

# Lecture 10

## Force distance curves II

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# Classical beam theory

## ■ Example 2



$$EI \frac{d^4 w(x)}{dx^4} = 0 \Rightarrow \frac{d^3 w(x)}{dx^3} = c_1 \Rightarrow \frac{d^2 w(x)}{dx^2} = c_1 x + c_2$$

$$\Rightarrow \frac{dw(x)}{dx} = \theta(x) = \frac{1}{2} c_1 x^2 + c_2 x + c_3$$

$$w(x) = \frac{1}{6} c_1 x^3 + \frac{1}{2} c_2 x^2 + c_3 x + c_4$$

*Boundary conditions*

$$w(0) = \theta(0) = 0 \quad EI \frac{d^2 w(L)}{dx^2} = 0 \quad EI \frac{d^3 w(L)}{dx^3} = -F$$

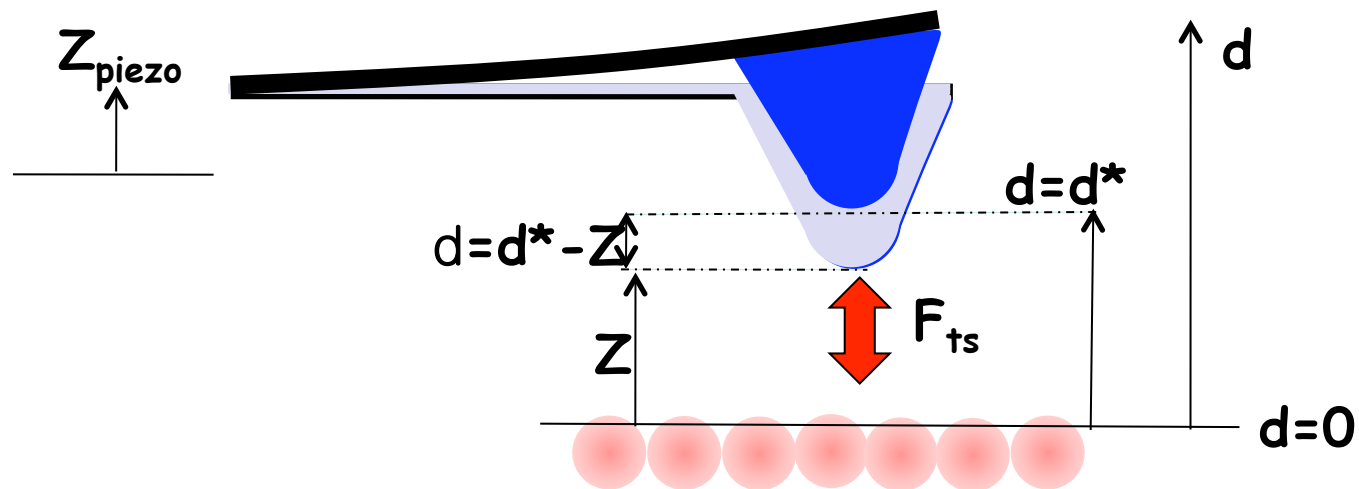
*(no point moment applied at  $x = L$ )*

$$\Rightarrow c_3 = c_4 = 0, \quad c_1 = -\frac{F}{EI}, \quad c_2 = \frac{FL}{EI}$$

$$w(L) = \delta = \frac{1}{3} \frac{FL^3}{EI}, \quad \theta(L) = \theta = \frac{1}{2} \frac{FL^2}{EI} \Rightarrow \frac{\theta}{\delta} = \frac{2}{3} L$$

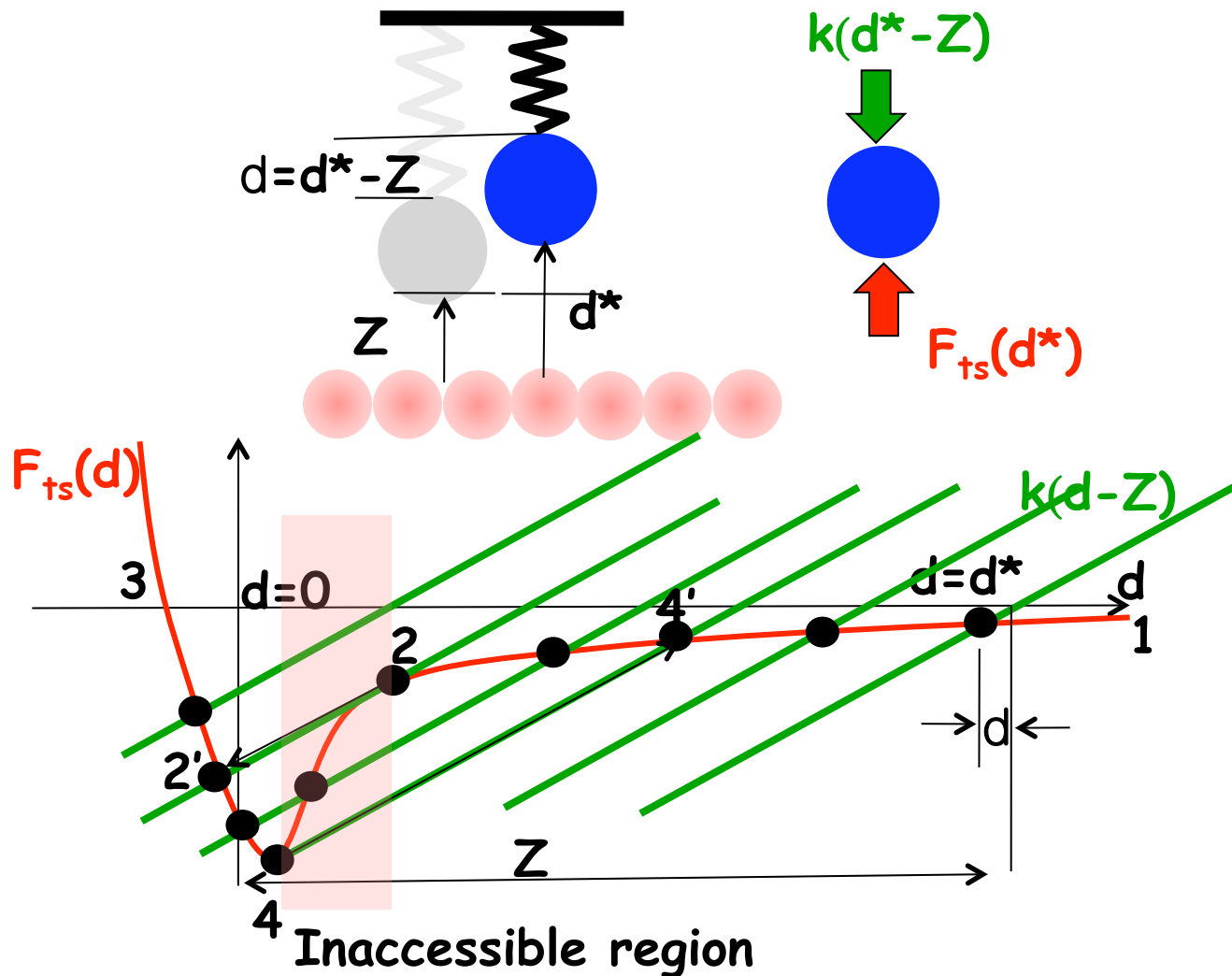
$F = k\delta$ , where  $k = \frac{3EI}{L^3}$  is the static bending stiffness of the cantilever

# Equilibrium positions during approach and retraction



- How do  $d^*$  and  $d$  change as  $Z$  is reduced during approach and then retracted?

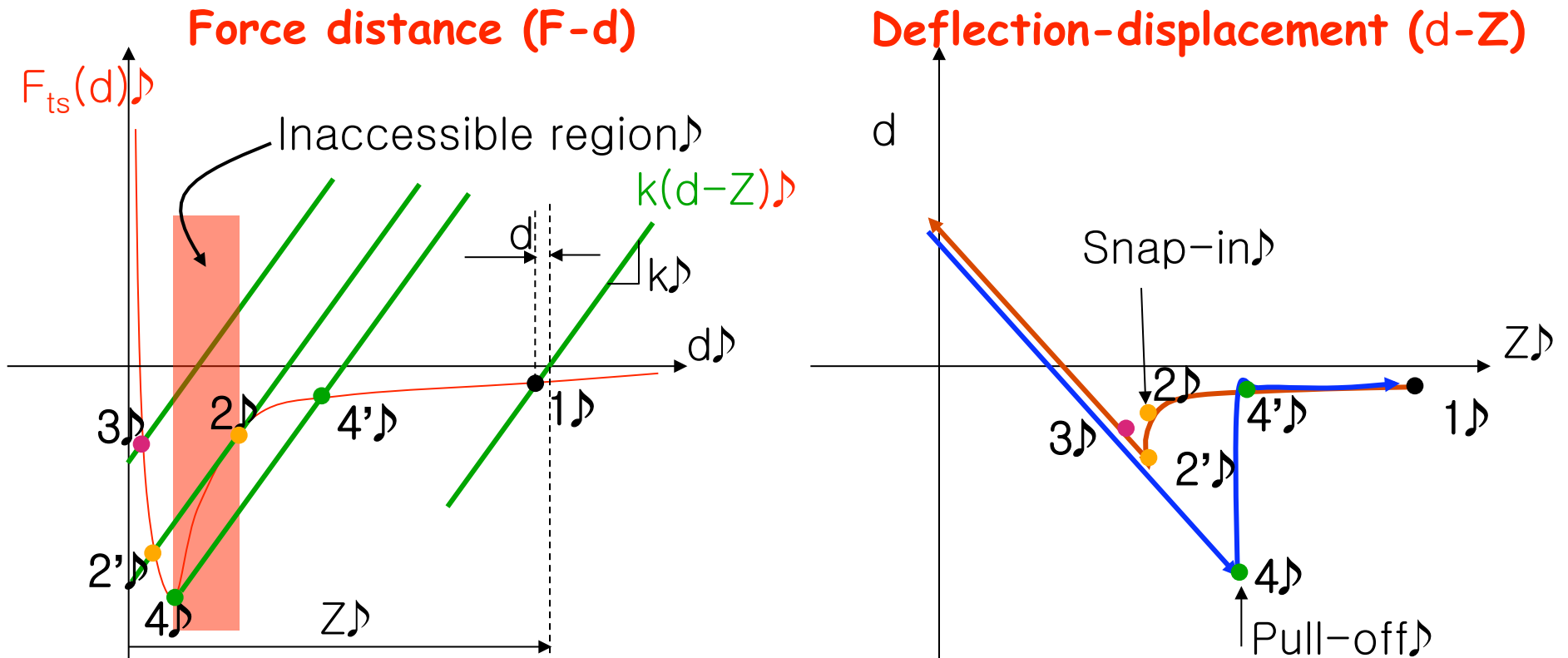
# Equilibrium positions during approach and retraction



2

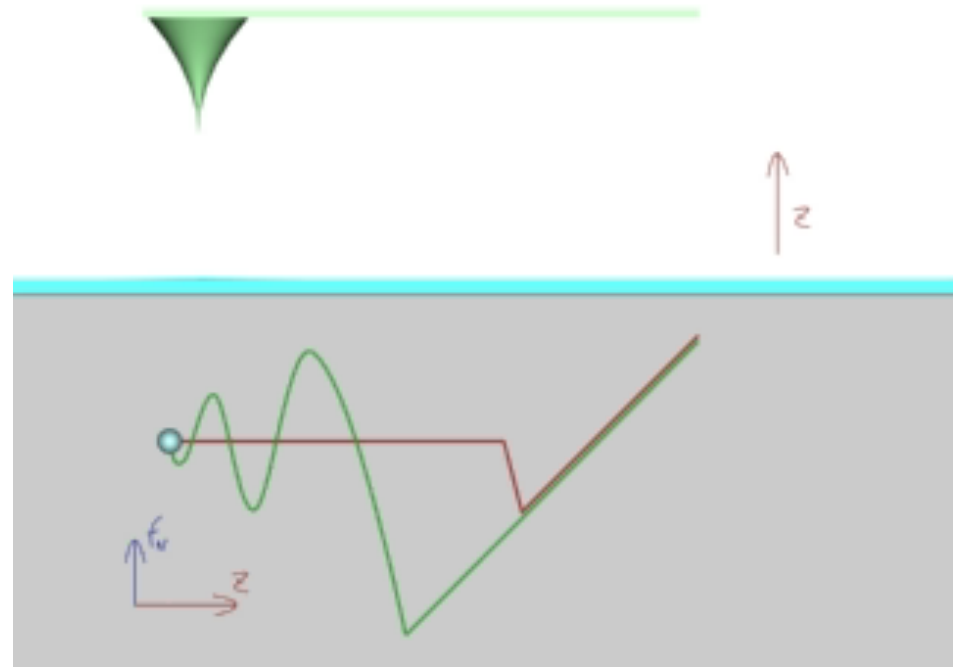
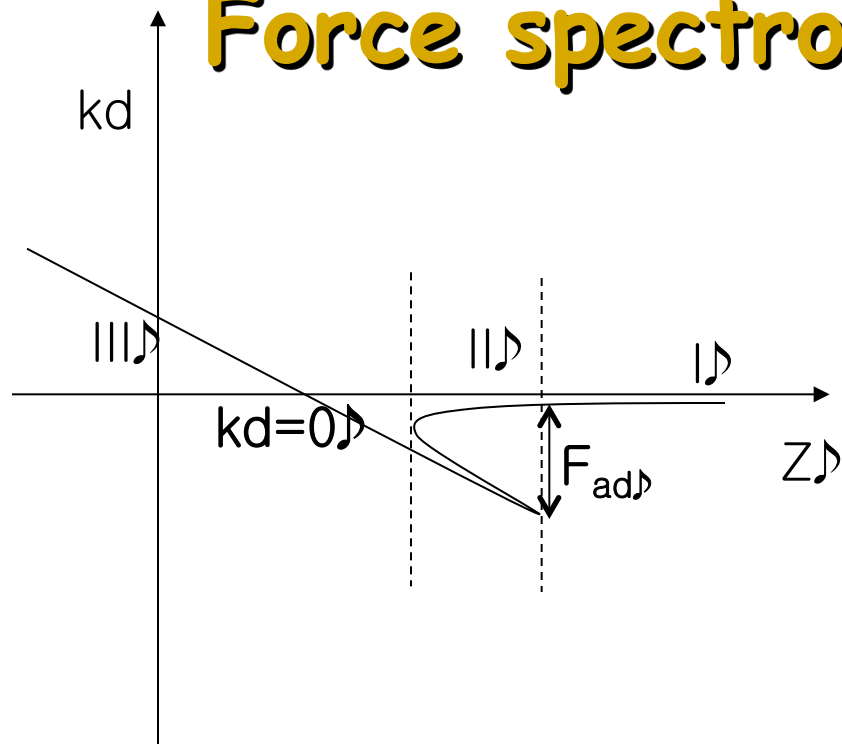
- With soft cantilevers (small  $k$ ) it is not possible to measure entire ' $d$ ' range

# Force distance & force-displacement



- Note that hysteresis occurs in the  $d$ - $Z$  curve between approach and retraction even though  $F_{ts}(d)$  is conservative

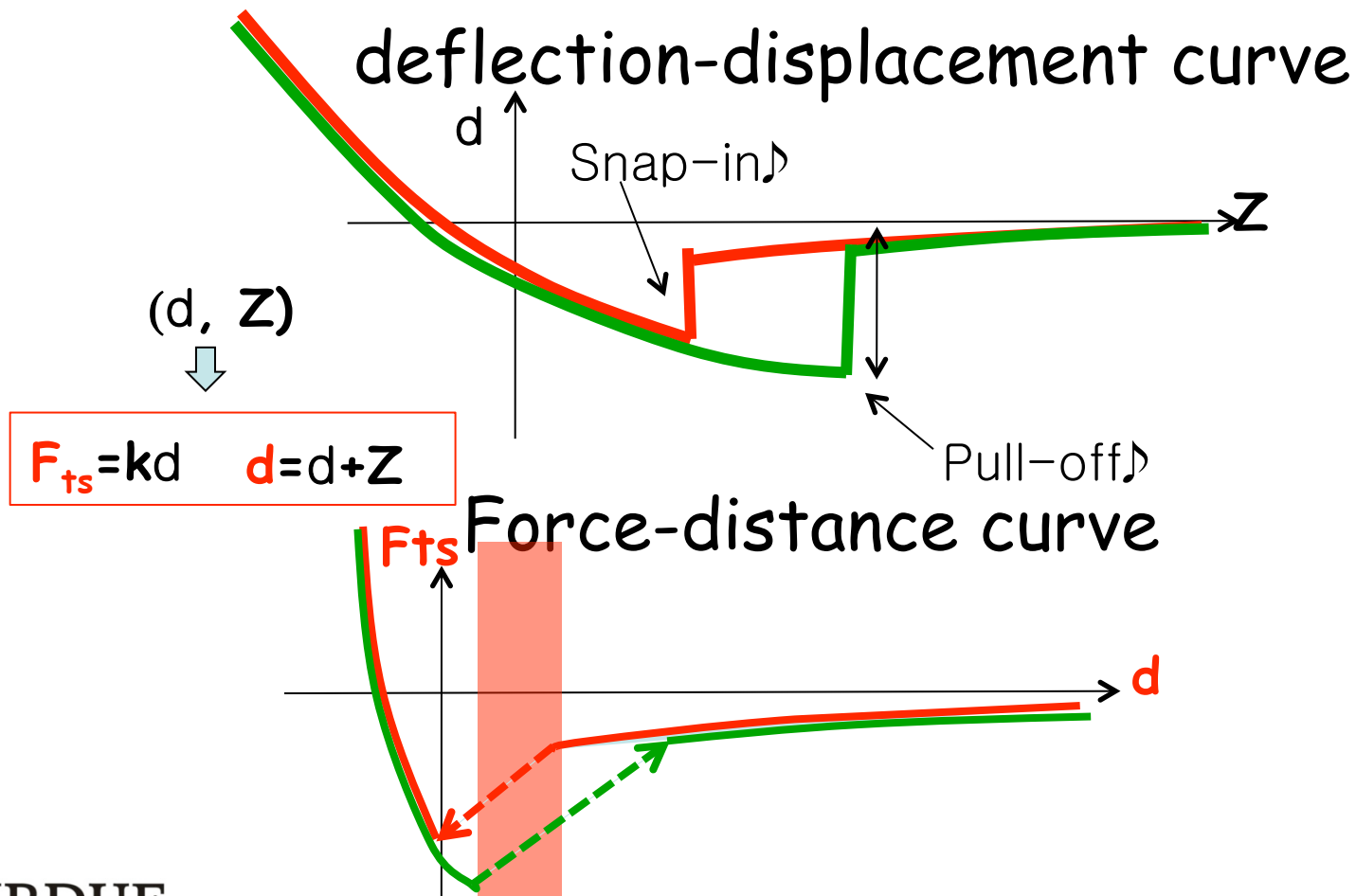
# Force spectroscopy - summary



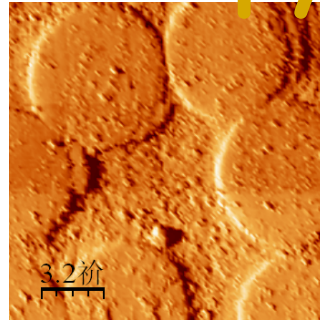
- Three distinct regions
- Pull-off force  $\sim F_{Ad}$  which can be converted to  $W_{132}$  (work of adhesion between two infinitely wide planes)
- Slope in III is good measure of repulsive forces (local elasticity)
- If  $k$  is known then from the static-force distance curve,  $F(d)$  can be calculated for all  $d$  except for inaccessible range near snap-in

# Force-displacement & force distance

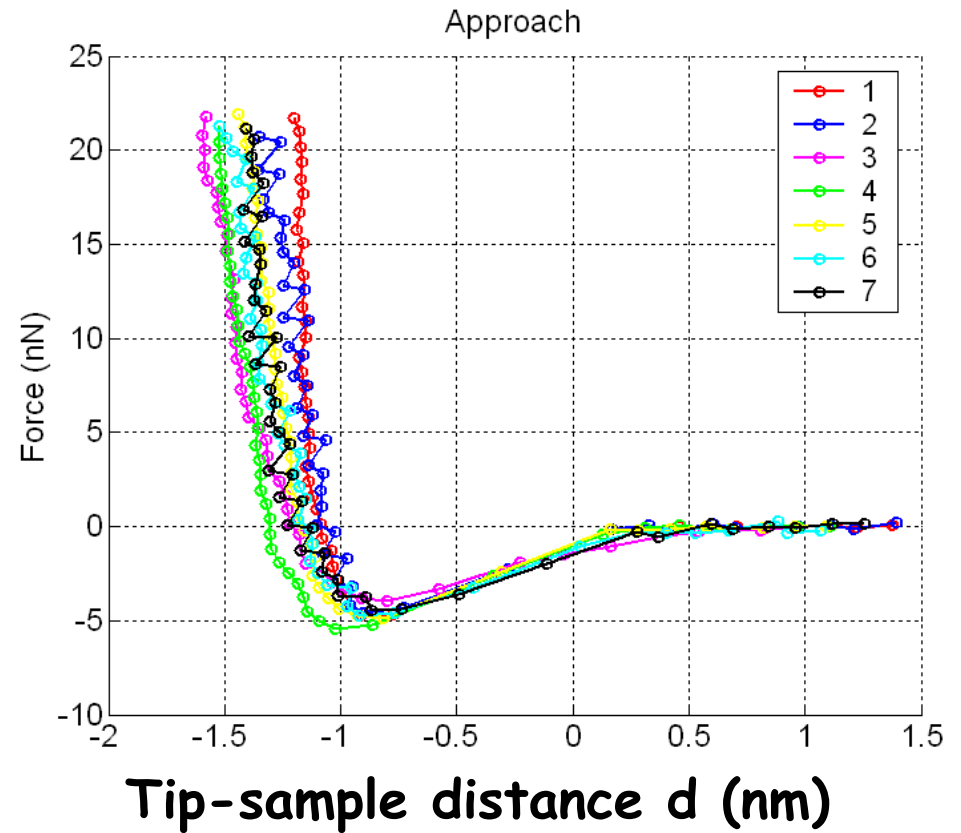
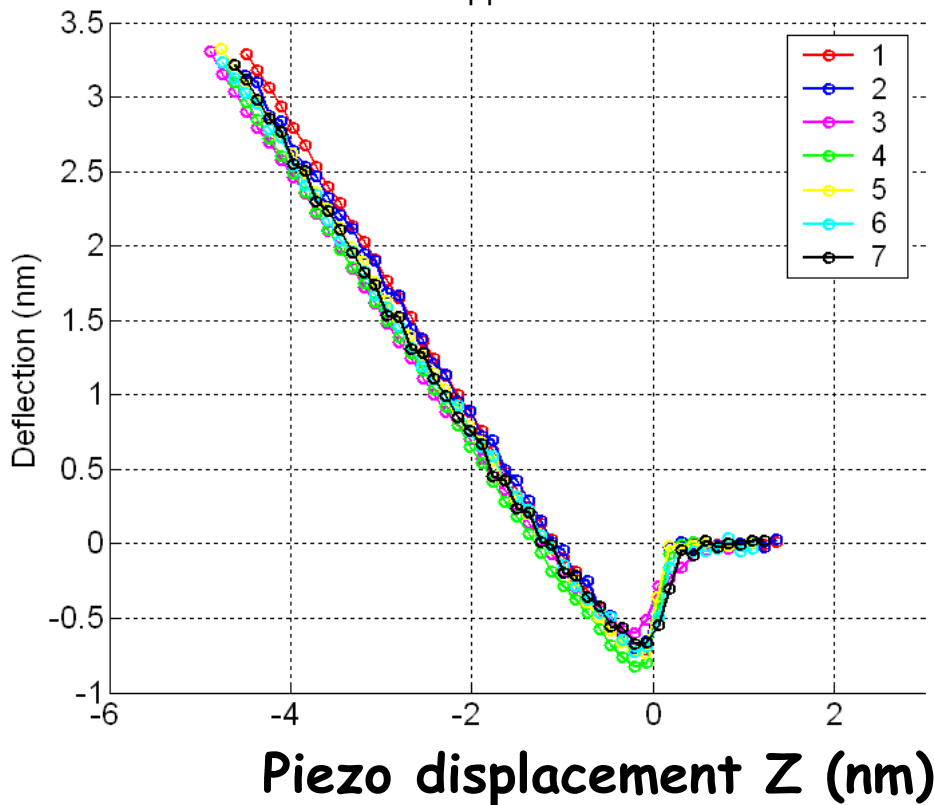
- In a typical d-Z experiment in AFM, the cantilever approaches/retracts from the sample while recording the cantilever deflection.
- However in force spectroscopy we are interested in converting this to a force-distance curve i.e.  $F_{ts}$  vs.  $d$ . How to convert?



# Force spectroscopy - an example



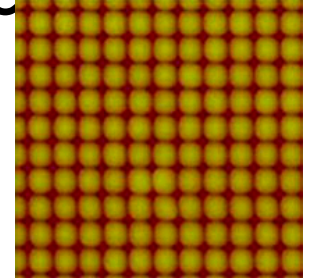
Convert deflection vs. displacement curves to force vs. distance (gap) curves



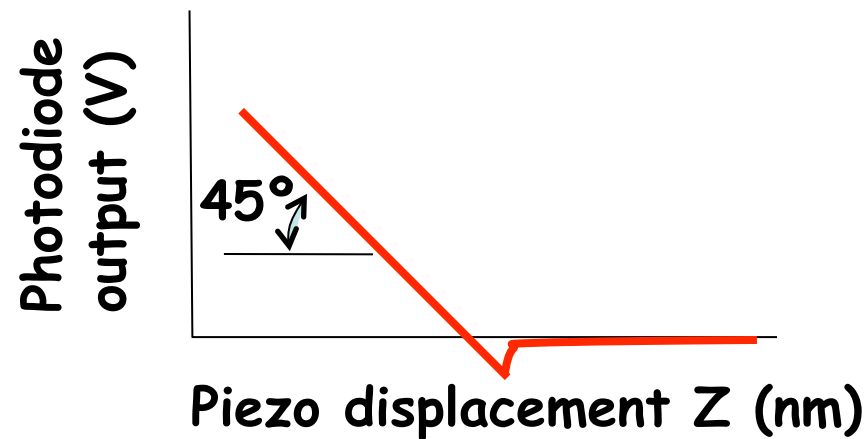


# Three important calibrations

- Z-piezo calibration: By scanning a sample of known height (calibration grating) in contact mode

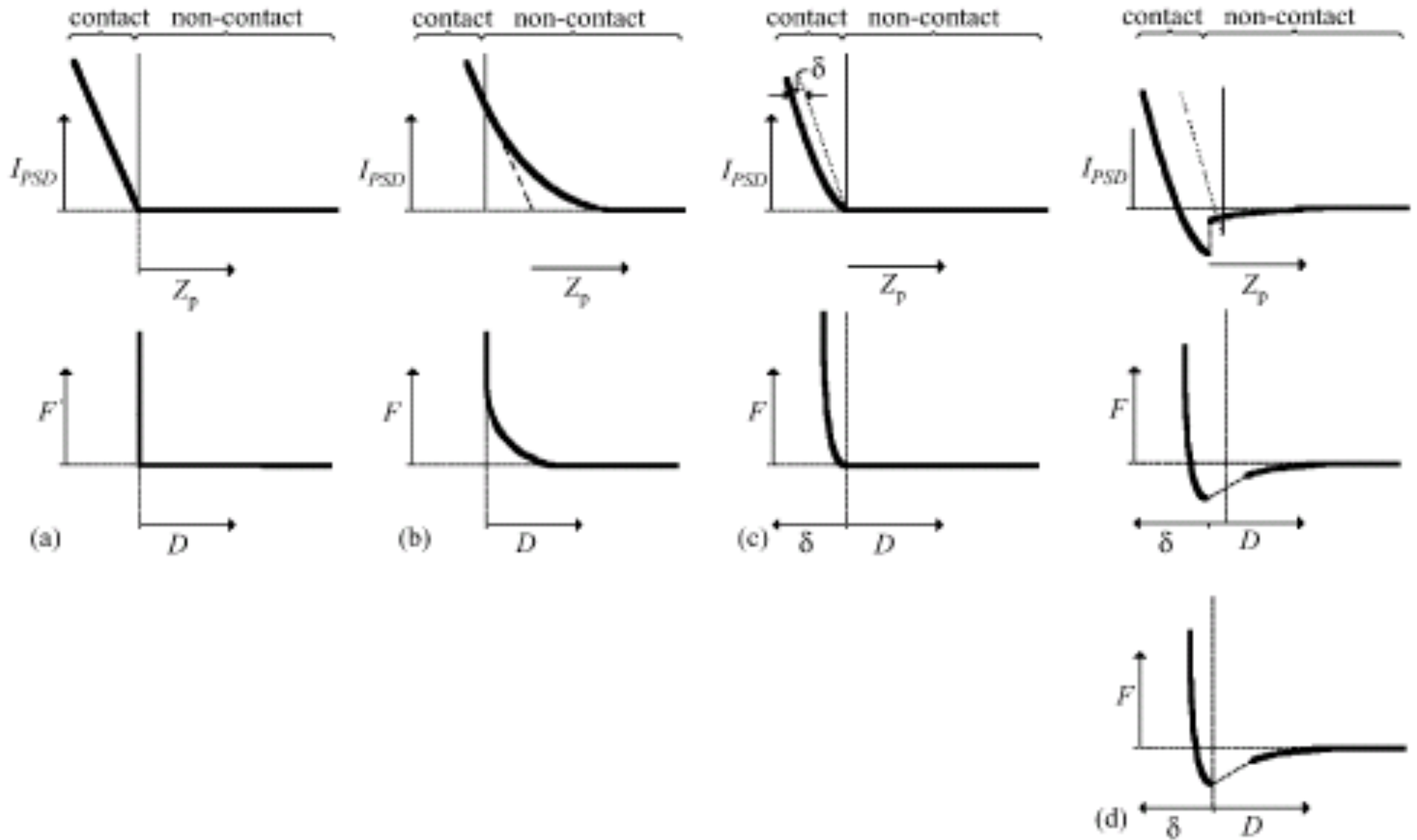


- Cantilever deflection calibration: d-Z curve on a hard sample

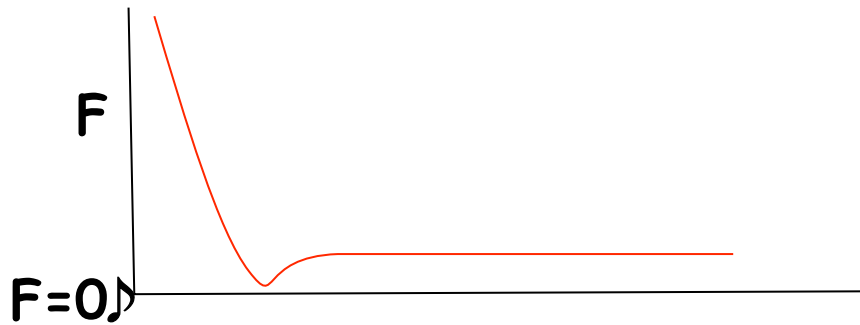


- Cantilever stiffness calibration (to be discussed later in class)

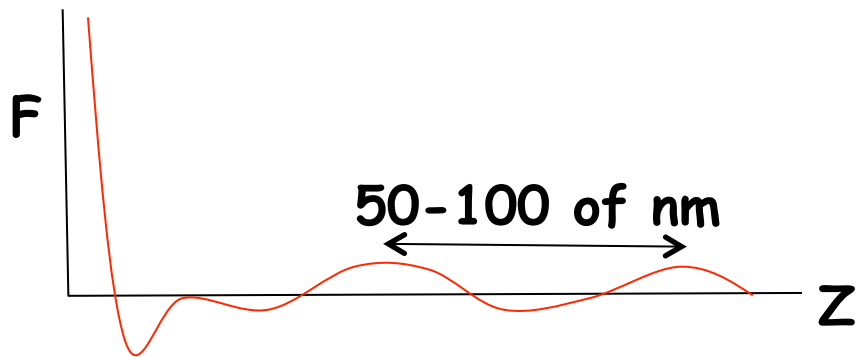
# Some other interactions



# Artefacts in F-Z curves

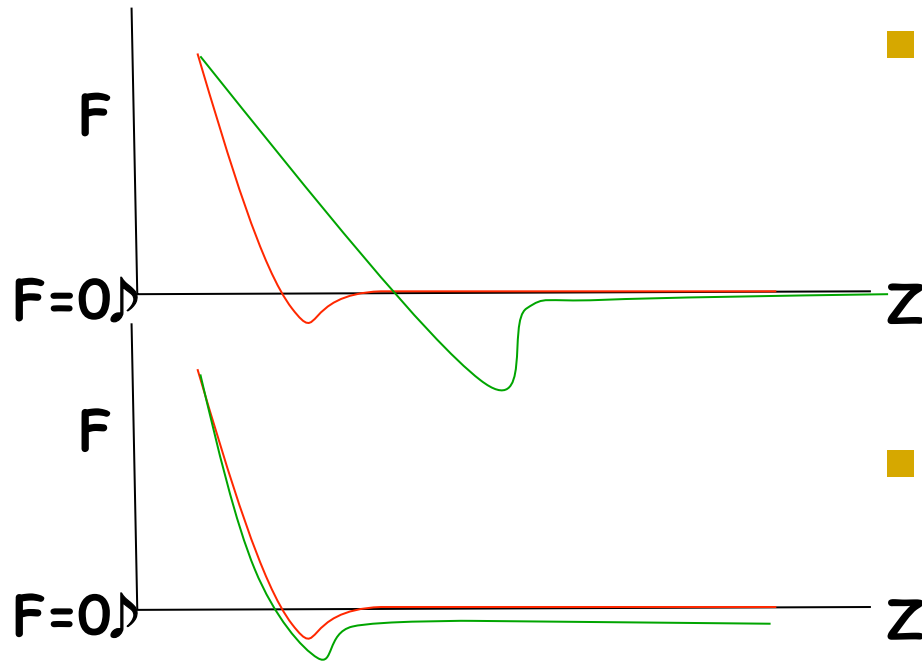


- Laser spot not centered in photodiode - recenter so that photodiode output Z is  $\sim 0$  far from sample



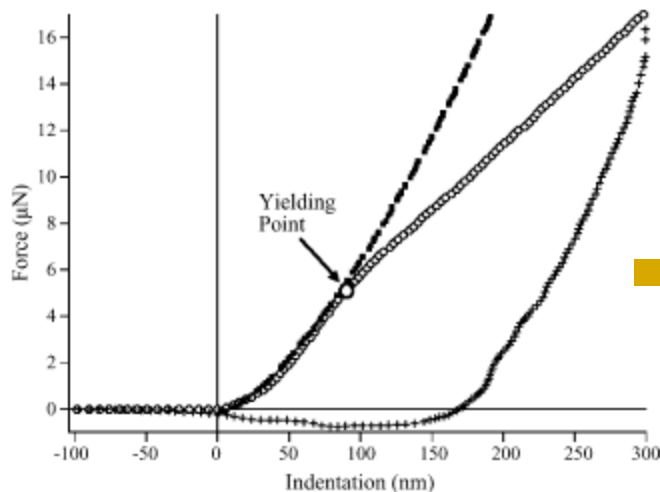
- Laser spot spilling over cantilever edge, reflecting off substrate interfering with signal back from cantilever - Focus spot better, or use non-coherent laser

# Artefacts in F-Z curves



- Z piezo hysteresis - warm up piezo first, use closed loop piezos

- Hydrodynamic drag - reduce speed



- Large indentation with plastic deformation - reduce force!

# Network for Computational Nanotechnology (NCN)

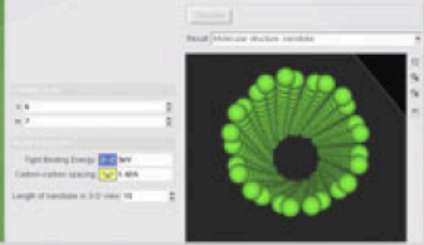


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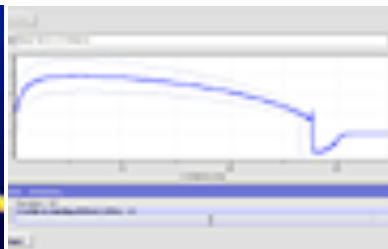
## CNTbands

electronic band structure for carbon nanotubes

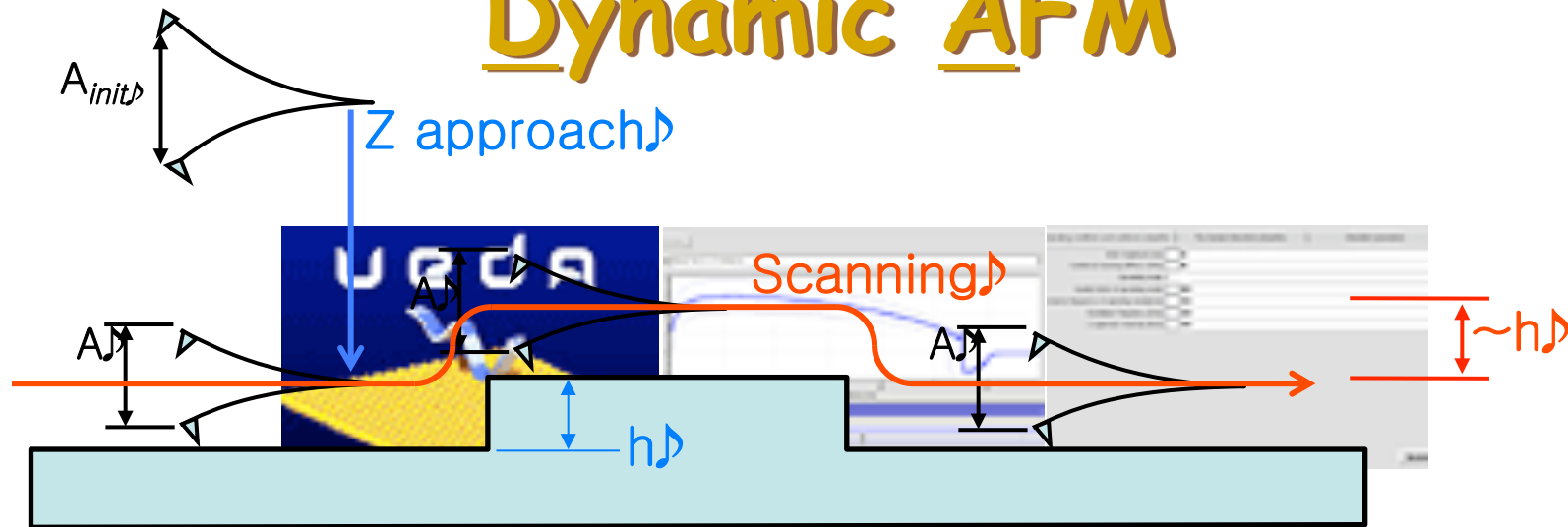


CNTbands is a Matlab script that computes  $E(k)$  and the density-of-states (DOS) vs. energy for a carbon nanotube specified by  $(n, m)$ . In addition, the script also computes some basic parameters of the nanotube such as diameter, number of hexagons in the unit cell, etc.

[Learn more](#) ▶

A screenshot of a table or data output window. The table has several columns and rows of numerical data. The text is small and difficult to read, but it appears to be a list of parameters and their corresponding values.

# VEDA- Virtual Environment for Dynamic AFM



- Many tools released on [www.nanohub.org](http://www.nanohub.org)<sup>1</sup>: F-Z, Dynamic Approach Curves, FM-approach, Amplitude Modulated Scanning
- Extension to liquids (natural, bimodal, higher harmonics etc)
- Accurate numerical simulations , convenient input parameters
- Freely accesible, web-based
- Simulations run off national teragrid and Purdue clusters
- Extensions to EFM, MFM, user specified forces on the horizon

## Next class

- VEDA tutorial (rm 236 ME building) taught by Daniel Kiracofe and John Melcher