor.

2) Small magnetic field means: $\mu_n B_z << 1$ $\quad \omega_c \tau << 1$ $\quad \omega_c = \frac{qB_z}{m^*}$

(parabolic bands)

3) Landau levels develop. $E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$

scattering

1) Landauer: $T(E) = \frac{\lambda(E)}{\lambda(E) + L}$

1) How is this equation derived?
2) How is mfp for backscattering related to the scattering time

$$\lambda(E) \propto \upsilon(E)\tau(E)$$

2) BTE: $\hat{C}f = \sum_{p'} S(\vec{p}', \vec{p}) f(\vec{p}')\left[1 - f(\vec{p})\right] - \sum_{p'} S(\vec{p}, \vec{p}') f(\vec{p})\left[1 - f(\vec{p}')\right]$

How is the transition rate, S(p,p') computed?

3) General:

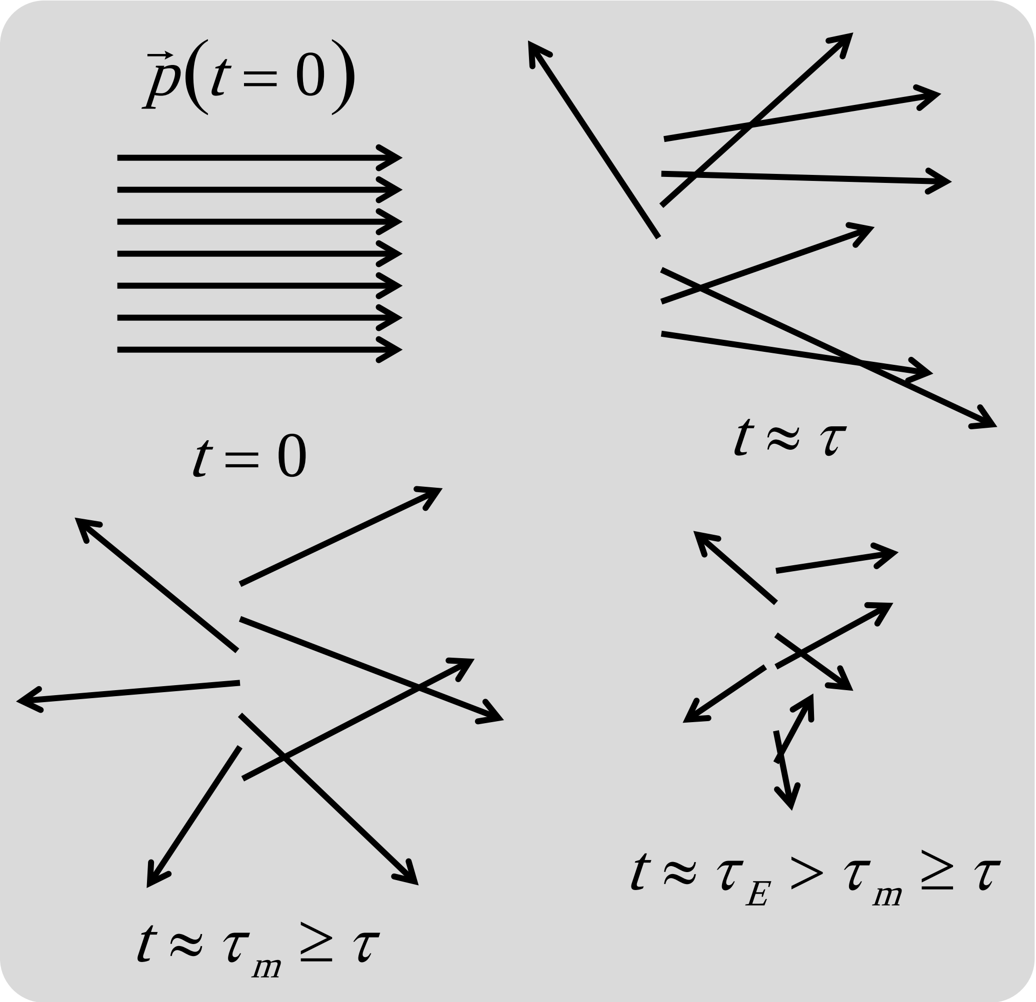
How do we simply describe the physical effects of scattering

L19: characteristic times

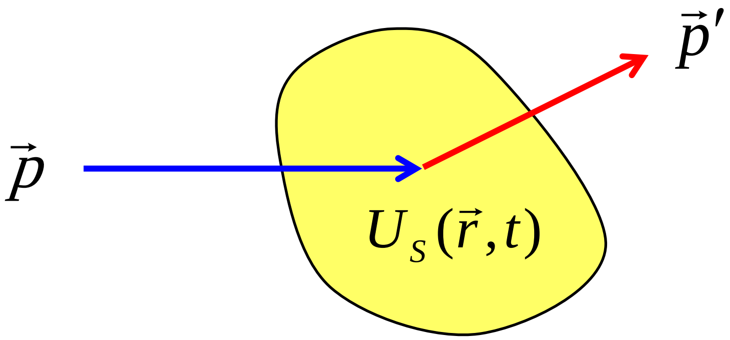
$$\frac{1}{\tau(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p}, \vec{p}')$$

$$\frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p}, \vec{p}') \frac{\Delta p_z}{p_z}$$

$$\frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p}, \vec{p}') \frac{\Delta E}{E_0}$$



L21: Fermi's Golden Rule



$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|H_{p'p}\right|^2 \delta(E' - E - \Delta E)$$

$$H_{\vec{p}',\vec{p}} = \int_{-\infty}^{+\infty} \psi_f^* U_S(\vec{r})\psi_i d\vec{r}$$

$$E' = E_0 + \Delta E$$

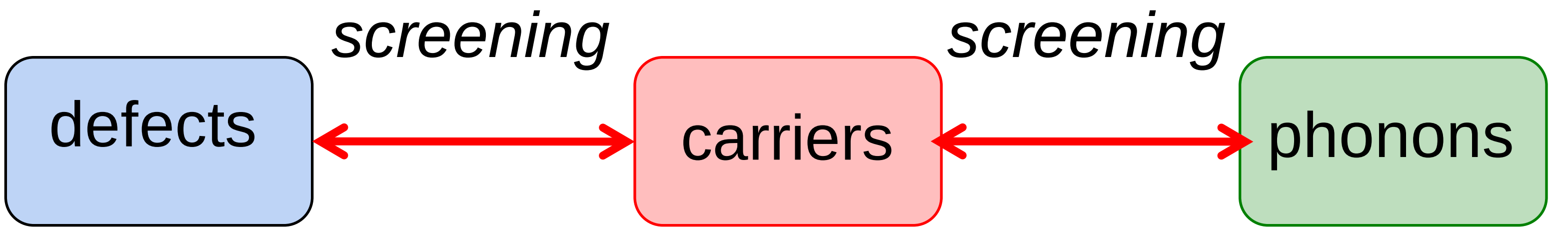
$\Delta E = 0$ for a static U_S

$\Delta E = \pm\hbar\omega$ for an oscillating U_S

$$\frac{1}{\tau(\vec{p})} = \sum_{\vec{p}'\uparrow} S(\vec{p},\vec{p}\,') \propto D_f(E)$$

For an electron with energy, *E*, its scattering rate is proportional to the density of final states at energy, *E* (1D, 2D, 3D)

scattering in semiconductors



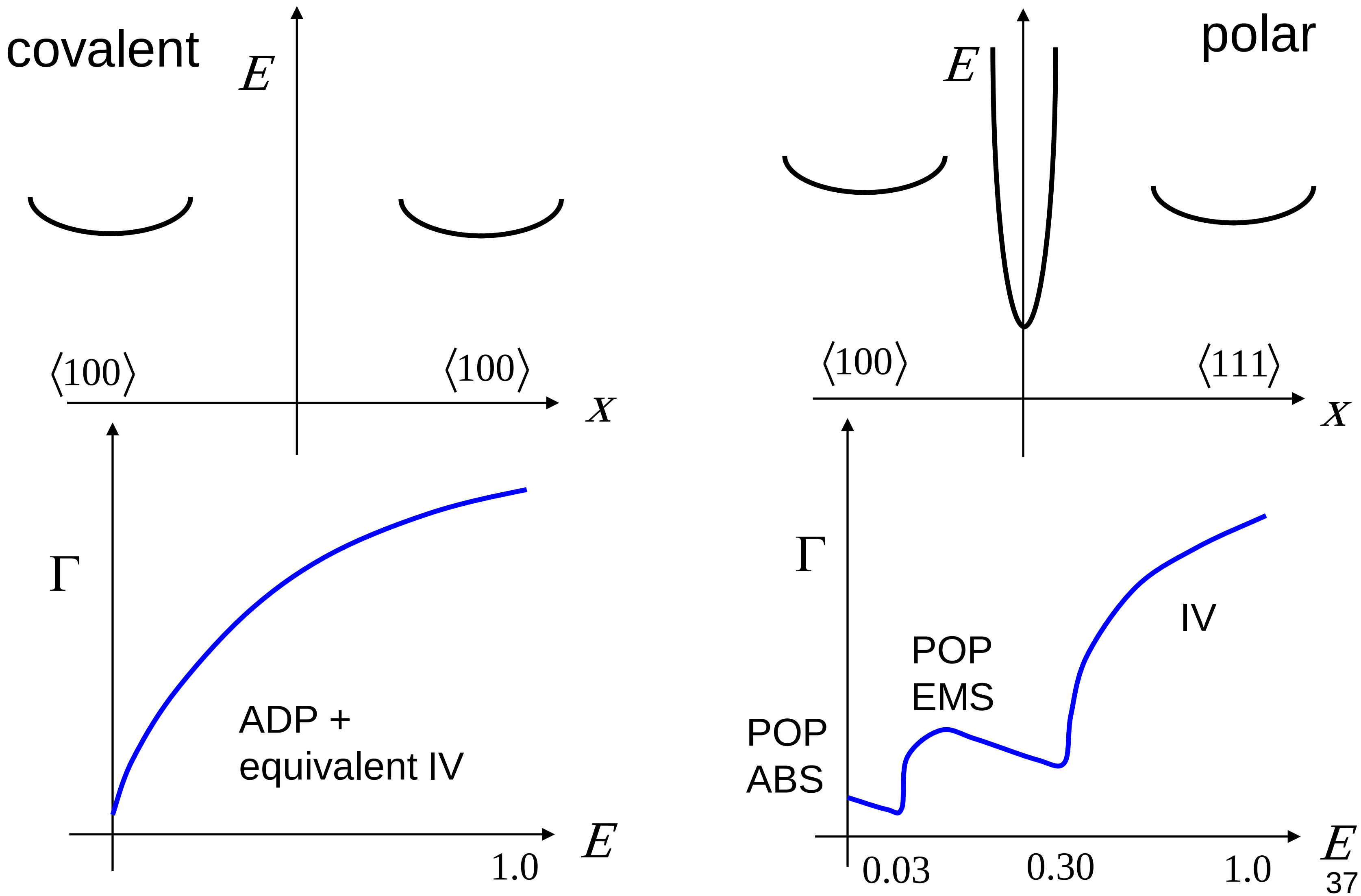
- ionized impurities
- neutral impurities
- dislocations
- surface roughness
- alloy

- electron-electron
- electron-plasmon
- electron-hole

- intravalley
  -ADP
  -ODP
  -POP
  -PZ

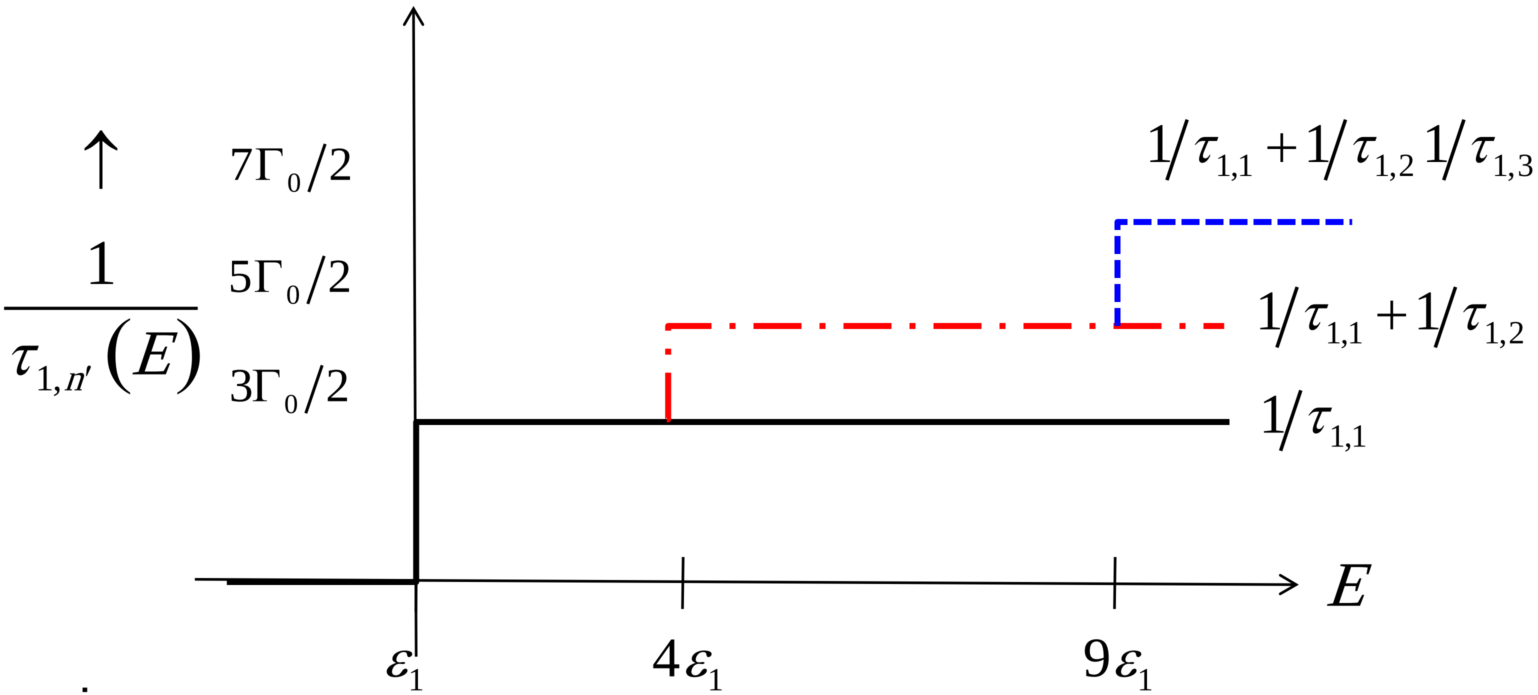
- intervalley
  -acoustic
  -optical

covalent vs. polar semiconductors



L26: scattering in quantum wells

$$\frac{1}{\tau_{n,n'}} = \frac{2\pi}{\hbar} U_{ac} \frac{D_{2D}(E)}{2} \frac{\left(2+\delta_{n,n'}\right)}{2} = \Gamma_0 \frac{\left(2+\delta_{n,n'}\right)}{2}$$



Note: energy is referenced to the bottom of the first subband.

L28: moments of the BTE

The quantities of interest to device researchers are moments of the distribution function:

$$n_\phi\left(\vec{r},t\right)=\sum_{\vec{p}}\phi\left(\vec{p}\right)f\left(\vec{r},\vec{p},t\right)$$

These quantities satisfy a continuity equation:

$$\frac{\partial n_\phi}{\partial t}=-\nabla\bullet\vec{F}_\phi+G_\phi-R_\phi$$

A clear prescription for generating a continuity (or balance) equation exists, ***but*** simplifying the resulting equations for use in practice is an art.

example: the drift-diffusion equation again

$$\phi(\vec{p}) = (-q)\frac{p_i}{m^*} \qquad n_\phi(\vec{r},t) = J_{ni}(\vec{r},t)$$

$$\vec{J}_n + \langle \tau_m \rangle \frac{\partial \vec{J}_n(\vec{r},t)}{\partial t} = nq\mu_n \vec{\mathcal{E}} + 2\mu_n \nabla \bullet \overleftrightarrow{W}$$

$$\mu_n = \frac{q\langle \tau_m \rangle}{m^*} \qquad W_{ij} = n\left\langle \frac{p_i \upsilon_j}{2} \right\rangle$$

assume:

i) *W* is diagonal
ii) near-equilibrium conditions
ii) slow variations in time and space

} DD equation

L29: four balance equations

$$\frac{\partial n(\vec{r},t)}{\partial t} = -\frac{d\left[J_{nx}/(-q)\right]}{dx}$$

0th moment of BTE

$$\langle \tau_m \rangle \frac{\partial J_{nx}(\vec{r},t)}{\partial t} + J_{nx} = nq\mu_n \mathcal{E}_x + 2\mu_n \frac{dW_{xx}}{dx}$$

1st moment of BTE

$$\frac{\partial W(\vec{r},t)}{\partial t} = -\frac{dF_{Wx}}{dx} + J_{nx}\mathcal{E}_x - \frac{(W - W_0)}{\langle \tau_E \rangle}$$

2nd moment of BTE

$$\langle \tau_{F_W} \rangle \frac{\partial F_W(\vec{r},t)}{\partial t} + F_{Wx} = -3n\frac{q\langle \tau_{F_W} \rangle}{m^*} u\mathcal{E}_x - \langle \tau_{F_W} \rangle \frac{dX_{xx}}{dx}$$

3rd moment of BTE

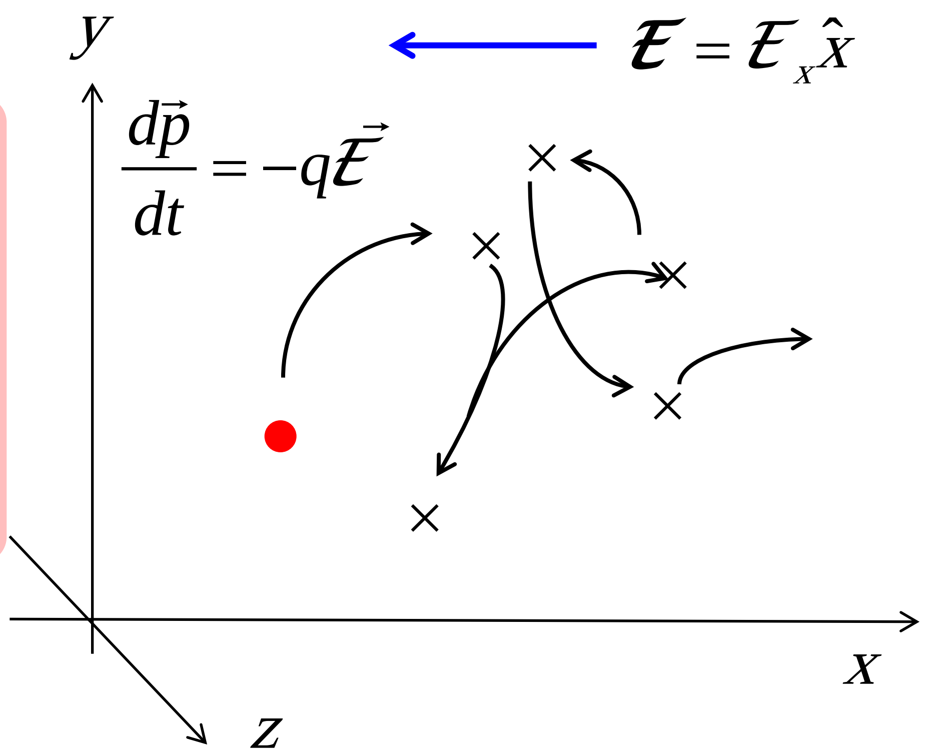
L31: Monte Carlo simulation

1) "free flight" for t_C seconds.
2) update $E(t_C^-)$ and $r\ (t_C^-)$
3) identify collision
4) update $E(t_C^+)$ and $p\ (t_C^+)$
5) Set $t = 0$ and repeat

r_1

r_2

r_3, r_4



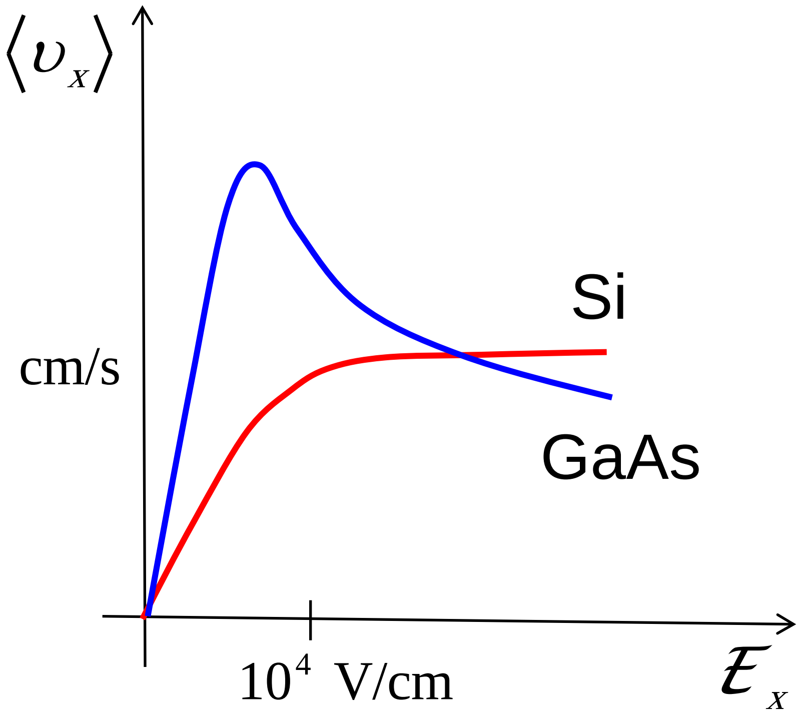
$$\mathcal{E} = \mathcal{E}_x \hat{x}$$

$$\frac{d\vec{p}}{dt} = -q\vec{\mathcal{E}}$$

1) free flights: semi-classical equations of motion

2) scattering: quantum mechanical transition rate $S(p,p')$

L32: hot carrier transport

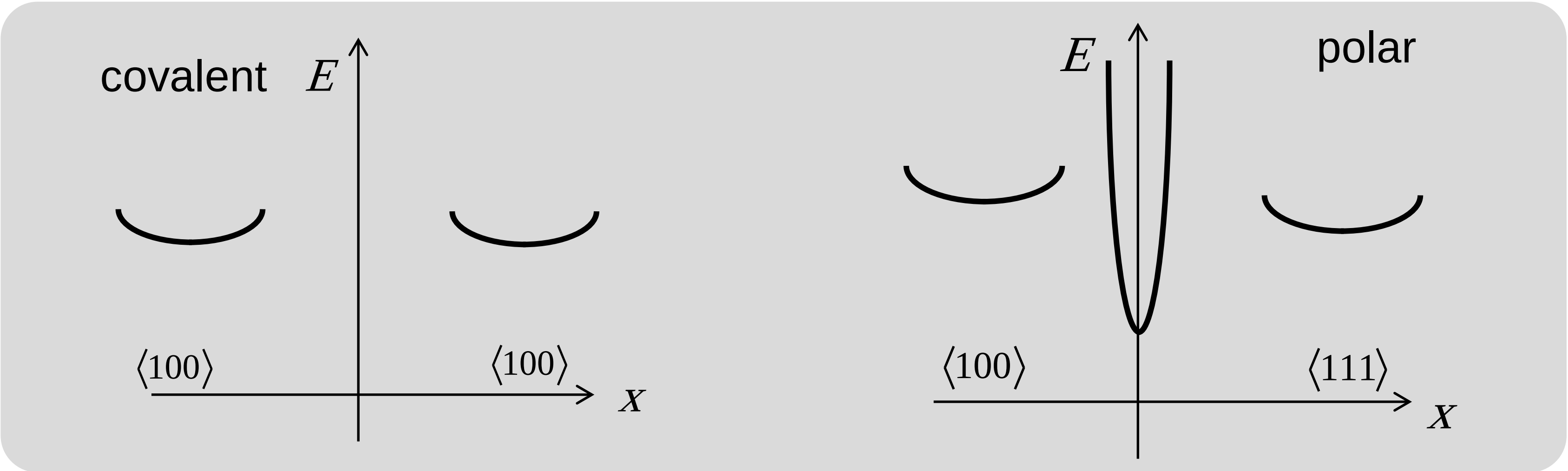


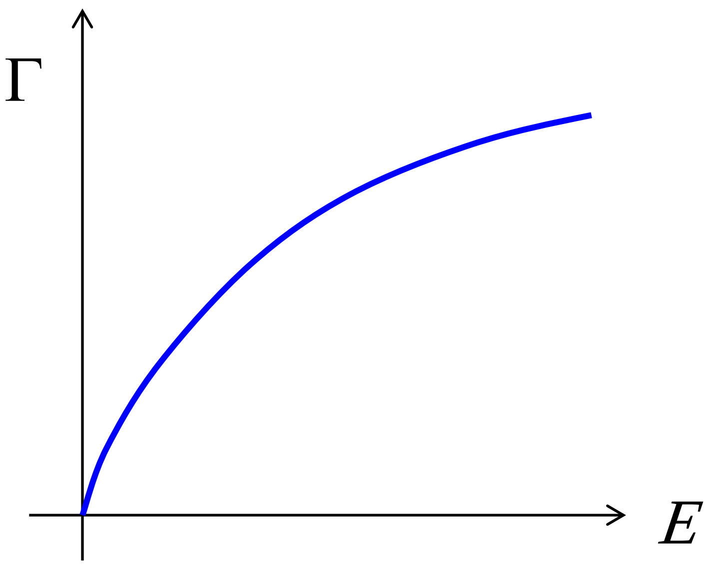
$$J_{nx}\mathcal{E}_x = \frac{n(u-u_0)}{\langle \tau_E \rangle}$$

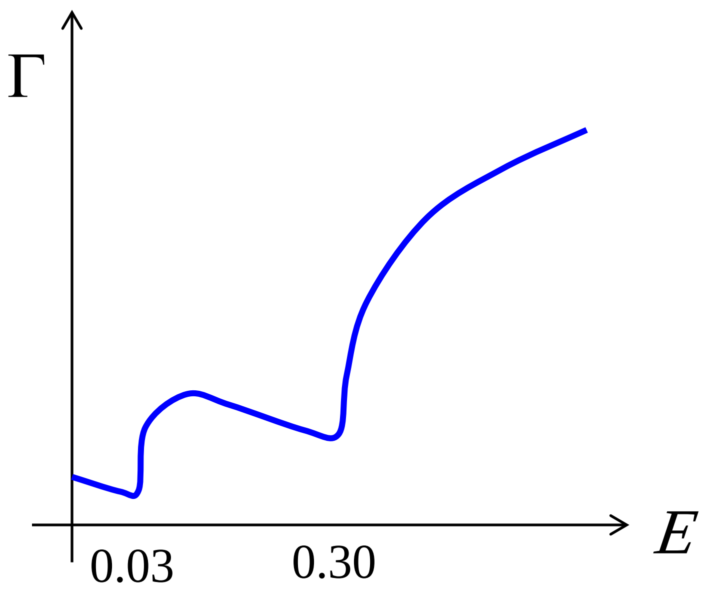
$$\upsilon_{dx} = -\mu_n \mathcal{E}_x = -\frac{q\langle \tau_m \rangle}{m^*}\mathcal{E}_x$$

$$\langle \tau_m \rangle \downarrow \quad \text{as} \quad u \uparrow$$

covalent vs. polar semiconductors







field-dependent DD equation

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} \quad \mu_n = q\langle \tau_m \rangle / m^* \quad D_n/\mu_n = 2u_{xx}/q$$

Goal: Find mobility and diffusion coefficient <u>without</u> solving BTE

In general, however: $\mu_n\left[f\left(\vec{r}, \vec{p}, t\right)\right] \quad D_n\left[f\left(\vec{r}, \vec{p}, t\right)\right]$

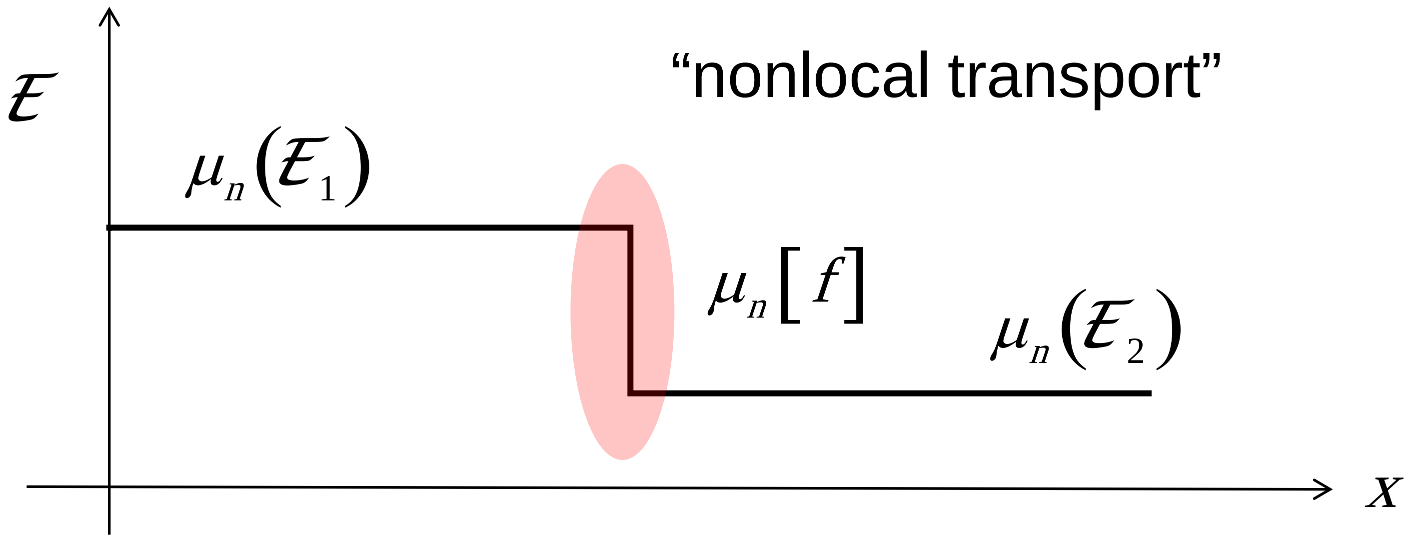
In a bulk semiconductor, *f* is determined by $\mathcal{E}$, so there is a one-to-one mapping between $\mathcal{E}$ and *f*.

$\mu_n(\mathcal{E}) \quad D_n(\mathcal{E})$

Electric ***field dependent*** mobility and diffusion coefficient.

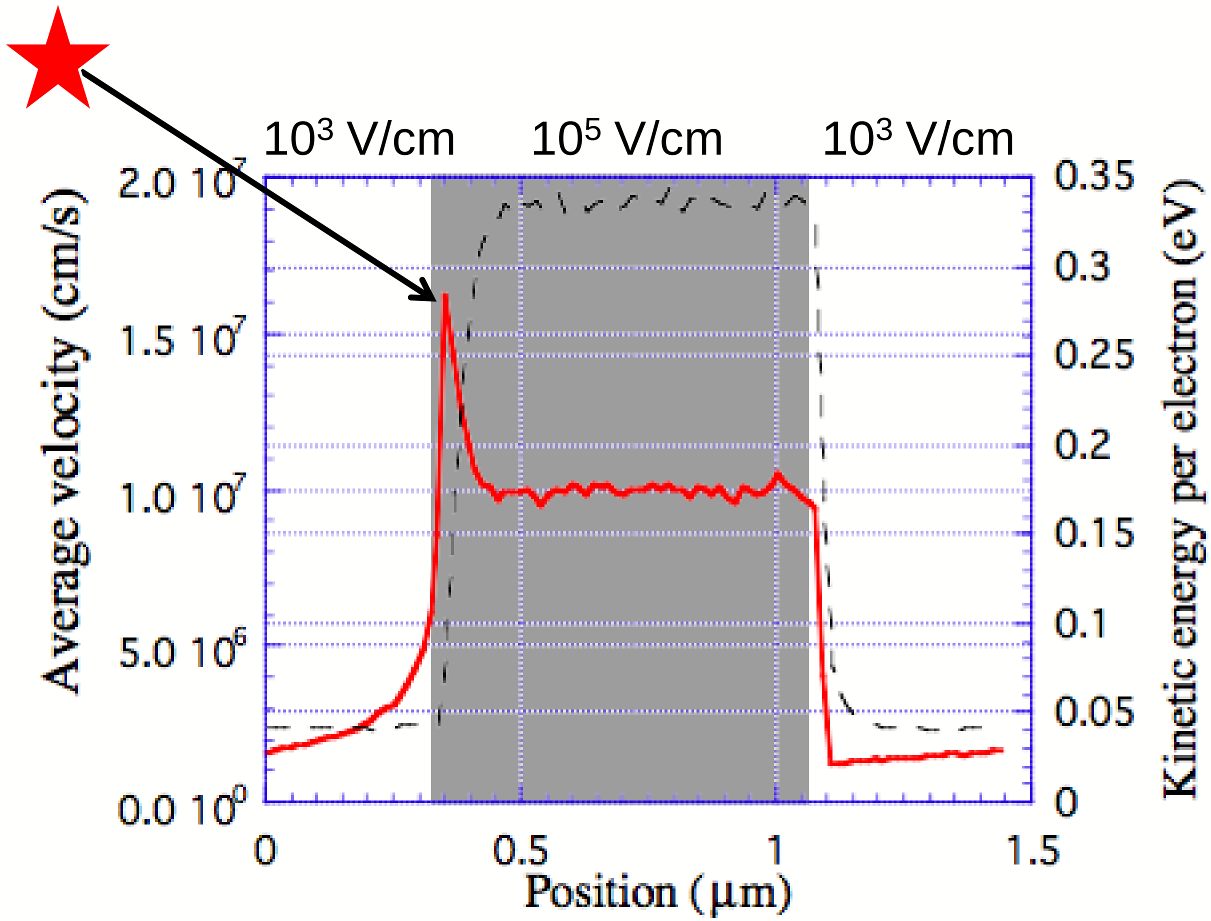
$$J_{nx} = nq\mu_n(\mathcal{E})\mathcal{E}_x + qD_n(\mathcal{E})\frac{dn}{dx}$$

L33: non-local transport



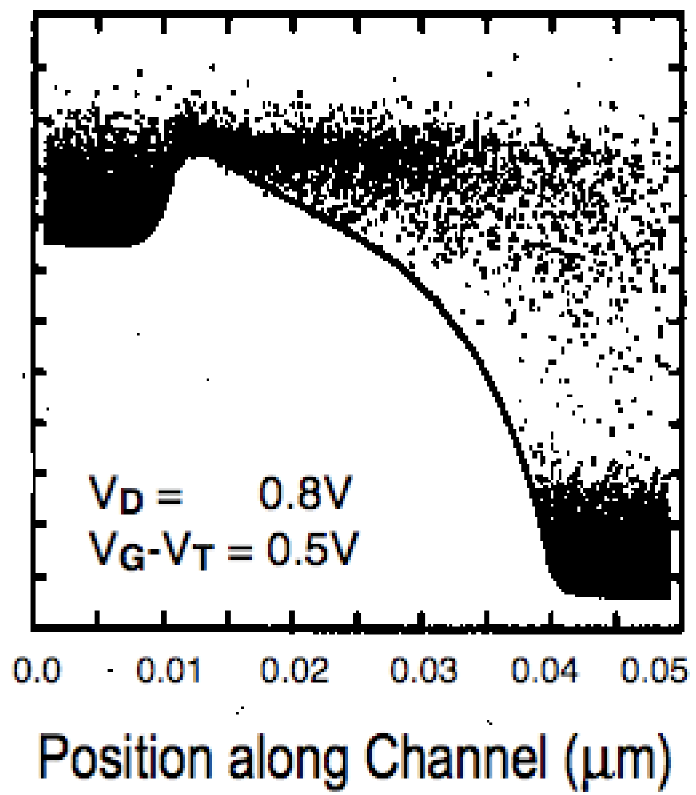
The concept of a field-dependent mobility applies only when the electric field changes slowly with position.

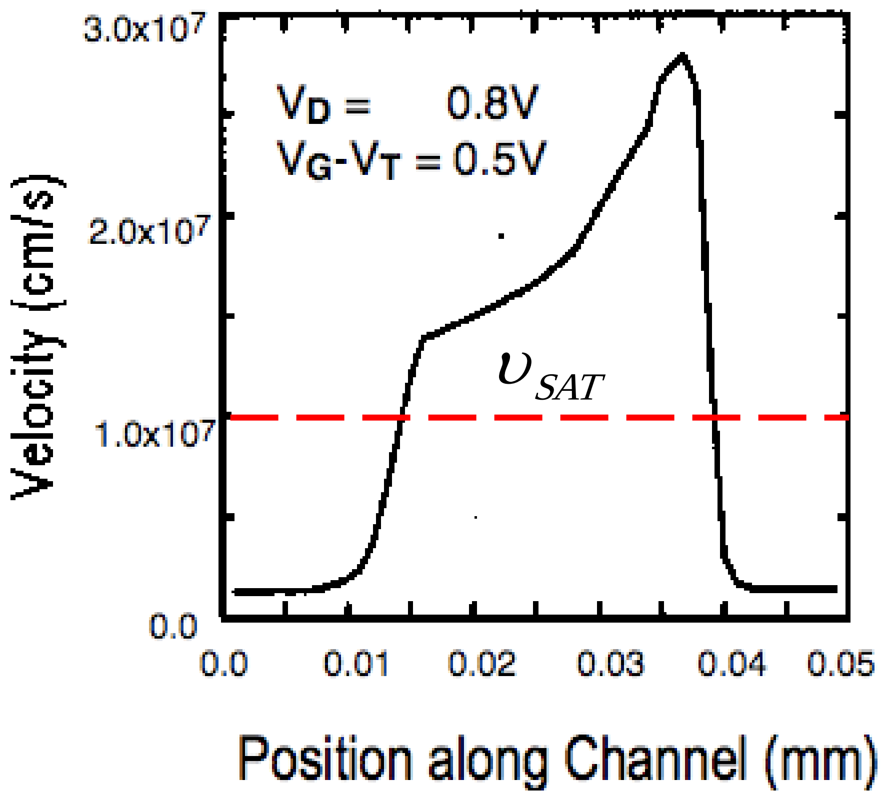
velocity overshoot



VO occurs in the presence of scattering when the energy relaxation time is longer than the momentum relaxation time.

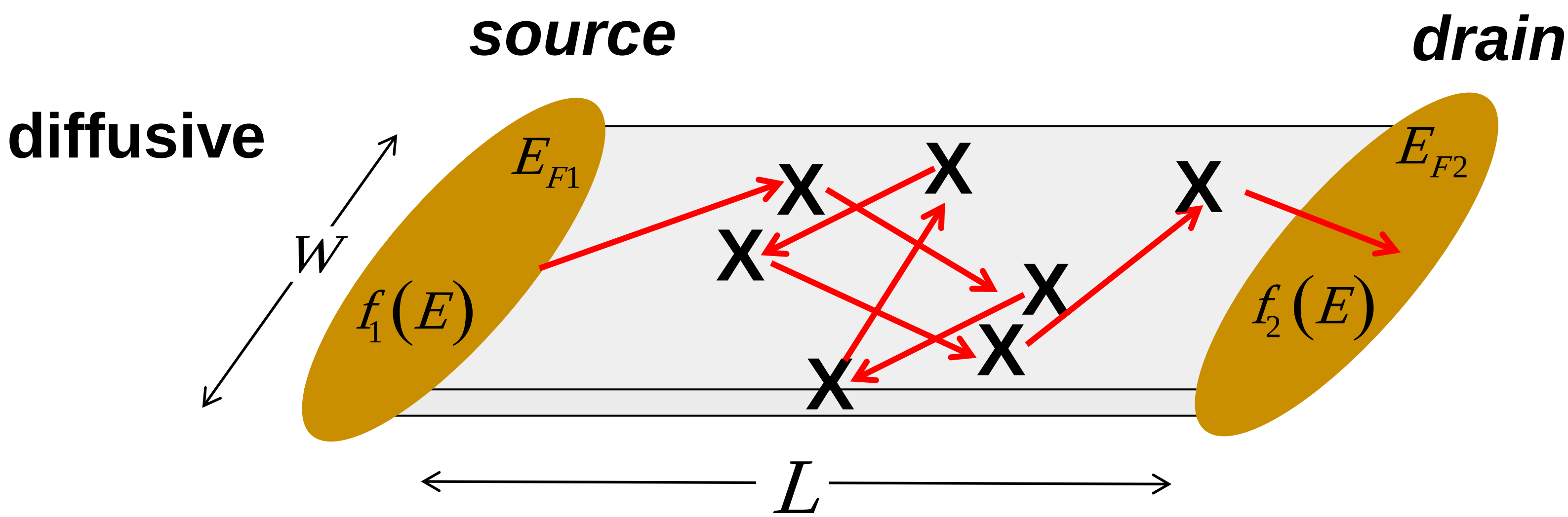
non-local transport in nanoscale MOSFETs

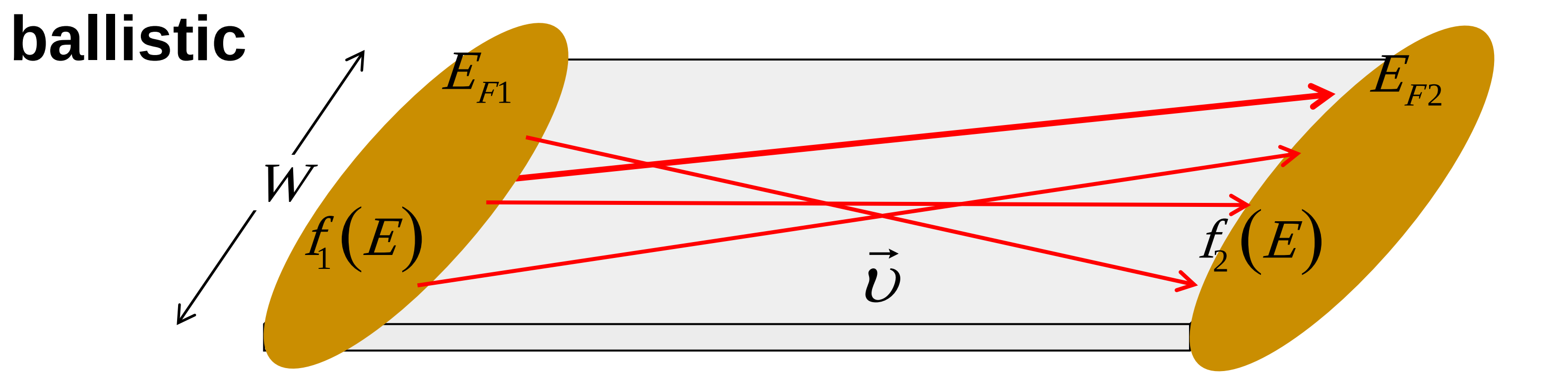




Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

ballistic vs. diffusive transport





(scattering from boundaries assumed to be negligible)

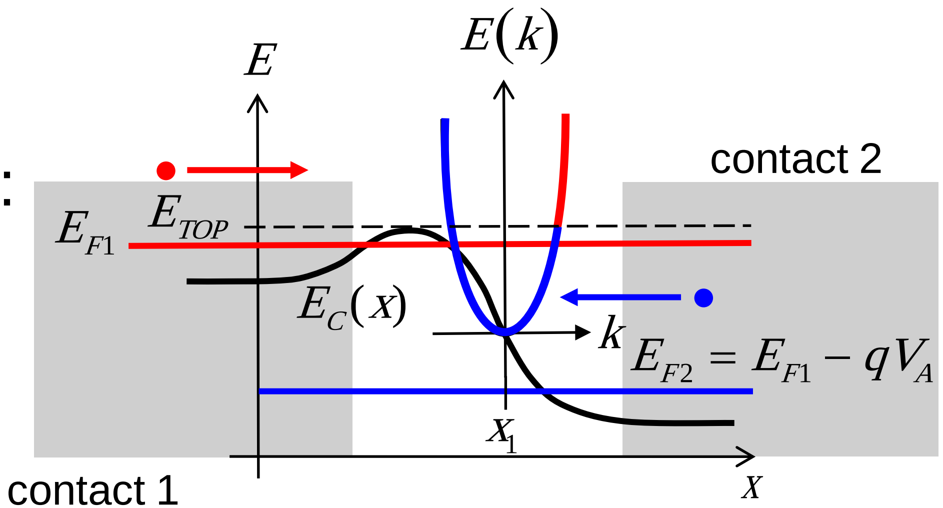
local density-of-states

$$D_{3D}(E) = \frac{1}{\Omega}\sum_{\mathbf{k}} \delta\left(E - E_{\mathbf{k}}\right)$$

Local density of states (L13):

$$D_i(E, x)$$

i = 1 or 2 for contact 1 or 2



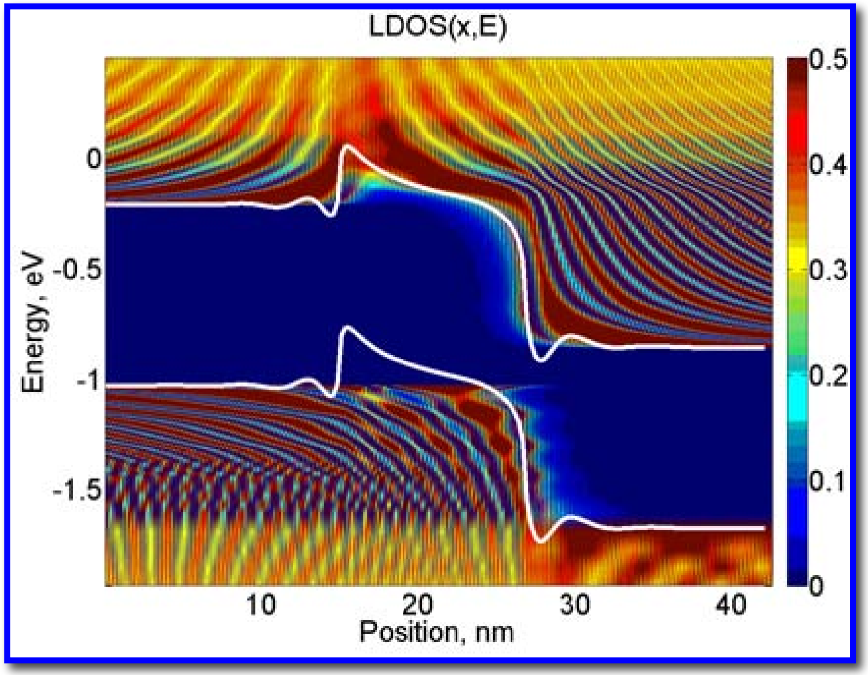
In a ballistic device (L3):

$$N(x) = \int \left[D_1(E,x) f_1(E) + D_2(E,x) f_2(E) \right] dE$$

quantum transport

$$D_{3D}(E) = \frac{1}{\Omega}\sum_{\mathbf{k}}\delta\left(E - E_k\right)$$

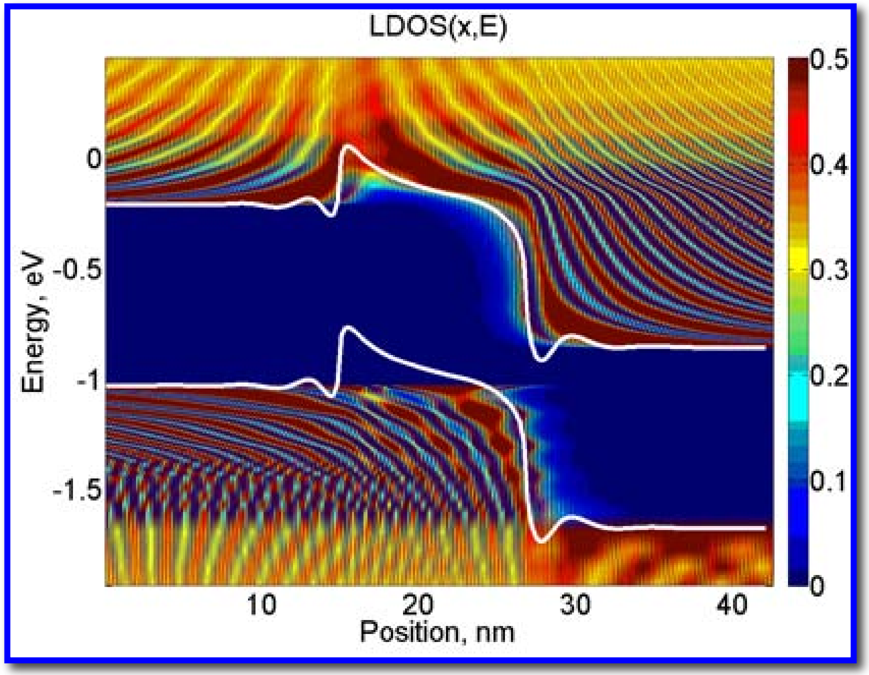
$$\rightarrow \psi^*\left(\vec{r}\right)\psi\left(\vec{r}\right)\sum_{\mathbf{k}}\delta\left(E - E_k\right)$$



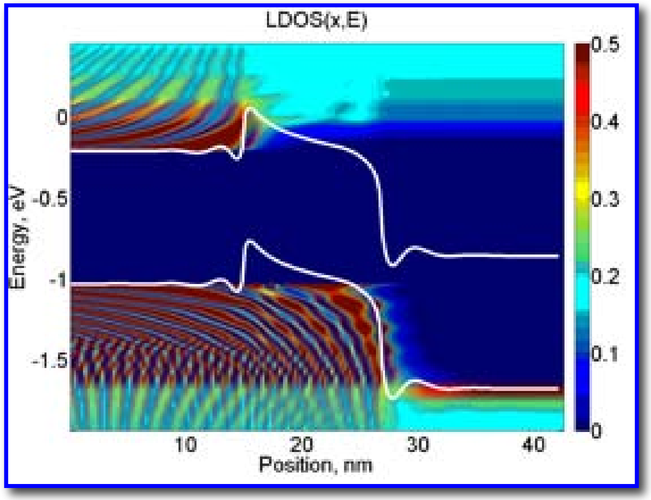
source + drain-injected LDOS in a carbon nanotube MOSFET

See: "Physics of Nanoscale MOSFETs," NCN Summer School, July 2008 http://nanohub.org/resources/5306

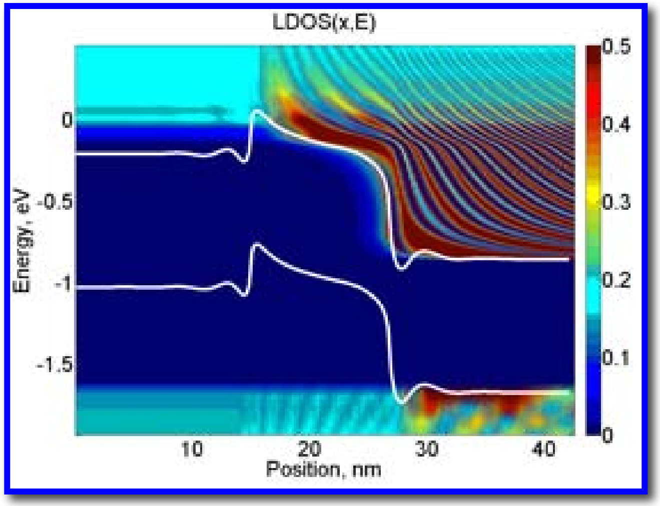
quantum transport



source + drain-injected LDOS in a carbon nanotube MOSFET



fillable by source



fillable by drain

the drift-diffusion equation

$$\vec{J}_p = -pq\mu_p \vec{\nabla} V - qD_p \vec{\nabla} p$$

1) Still describes transport in a very large number of cases

2) ECE-656 has taught you how to relate mobility and diffusion coefficient to material parameters and it suggests how we can engineer these parameters with strain and quantum confinement.

3) We have also learned about the assumptions underlying the DD equation – (basically slow variations in time and space and applications to devices that are many mfp's long).

4) We also learned that some problems cannot be treated by the DD equation – ballistic transport, non-local semiclassical transport, and quantum transport.

5) Finally, we learned some new techniques and how they related to the DD equations (Landauer approach, BTE, and an introduction to quantum transport.