ECE650: Reliability Physics of Nano-Transistors
Lecture 3: Reliability as a Threshold Problem

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outline of lecture 3

1. Reliability as a Threshold Problem:
   Difference between Empirical and Physical Models

2. ‘Blind Fish in a Waterfall problem’ as a prototype of Reliability Issues: Accelerated Testing

3. ‘Blind Fish in a Waterfall problem’ as a prototype of Reliability Issues: Statistical Distribution

3. Four elements of Physical Reliability

4. Conclusions
oxide degradation/breakdown/Statistics

\[ p = q^M = \left( \frac{t}{t_0} \right)^{M \alpha} \]

Theory of Accelerated Testing

Nonlinear projection for processes with threshold

Empirical projection is very difficult, if not impossible...
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Nonlinear Projection: an illustrative example

How far away from the trap-site do I need to inject the particles so that after TBD sec of diffusion no more than y percent of particles is lost?
**Accelerated Testing: Empirical Approach**

Absorption site

\[ p = 0.5(1 + \Delta) \quad q = 0.5(1 - \Delta) \]

\[ T_{av} = \frac{N_f f_j}{N} = \sum f_j f_j \]

\[ f = \frac{N_j}{N} \]

\[ p \sim 1 \]

\[ \text{decreasing } p \]

\[ \ln T_{av} \text{ years} \]

\[ v_{\text{safe}} \]

\[ p \text{ or velocity} \]

\[ \text{time} \]

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**Average Arrival Time Distribution**

\[ T(x) = \tau + [q \times T(x - \delta)] + [p \times T(x + \delta)] \]

Time to absorption after being injected at \( x \) ....

\[ \frac{T(x)}{2} - \frac{1}{2} (T(x + \delta) + T(x - \delta)) \frac{\Delta (T(x + \delta) - T(x - \delta))}{\delta^2} = \frac{\tau}{\delta^2} = 0 \]

\[ \frac{d^2 T}{dx^2} + \frac{2}{v} \frac{dT}{dx} + \frac{2}{D} = 0 \]

\[ v \equiv \frac{\delta}{2\Delta} \quad D \equiv \frac{\delta^2}{\tau} \]

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Average Arrival Time Distribution

\[ d^2T + \frac{2}{v} \frac{dT}{dx} + \frac{2}{D} = 0 \]

\[ T(x) = C_1 - C_2 x \]

\[ \frac{\tau}{\delta \Delta} (x_m - x) = \frac{\tau}{2} \left( \frac{x_m - x}{p - 0.5} \right) \]

Average lifetime diverges at small velocity, v or p -> 0

Physical vs. Empirical Projection

\[ T(x) = \frac{\tau}{2\delta} \left[ \frac{x_m - x}{p(v) - 0.5} \right] \]

- Empirical meas. & comp. simulation would not do
- Prediction is possible because t=0- is smooth
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Diverging Average vs. Statistical Distribution

Average is not enough, Statistics is critical
Derivation Of “Fishy” (or BFRW) Distribution

\[
\frac{\partial P}{\partial t} = D \frac{\partial^2 P}{\partial x^2} \quad P(x, t = 0) = \delta(x - x_0) \\
P(x, 0) = 0
\]

\[
P(x, t) = (4\pi Dt)^{-1/2} \left[ e^{-(x - x_0)^2/4Dt} - e^{-(x + x_0)^2/4Dt} \right]
\]

\[
\int_0^t f(\tau)d\tau + \int_0^L P(x, t)dx = 1 \Rightarrow f(t) = \frac{x_0}{\sqrt{4\pi Dt^3}} e^{-x_0^2/4Dt}
\]

Empirical vs. Physical Distribution

\[
f_G(t) = \frac{t^{k-1}e^{-t/\theta}}{\Gamma(k)\theta^k} \quad T_{\text{avg}} = k\theta
\]
**Long Tail of a Distribution**

\[
T(x_0) = \int_0^\infty t f(t) dt
\]

\[
= \int_0^\infty \frac{x_0 t}{\sqrt{4\pi Dt^3}} e^{-x_0^2 / 4Dt} dt \to \infty
\]

\[
T(x_0) = \int_0^\infty t f(t) dt
\]

\[
= \int_0^\infty t \times t^{-3/2} dt \to \infty
\]

Although there is no average, a huge fraction of field-return can occur within short period of time.

**Aside: long tail of a distribution**

1D model for ......

- field-return of components
- charge loss in Nanocrystal Flash
- release of proteins from inside the cells, etc.
- Drug release from capsules, etc.
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Four Elements of Physical Reliability

1. Theory of Stress Acceleration
   \[ T(x) = \frac{\tau}{2\delta} \left( \frac{x_m - x}{p(v) - 0.5} \right) \]
2. Theory of Stochastic Distribution
   \[ PDF = \frac{x_o}{\sqrt{4\pi Dt}} e^{-x^2/(4Dt)} \]
3. Characterization \( D, \delta, \tau \)
4. Analysis of Statistical Data

* Realistic Specification
Conclusions

We have discussed the difference between empirical vs. physical models and have demonstrated how the presence of thresholds makes the acceleration model inherently nonlinear.

Statistical Distribution is Physical. And physics based distributions differ significantly from empirical presumption about such distributions (Gaussian).

Reliability problems are too complex to be exclusively predicted from first principles. Characterization experiments is a must.