

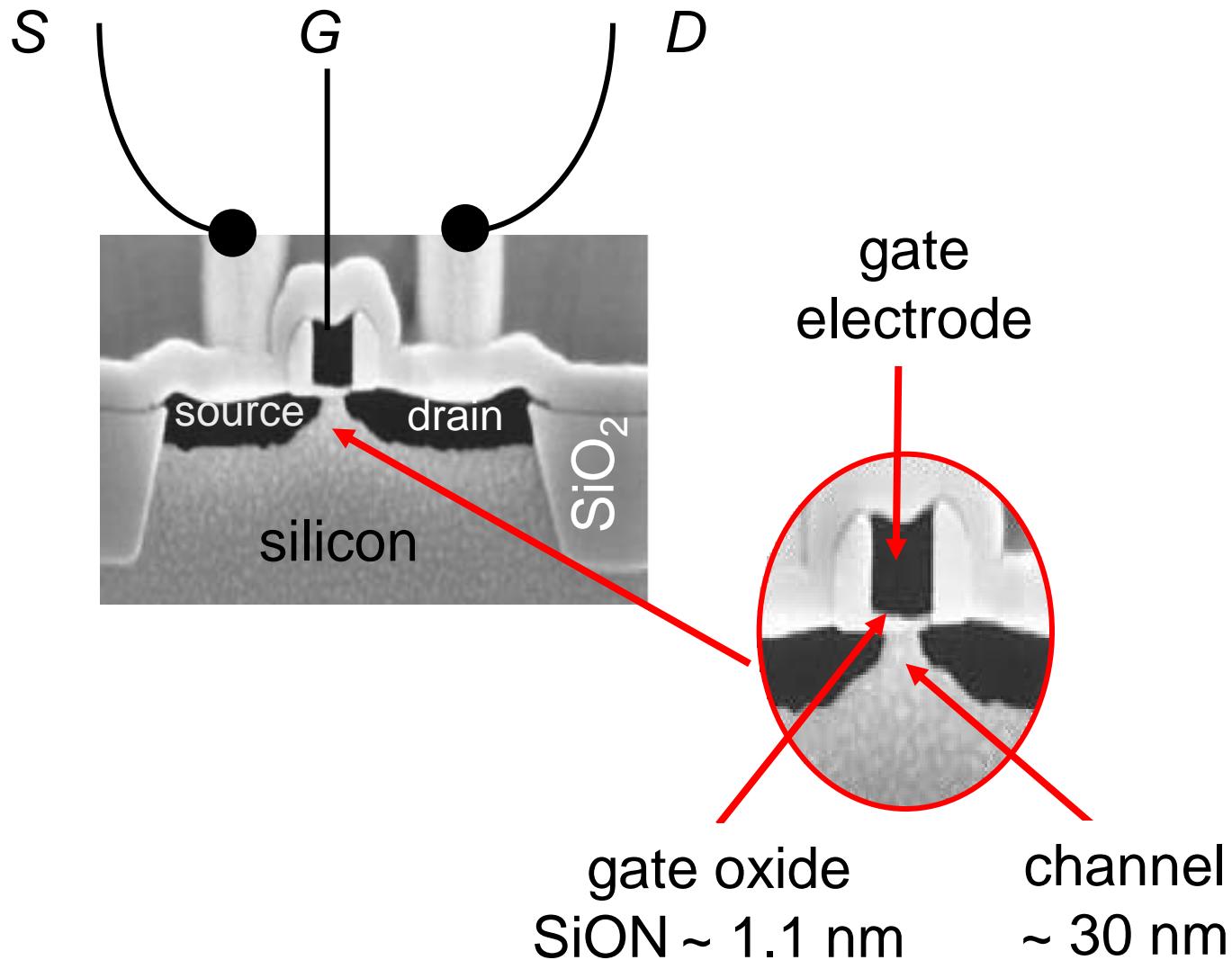
NCN@Purdue "Electronics from the Bottom Up" Summer School: July 12-16, 2010

# Nanotransistors: A Bottom Up View

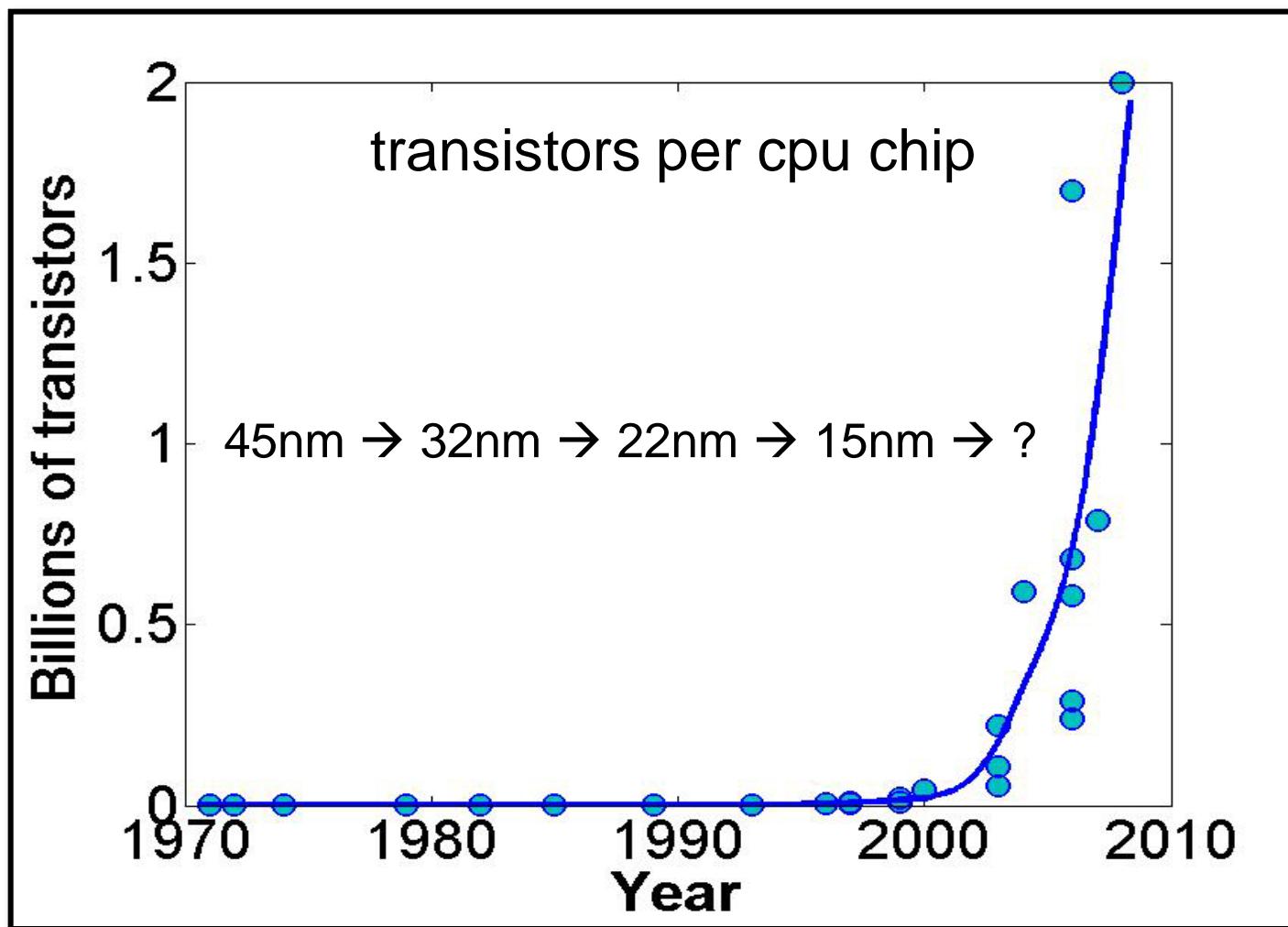
**Mark Lundstrom**

Electrical and Computer Engineering  
and  
Network for Computational Nanotechnology  
Birck Nanotechnology Center  
Purdue University, West Lafayette, Indiana USA

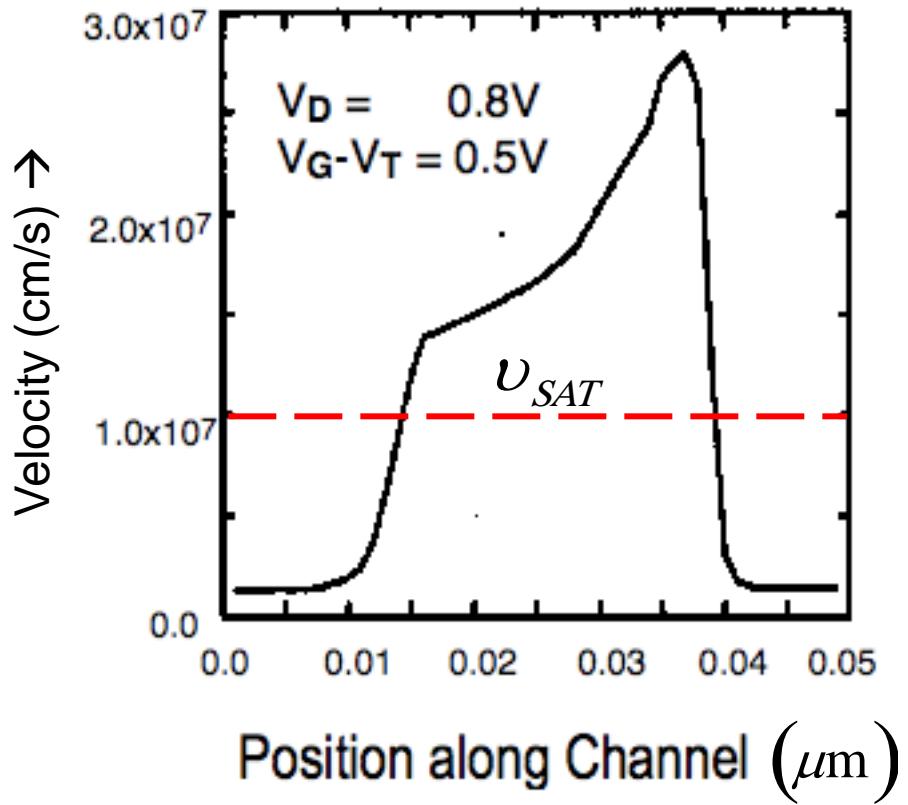
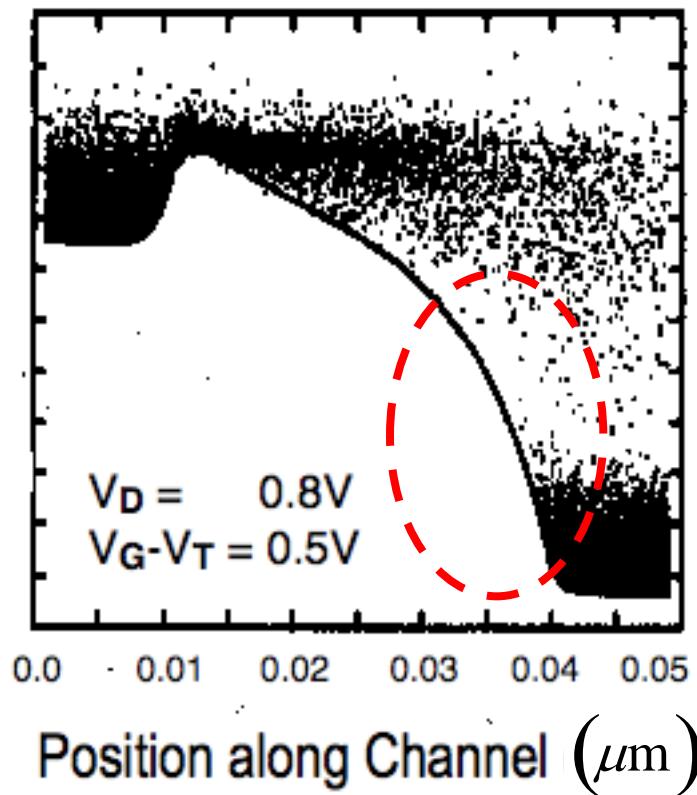
# nanoscale MOSFETs 2010



# microelectronics → nanoelectronics

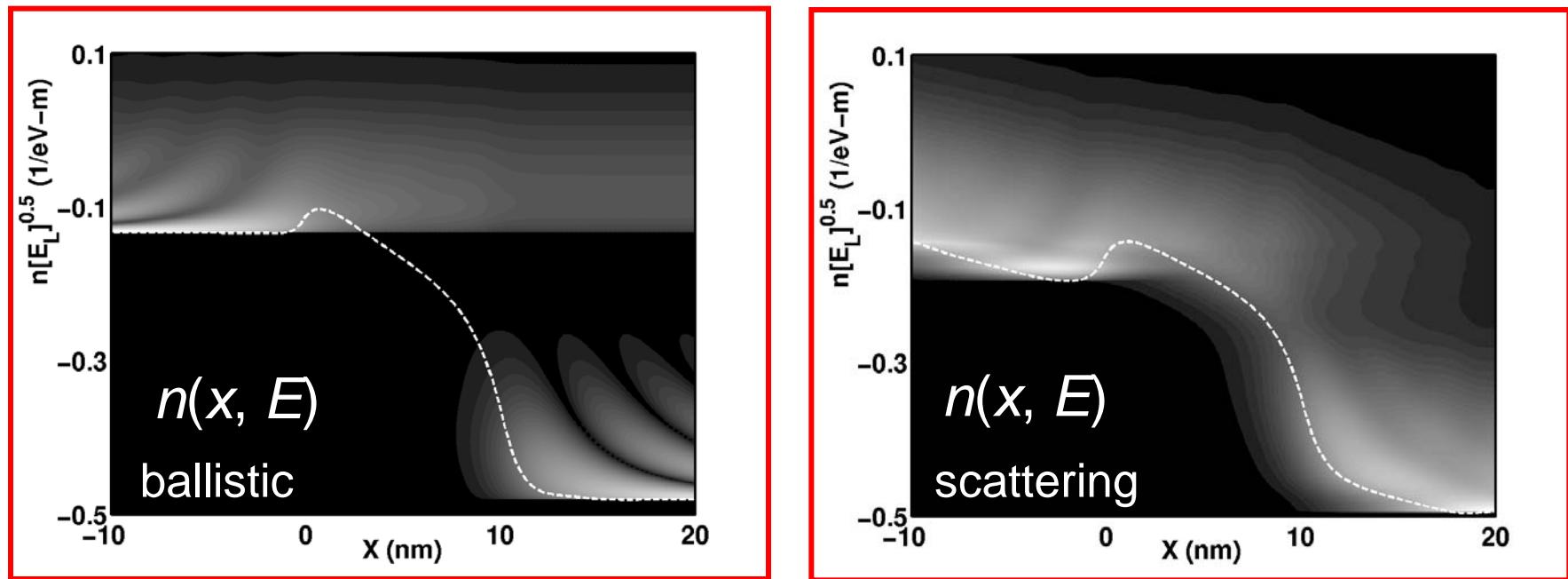


# carrier transport nanoscale MOSFETs



D. Frank, S. Laux, and M. Fischetti, Int. Electron Dev. Mtg., Dec., 1992.

# quantum transport



(10nm channel length, double gate, Si MOSFET simulated with nanoMOS.)

# understanding

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“It is nice to know that the computer understands the problem, but I would like to understand it too.”

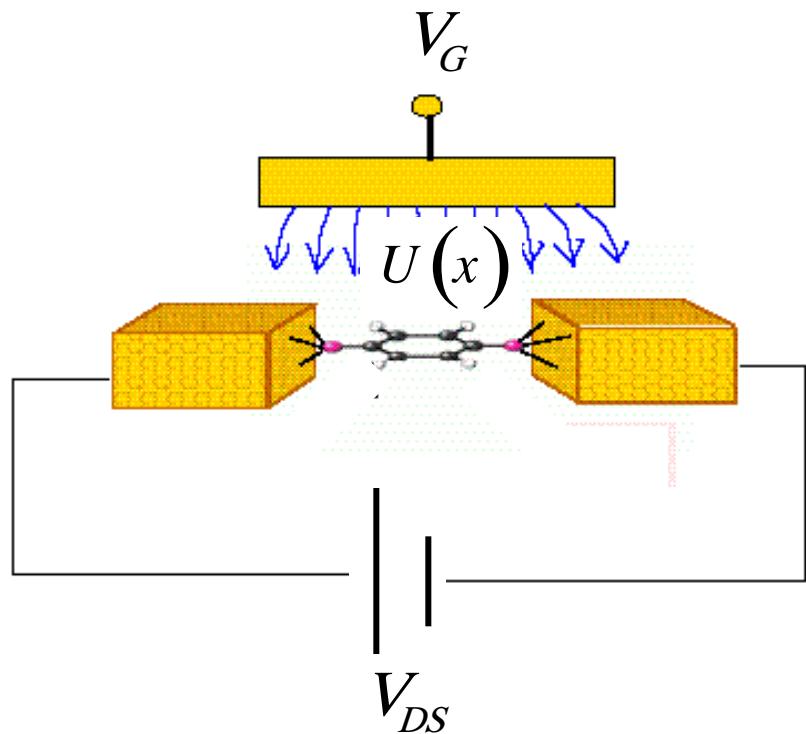
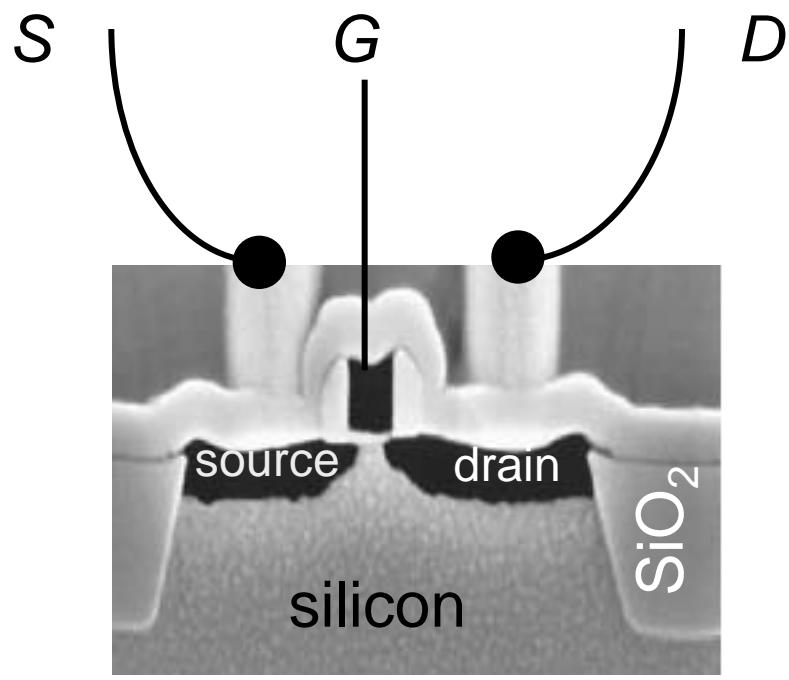
Eugene Wigner

# objectives of this lecture

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- 1) Describe a simple, physical picture of the nanoscale MOSFET (complement, not replace simulations).
- 2) Discuss ballistic limits and velocity saturation in nanotransistors.
- 3) Compare to experimental results for Si and III-V FETs.
- 4) Discuss scattering in nano-MOSFETs.

# bottom up approach



A. W. Ghosh, et al., *Nano Lett.*, **4**, 565, 2004.

# outline

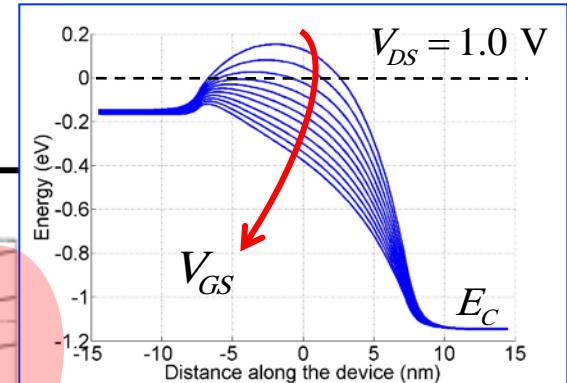
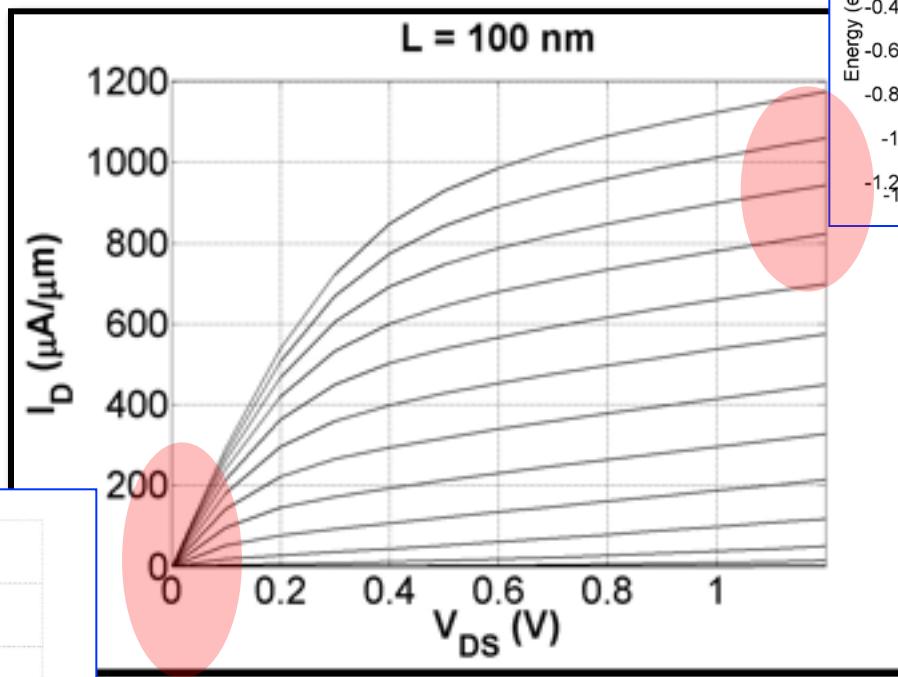
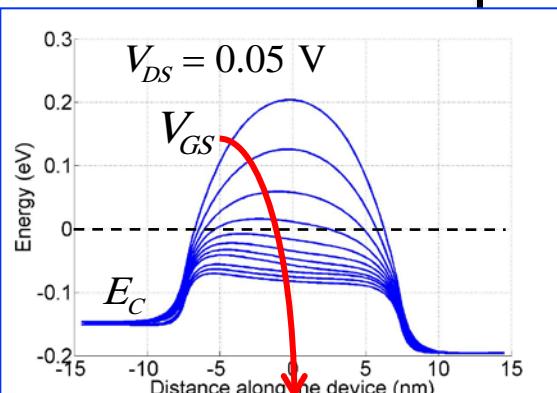
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- 1) Introduction
- 2) The nano-MOSFET
- 3) The ballistic MOSFET
- 4) Scattering in nano-MOSFETs
- 5) Summary

# how transistors work

2007 N-MOSFET

electron energy  
vs. position



electron energy  
vs. position

(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

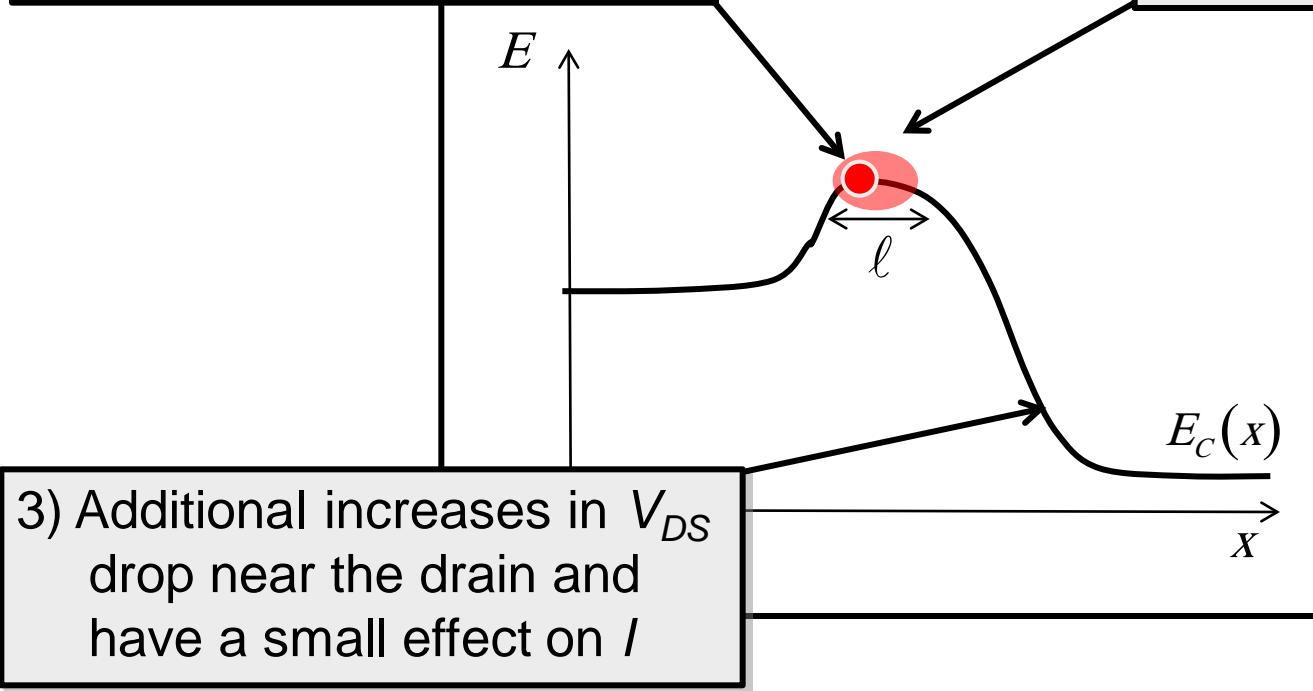
E.O. Johnson, "The IGFET: A Bipolar Transistor in Disguise,"  
*RCA Review*, 1973

# MOSFETs are barrier controlled devices

1) "Well-tempered MOSFET"

$$Q_I(0) \approx C_{ox}(V_G - V_T)$$

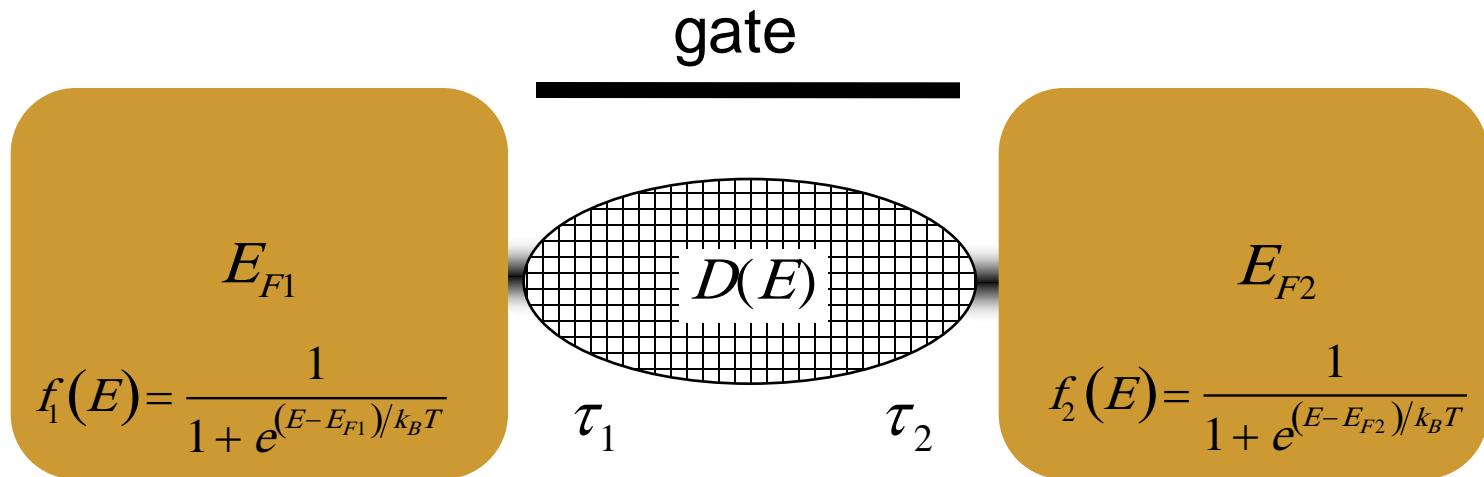
2) region under strong control of gate



M. Lundstrom, *IEEE EDL*, **18**, 361,  
1997.

A. Khakifirooz, O. M. Nayfeh, D. A. Antoniadis, *IEEE TED*, **56**, pp. 1674-1680, 2009.

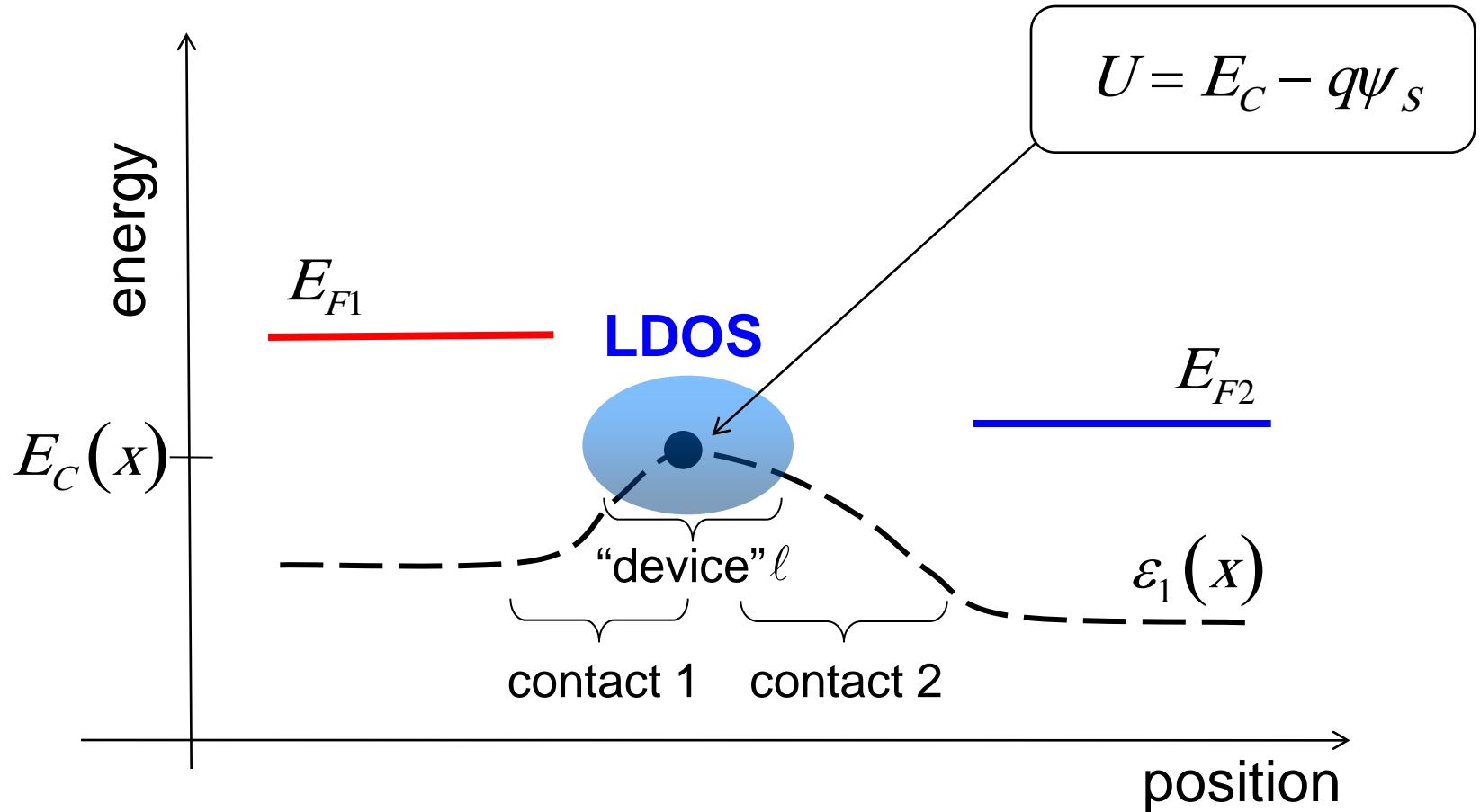
# current flows when the Fermi-levels are different



$$N = \int \frac{D(E)}{2} [ f_1(E) + f_2(E) ] dE$$

$$I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE$$

# “top of the barrier model”

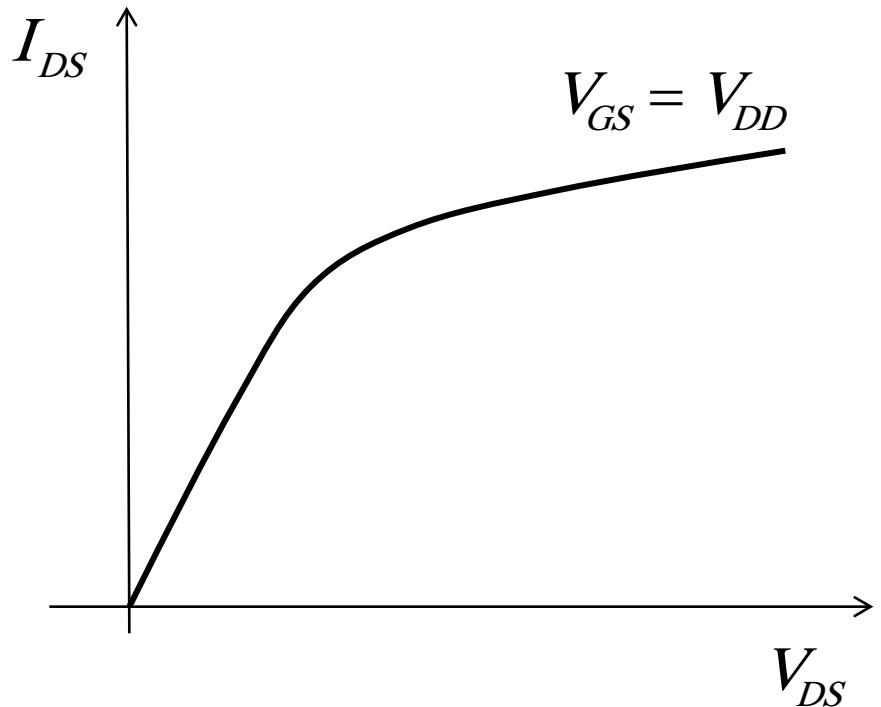


# outline

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- 1) Introduction
- 2) The nano-MOSFET
- 3) The ballistic MOSFET**
- 4) Scattering in nano-MOSFETs
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# ballistic MOSFET



$$I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE$$

$$T(E) = 1$$

$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T_e}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F1} + qV_{DS})/k_B T_e}}$$

$$M(E) = ?$$

+ MOS electrostatics

# number of modes, $M(E)$

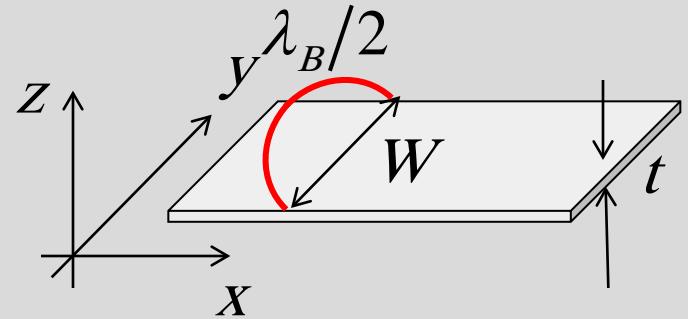
$$M_{2D}(E) = W \frac{h}{4} \langle v_x^+ \rangle D_{2D}(E)$$

assume parabolic bands:

$$v = \sqrt{2(E - E_C)/m^*}$$

$$\langle v_x^+ \rangle = 2v/\pi$$

$$D_{2D}(E) = g_V m^*/\pi\hbar^2$$



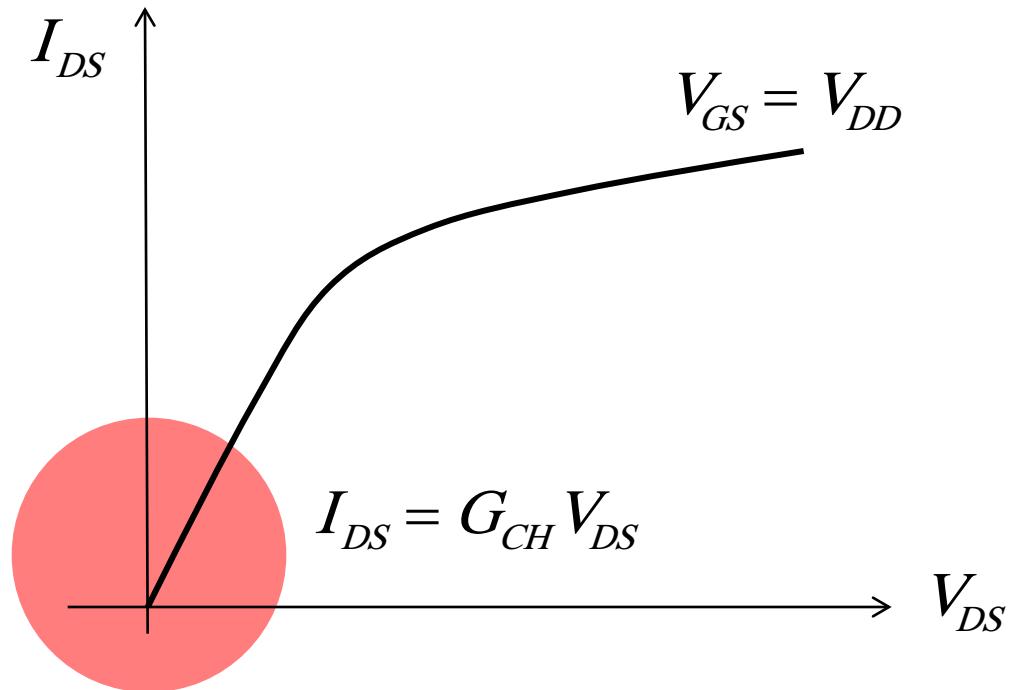
lowest mode

$$E(k) = \hbar^2 k^2 / 2m^*$$

$$k = 2\pi/\lambda_B \quad M = \frac{W}{\lambda_B/2}$$

$$M_{2D}(E) = g_V W \sqrt{2m^*(E - E_C)} / \pi\hbar$$

# ballistic MOSFET: linear region



near-equilibrium  $f_1 \approx f_2$

$$\begin{aligned} I &= \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE \rightarrow \\ &= \left[ \frac{2q^2}{h} \int T(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \right] V_{DS} \end{aligned}$$

# linear region with MB statistics

$$G_{CH} = \frac{2q^2}{h} \int_{E_C}^{\infty} T(E) M_{2D}(E) \left( -\frac{\partial f_0}{\partial E} \right) dE$$

Boltzmann statistics:

$$-\frac{\partial f_0}{\partial E} = \frac{f_0}{k_B T_e}$$

$$G_{CH} = W q n_S \frac{v_T}{2 k_B T_e / q}$$

$$qn_S \approx C_{ox} (V_{GS} - V_T)$$

$$G_{CH} = W C_{ox} \frac{v_T}{2 k_B T_e / q} (V_{GS} - V_T)$$

$$M_{2D}(E) = g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi \hbar}$$

$$T(E) = 1 \quad (\text{ballistic})$$

$$\begin{aligned} n_S &= N_{2D} e^{(E_F - E_C)/k_B T_e} \\ &= g_V \frac{m^* k_B T_e}{\pi \hbar^2} e^{(E_F - E_C)/k_B T_e} \end{aligned}$$

$$v_T = \sqrt{\frac{2 k_B T_e}{\pi m^*}}$$

# linear region at $T_e = 0K$

---

$$G_{CH} = \frac{2q^2}{h} \int_{E_C}^{\infty} T(E) M_{2D}(E) \left( -\frac{\partial f_0}{\partial E} \right) dE$$

Degenerate statistics:  $-\frac{\partial f_0}{\partial E} = \delta(E_F)$

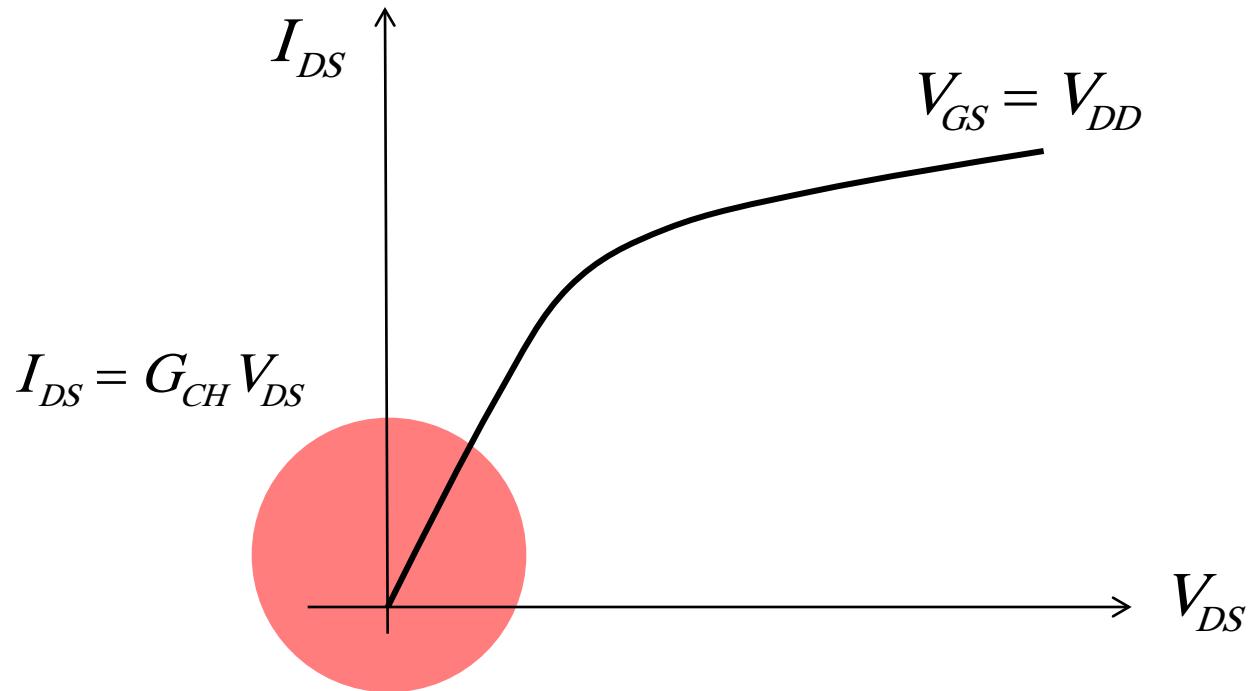
$$G_{CH} = \frac{2q^2}{h} M(E_F)$$

$$M_{2D}(E) = g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi \hbar}$$

$$T(E) = 1 \quad (\text{ballistic})$$

$$n_s = g_V \frac{m^* k_B T_e}{\pi \hbar^2} (E_F - E_C)$$

# ballistic MOSFET: linear region



near-equilibrium     $f_1 \approx f_2$

$$I_{DS} = G_{CH} V_{DS}$$

$$I_{DS} = WC_{ox} \frac{v_T}{2k_B T_e/q} (V_{GS} - V_T) V_{DS}$$

## aside: relation to conventional expression

ballistic MOSFET

$$I_{DS} = WC_{ox} \frac{v_T}{2k_B T_e / q} (V_{GS} - V_T) V_{DS}$$

conventional MOSFET

$$I_{DS} = \frac{W}{L} C_{ox} \mu_n (V_{GS} - V_T) V_{DS}$$

$$I_{DS} = \frac{W}{L} C_{ox} \frac{v_T L}{2k_B T_e / q} (V_{GS} - V_T) V_{DS}$$

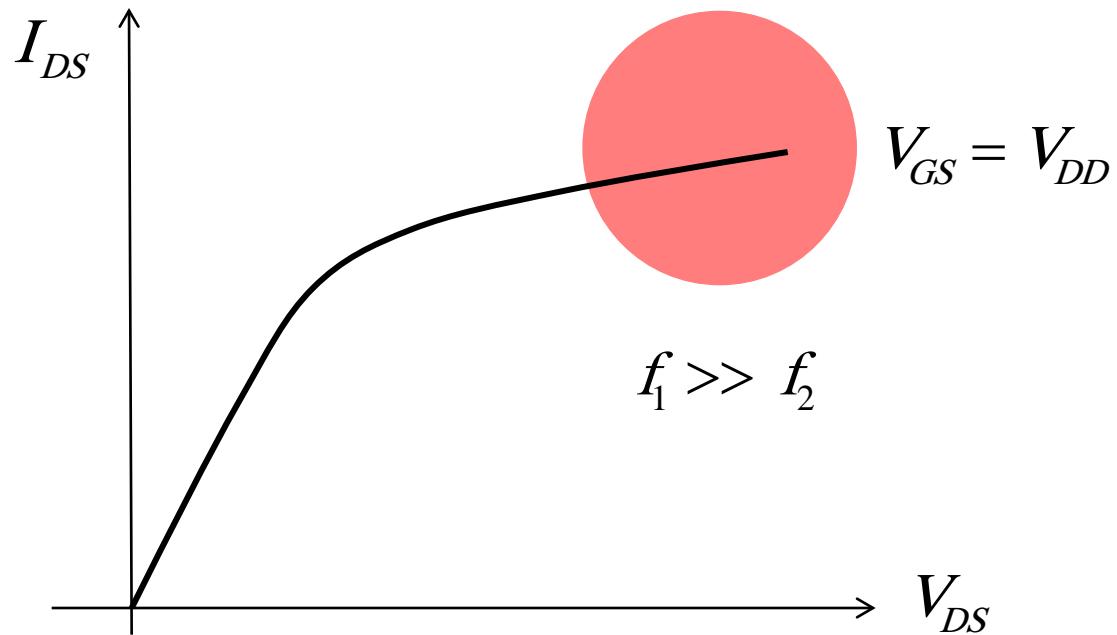
$$D_n = \frac{v_T \lambda_0}{2} \quad \mu_n = \frac{D_n}{(k_B T_e / q)} \quad \mu_B \equiv \frac{v_T L}{2 k_B T_e / q} \quad \text{"ballistic mobility"}$$

$$I_{DS} = \frac{W}{L} C_{ox} \mu_B (V_{GS} - V_T) V_{DS}$$

$$\mu_n \rightarrow \mu_B$$

# ballistic MOSFET: on-current

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$$I_{DS} = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE \rightarrow I_{DS} = \frac{2q}{h} \int T(E) M(E) f_1 dE$$

# saturated region with MB statistics

$$I_{DS} = \frac{2q^2}{h} \int_{E_C}^{\infty} T(E) M_{2D}(E) f_1(E) dE$$

Boltzmann statistics:  $f_1 \approx e^{-(E_C - E_F)/k_B T}$

$$I_{DS} = W q n_S v_T$$

$$I_{DS} = W C_{ox} v_T (V_{GS} - V_T)$$

$$M_{2D}(E) = g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi \hbar}$$

$$T(E) = 1$$

$$\begin{aligned} n_S &= \frac{N_{2D}}{2} e^{(E_F - E_C)/k_B T_e} \\ &= g_V \frac{m^*}{2\pi\hbar^2} e^{(E_F - E_C)/k_B T_e} \end{aligned}$$

$$v_T = \sqrt{\frac{2k_B T_e}{\pi m^*}} = \langle v_x^+ \rangle$$

## aside: carrier densities at the top of the barrier

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$$n_s = \int \left[ \frac{D_{2D}(E)}{2} f_1(E) + \frac{D_{2D}(E)}{2} f_2(E) \right] dE$$

1) low  $V_{DS}$  (near-equilibrium):

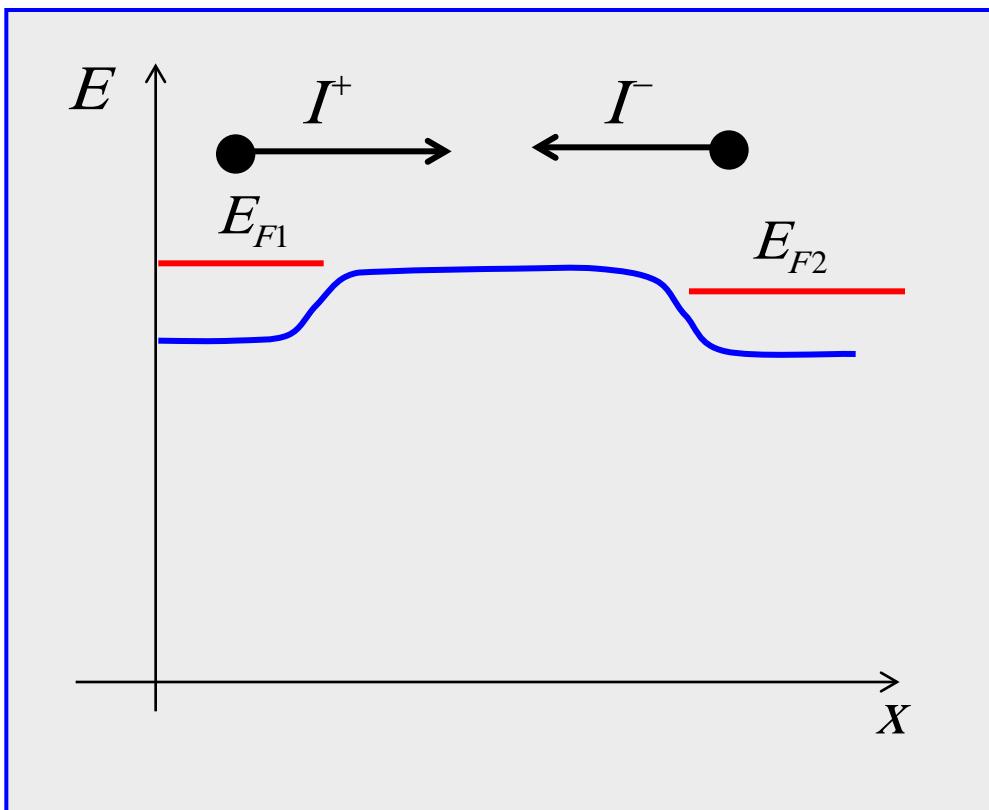
$$f_1 \approx f_2 \approx f_0 \quad n_s \approx \int D_{2D}(E) f_0(E) dE = N_{2D} e^{(E_C - E_F)/k_B T_e}$$

2) high  $V_{DS}$  (far from equilibrium):

$$f_1 \approx f_0; \quad f_2 \approx 0 \quad n_s \approx \int \frac{D_{2D}(E)}{2} f_0(E) dE = \frac{N_{2D}}{2} e^{(E_C - E_F)/k_B T_e}$$

But.....  $n_s \approx C_{ox}(V_{GS} - V_T)$  in both cases!

# under low $V_{DS}$



$$I^+ = qn_S^+ v_T \quad I^- = qn_S^- v_T$$

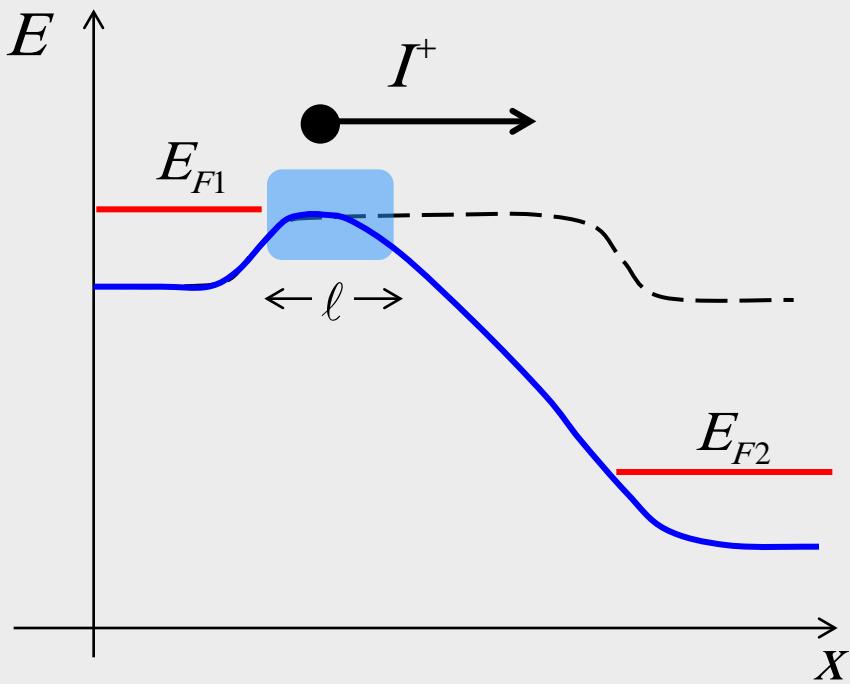
$$I^+ ; \quad I^- \quad n_S^+ ; \quad n_S^-$$

Under low drain bias, both positive and negative velocity states are occupied. The total density is given by MOS electrostatics:

$$n_S \approx C_{ox} (V_{GS} - V_T)$$

# under high $V_{DS}$

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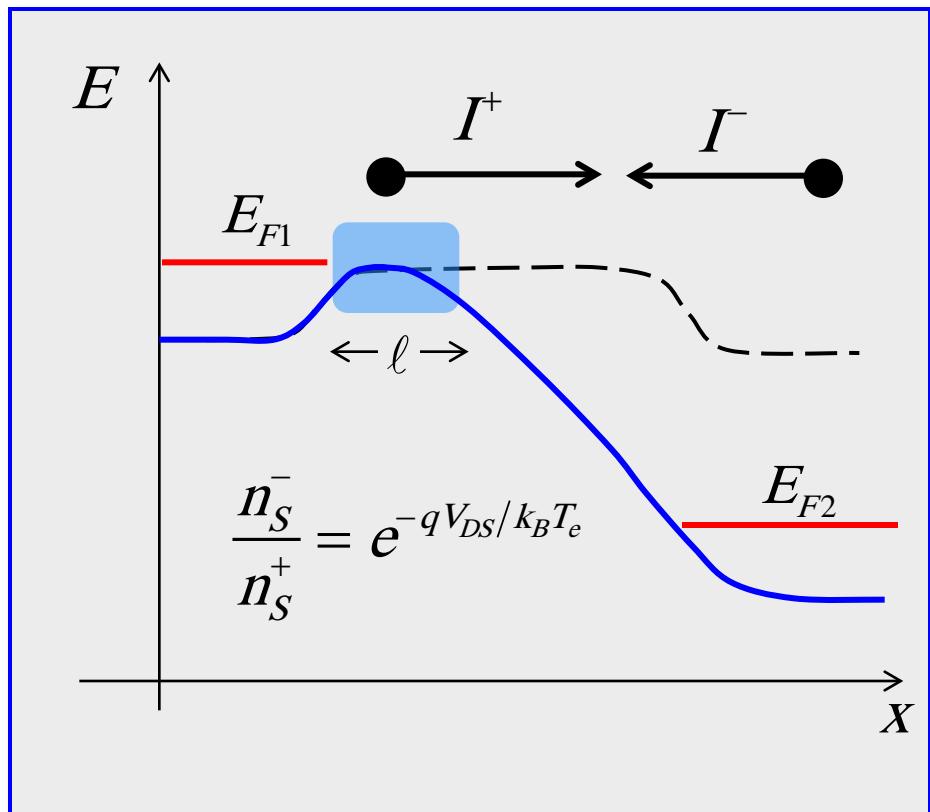
$$I^+ = qn_S^+ v_T$$

$$I^- \approx 0 \quad n_S^- \approx 0$$

Under high drain bias, only positive negative velocity states are occupied, **but** the total density is still given by MOS electrostatics:

$$n_S \approx C_{ox} (V_{GS} - V_T)$$

# velocity saturation under high $V_{DS}$



Velocity saturates in a ballistic MOSFET but at the top of the barrier, where  $E$ -field = 0.

$$I^+ = qn_S^+v_T \quad I^- = qn_S^-v_T$$

$$\langle v \rangle = \frac{I_{DS}}{qn_S} = \frac{(I^+ - I^-)}{qn_S}$$

$$\langle v \rangle = v_T \frac{\left(1 - e^{-qV_{DS}/k_B T_e}\right)}{\left(1 + e^{-qV_{DS}/k_B T_e}\right)}$$

low  $V_{DS}$ :

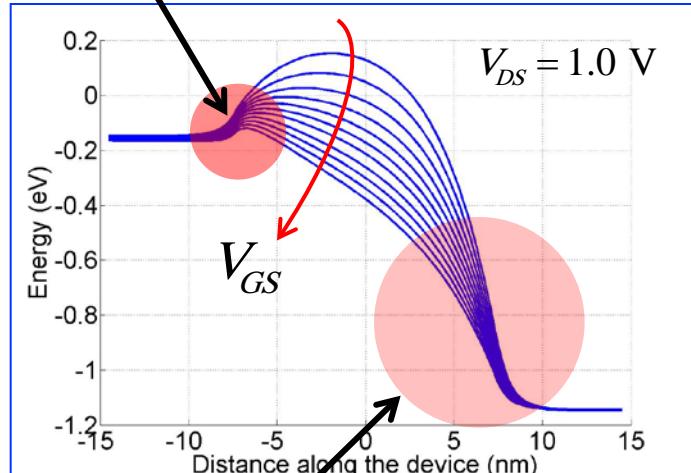
$$\langle v \rangle \propto V_{DS}$$

high  $V_{DS}$ :

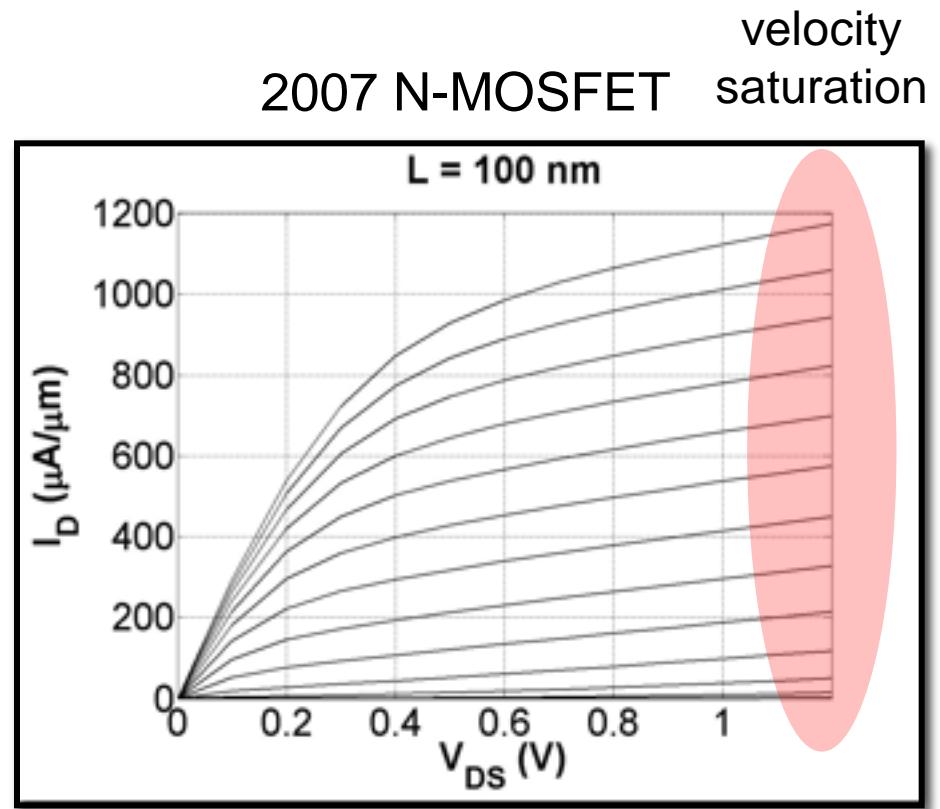
$$\langle v \rangle \rightarrow v_T$$

# velocity saturation in a ballistic MOSFET

$$v = v_T \approx 1.2 \times 10^7 \text{ cm/s}$$

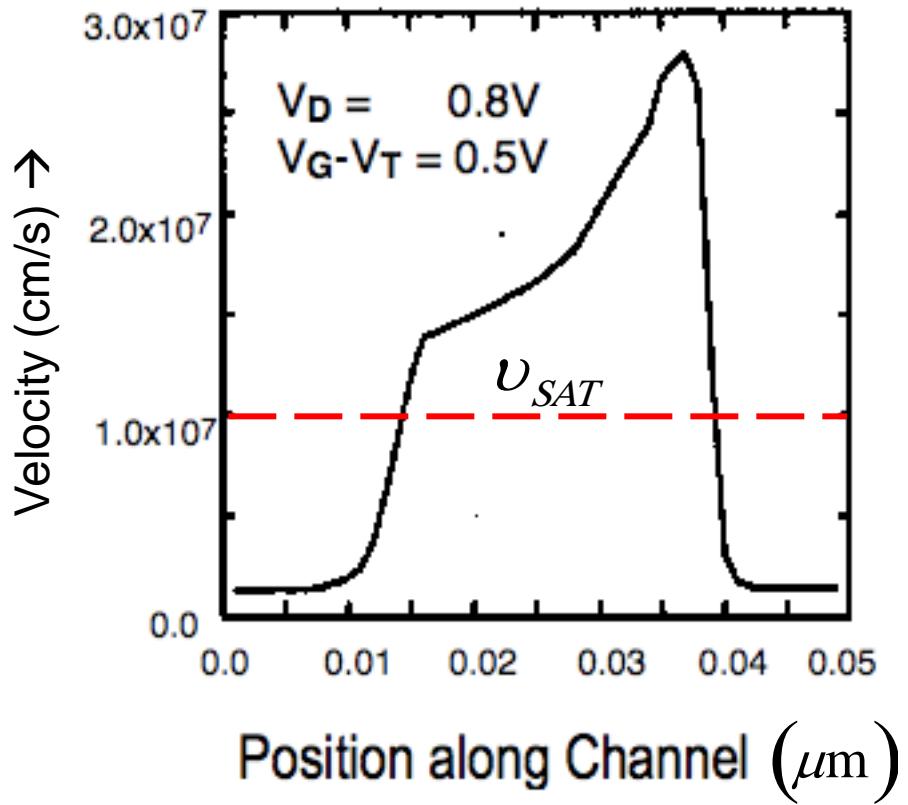
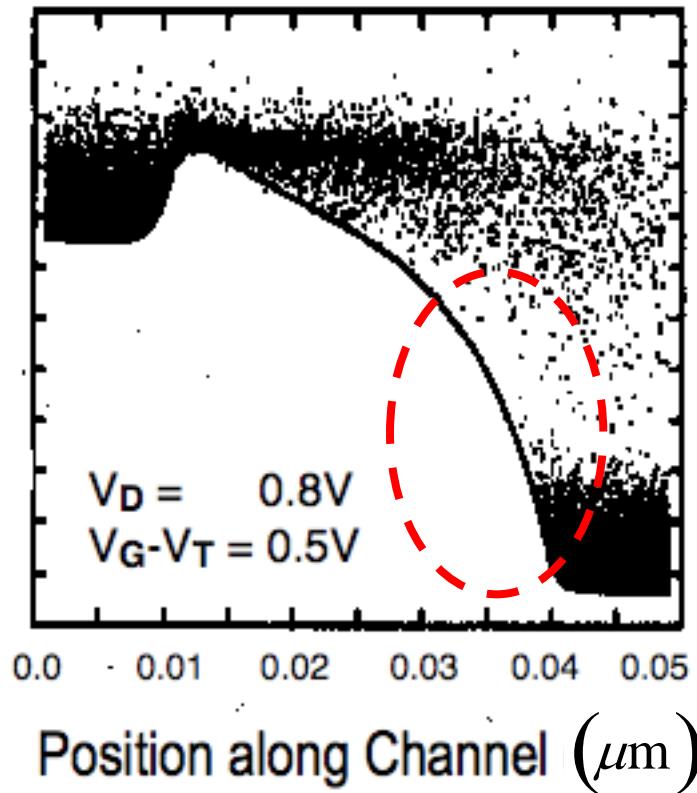


$$v = v_{sat} \approx 1.0 \times 10^7 \text{ cm/s}$$



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

# carrier transport nanoscale MOSFETs



D. Frank, S. Laux, and M. Fischetti, Int. Electron Dev. Mtg., Dec., 1992.

## aside: relation to conventional expression

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ballistic MOSFET

$$I_{ON} = WC_{ox}v_T(V_{GS} - V_T)$$

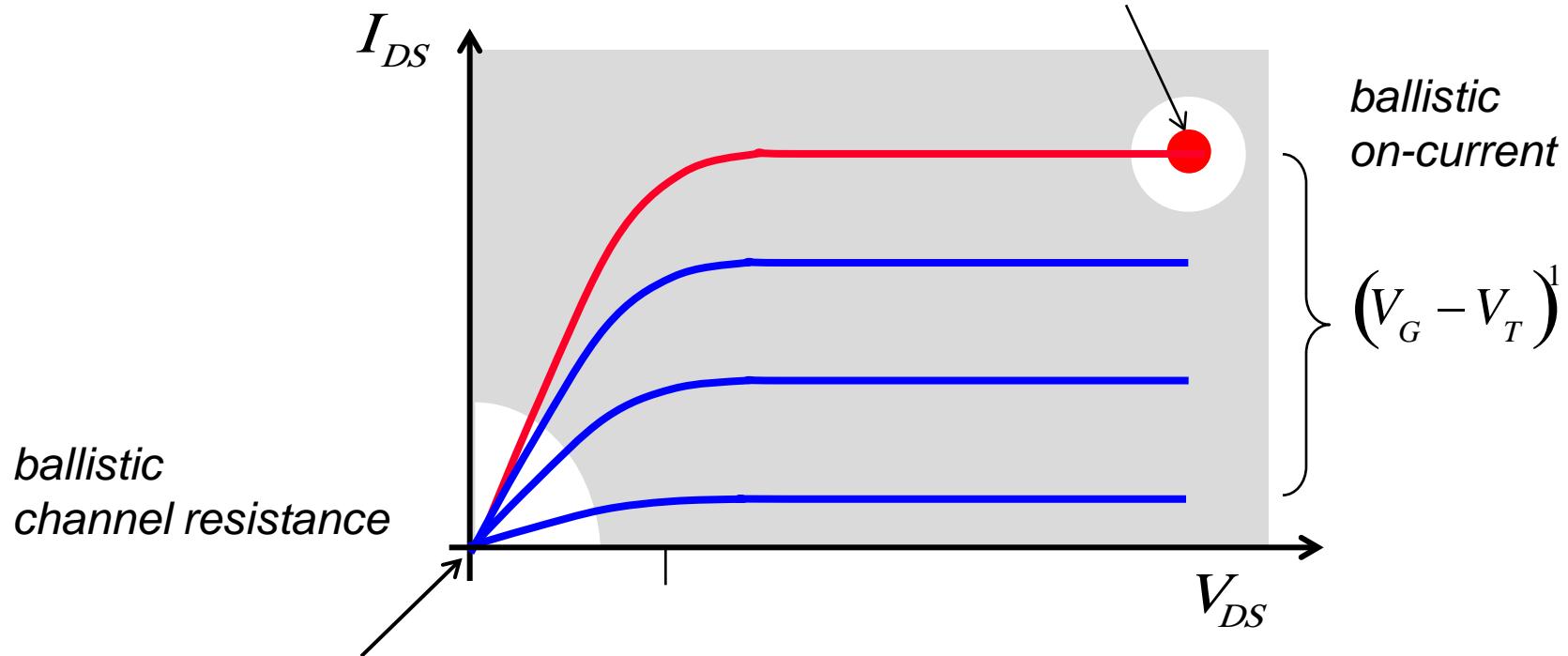
conventional MOSFET

$$I_{DS} = WC_{ox}v_{sat}(V_{GS} - V_T)$$

$$v_{sat} \rightarrow v_T$$

# the ballistic IV (Boltzmann statistics)

$$I_{DS}(\text{on}) = Wv_T C_{ox} (V_{GS} - V_T)$$



$$I_{DS} = R_{CH} V_{DS} = WC_{ox} \frac{v_T}{(2k_B T_e/q)} (V_{GS} - V_T) V_{DS}$$

## comparison with experiment: silicon

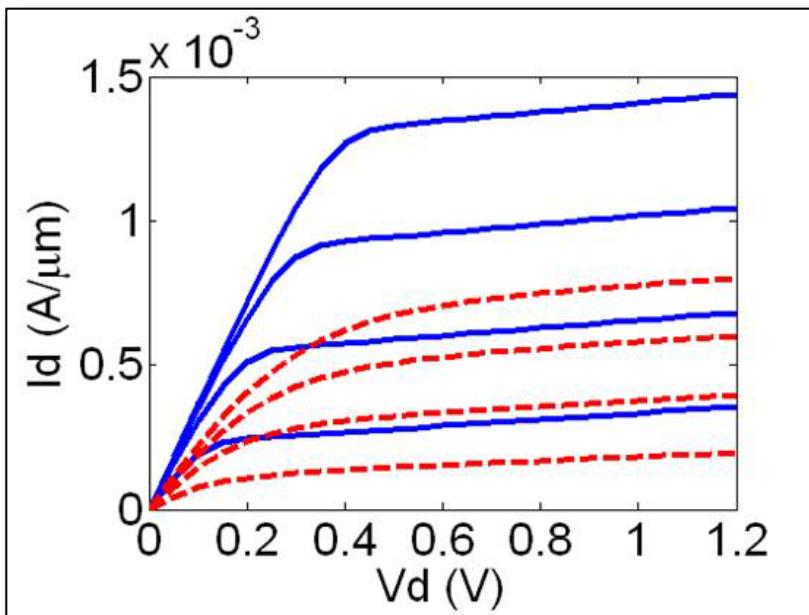
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$$L_G = 60 \text{ nm}$$

$$I_{Dlin}/I_{ballistic} \approx 0.2$$

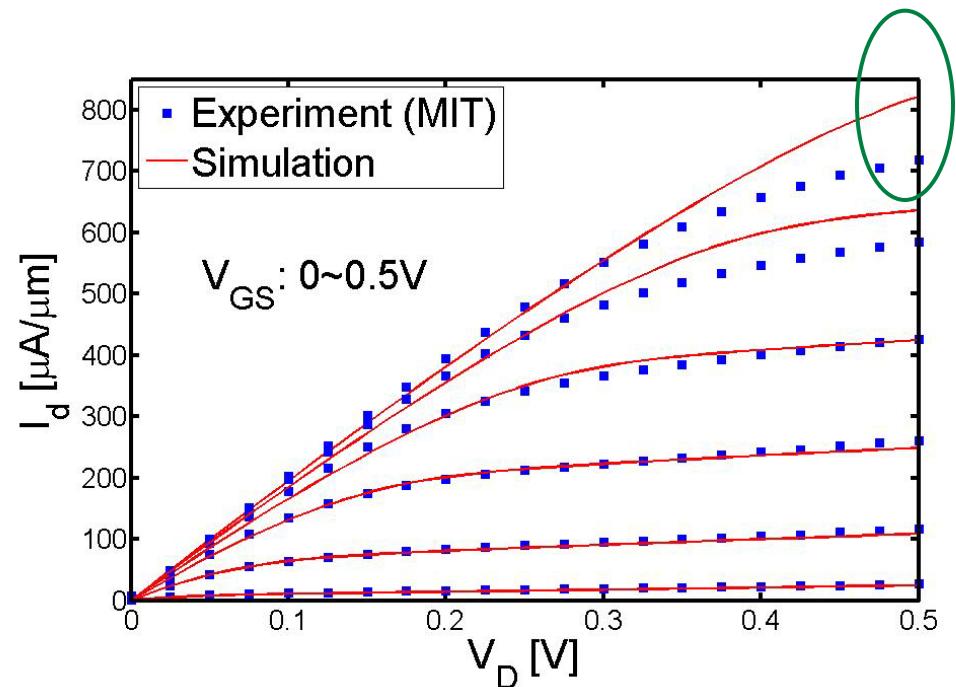
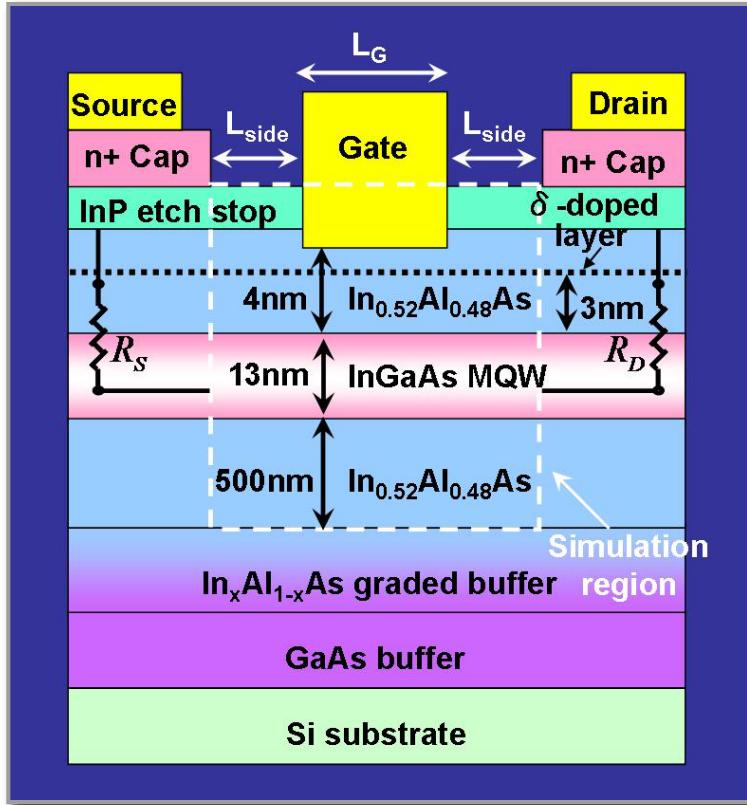
$$I_{ON}/I_{ballistic} \approx 0.6$$

- Si MOSFETs deliver > one-half of the ballistic on-current.
- MOSFETs operate closer to the ballistic limit under high  $V_{DS}$ .



C. Jeong, et al. "Backscattering analysis of Si MOSFETs," *IEEE Trans. Electron Dev.*, 56, 2762, 2009.

# comparison with experiment: InGaAs HEMTs



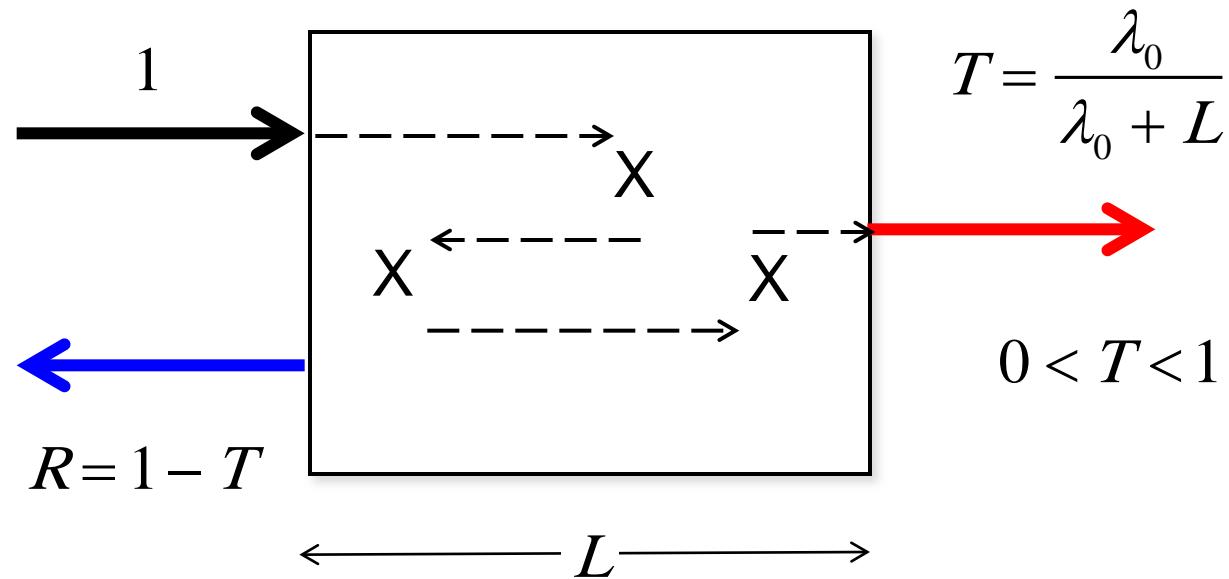
Jesus del Alamo group (MIT)

# outline

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- 1) Introduction
- 2) The nano-MOSFET
- 3) The ballistic MOSFET
- 4) Scattering in nano-MOSFETs**
- 5) Summary

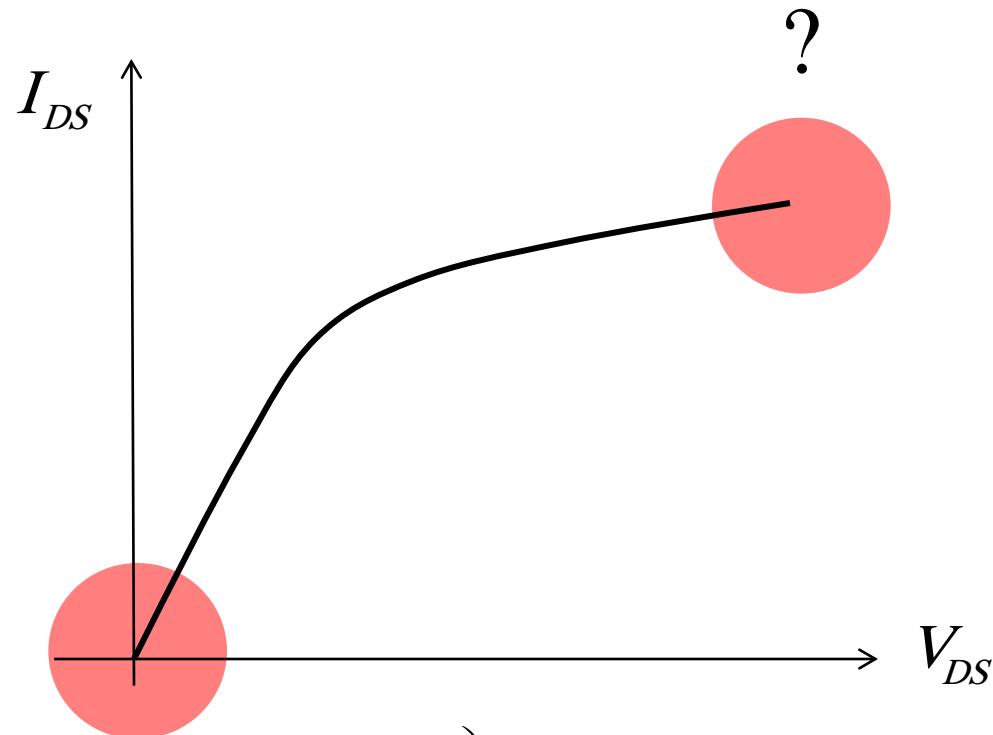
# transmission and carrier scattering



$\lambda_0$  is the mean-free-path for backscattering

$$I_{DS} \rightarrow TI_{DS} ?$$

# the quasi-ballistic MOSFET



$$I_D = T \left( W C_{ox} (V_{GS} - V_T) \frac{v_T}{(2k_B T_e / q)} \right) V_{DS} \quad \rightarrow T_{lin} \approx 0.2$$

# on current and transmission

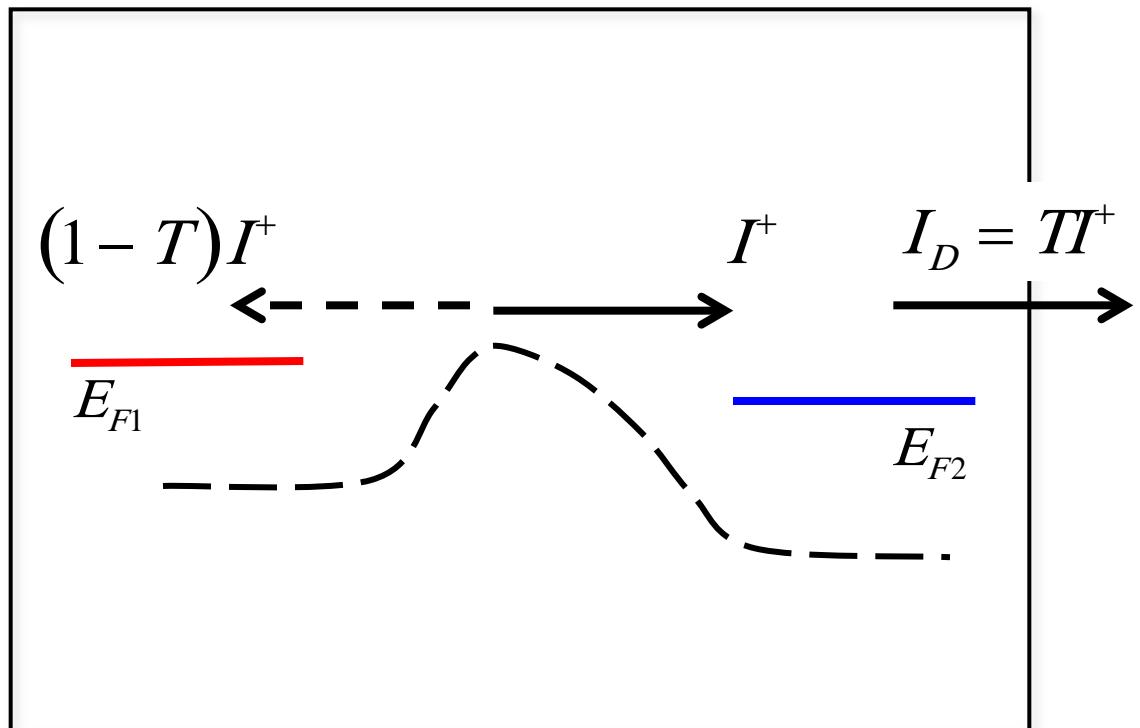
$$Q_i(0) = \frac{I_{BALL}^+}{Wv_T}$$

$$Q_i(0) = \frac{I^+ + (1-T)I^+}{Wv_T}$$

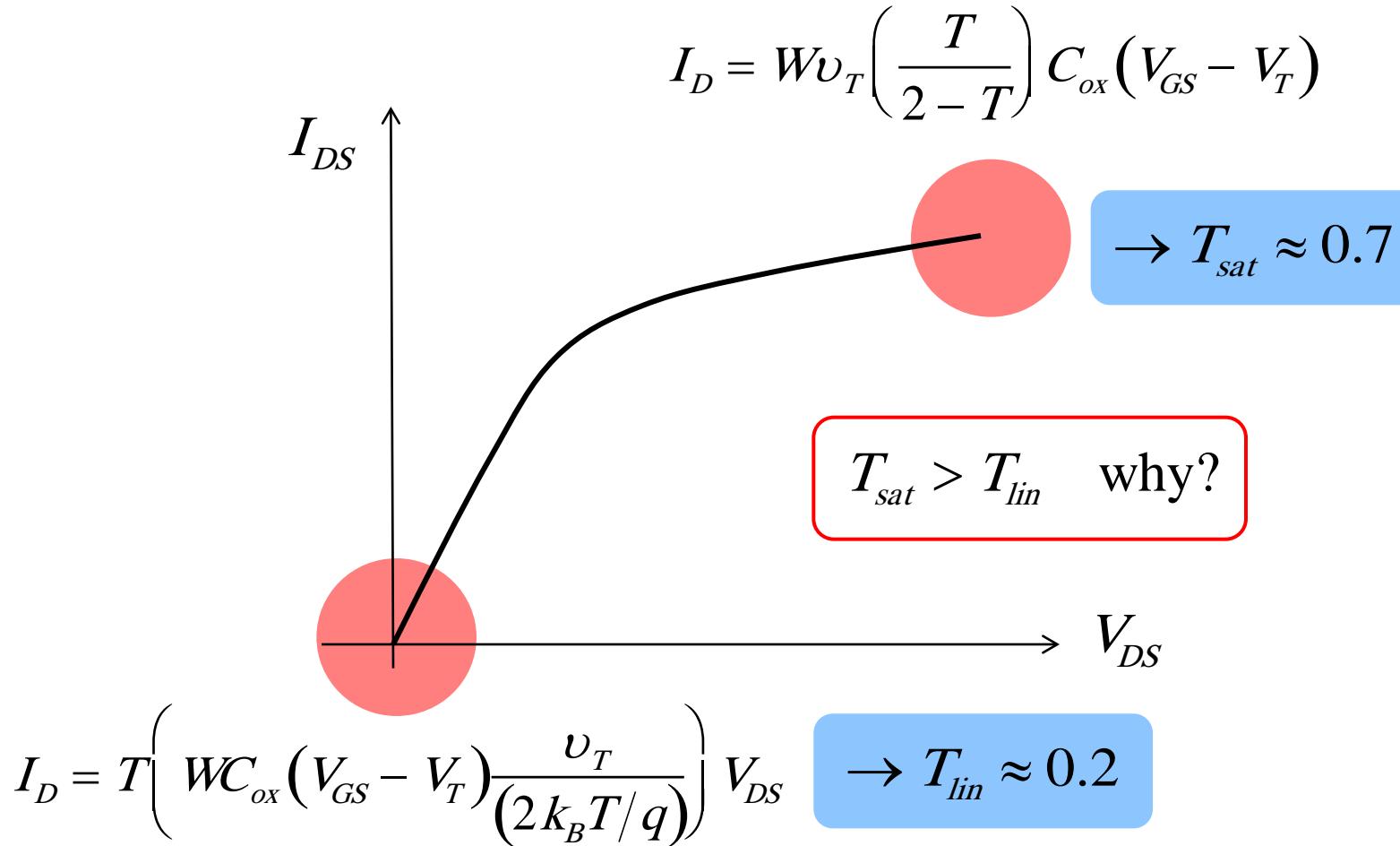
But the charge is the same in both cases.

$$I^+ = \frac{I_{BALL}^+}{(2-T)}$$

$$I_{ON} = \left( \frac{T}{2-T} \right) I_{BALL}$$

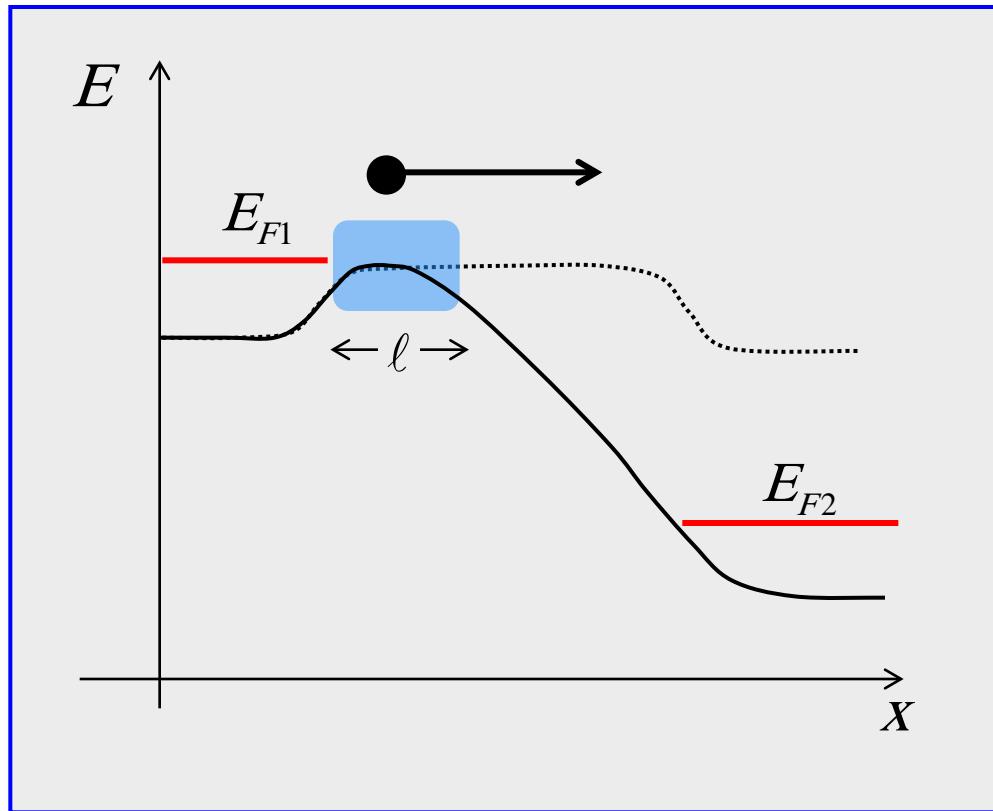


# the quasi-ballistic MOSFET



# scattering under high $V_{DS}$

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$$T_{lin} = \frac{\lambda_0}{\lambda_0 + L}$$

$$L \rightarrow \infty$$

$$T_{sat} = \frac{\lambda_0}{\lambda_0 + \infty}$$

$$\ell \ll L$$

$$T_{sat} > T_{lin}$$

# connection to traditional model (low $V_{DS}$ )

---

$$I_D = \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_T) V_{DS}$$

$$I_D = T \left( W C_{ox} (V_{GS} - V_T) \frac{\nu_T}{(2k_B T_e/q)} \right) V_{DS} \quad T = \frac{\lambda_0}{\lambda_0 + L}$$

$$I_D = \frac{W}{L + \lambda_0} \mu_n C_{ox} (V_{GS} - V_T) V_{DS}$$

$$I_D = \frac{W}{L} \mu_{app} C_{ox} (V_{GS} - V_T) V_{DS}$$

$$1/\mu_{app} = 1/\mu_n + 1/\mu_B$$

# connection to traditional model (high $V_{DS}$ )

---

$$I_D = WC_{ox}v_{sat}(V_{GS} - V_T)$$

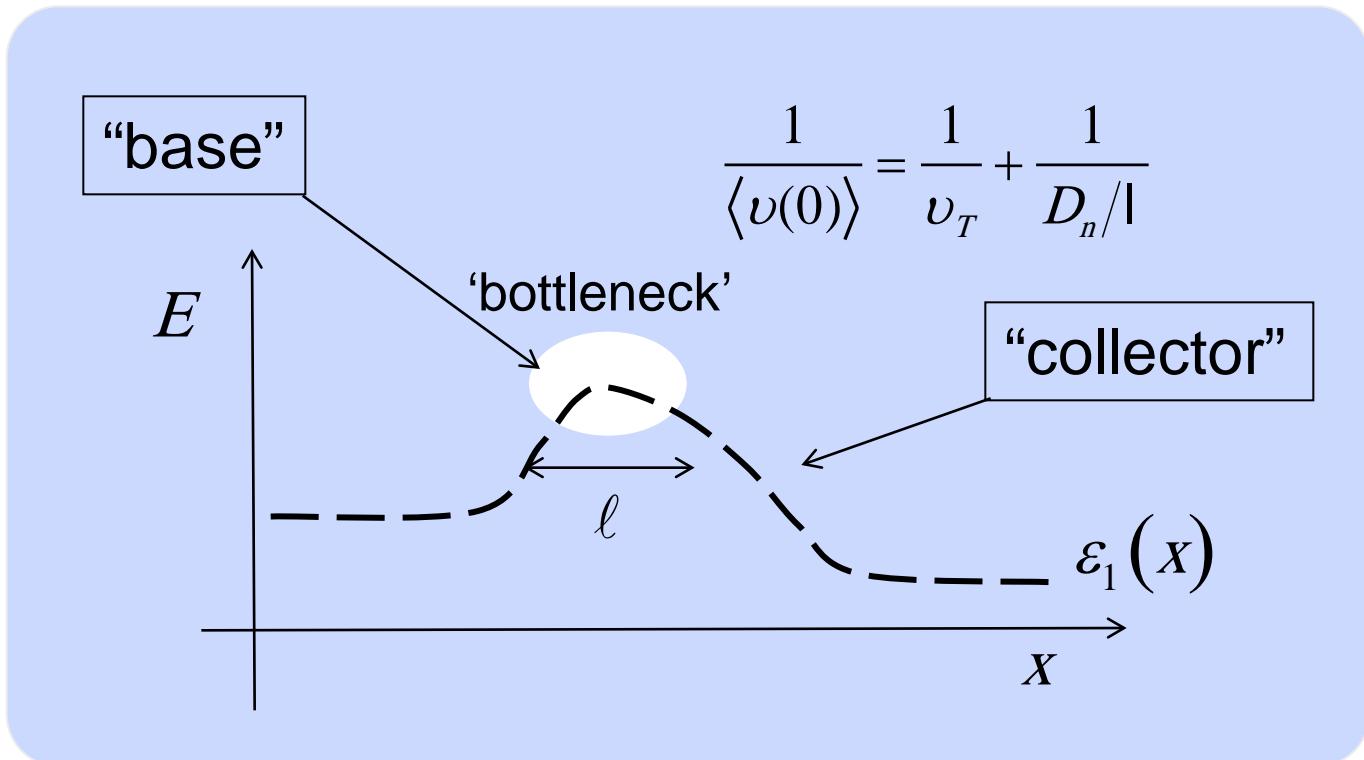
$$I_D = WC_{ox}(V_{GS} - V_T)v_T\left(\frac{T}{2-T}\right)$$

$$I_D = W\left[\frac{1}{v_T} + \frac{1}{(D_n/l)}\right]^{-1} C_{ox}(V_{GS} - V_T)$$

*how do we interpret this result?*

# the MOSFET as a BJT

$$I_D = W \langle v(0) \rangle C_{ox} (V_{GS} - V_T)$$



# outline

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- 1) Review
- 2) The nano-MOSFET
- 3) The ballistic MOSFET
- 4) Scattering in nano-MOSFETs
- 5) Summary

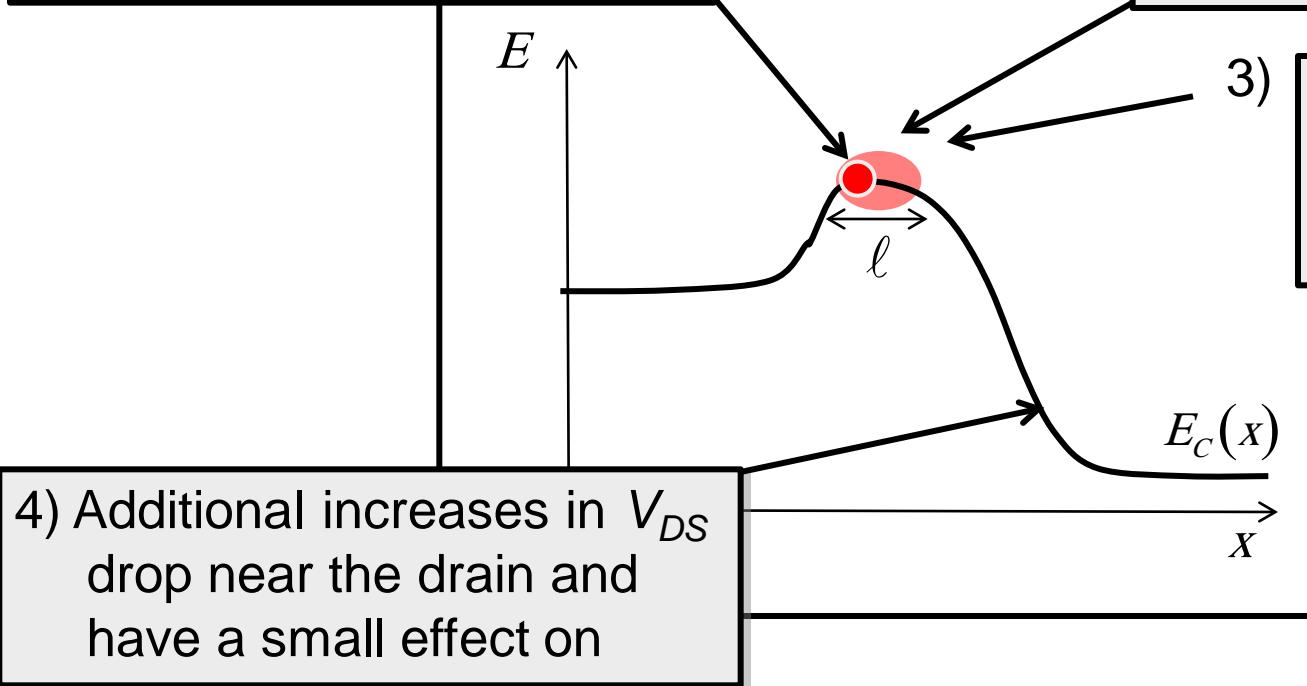
# MOSFETs are barrier controlled devices

1) "Well-tempered MOSFET"

$$Q_I(0) \approx C_G(V_G - V_T)$$

2) "bottleneck" for current

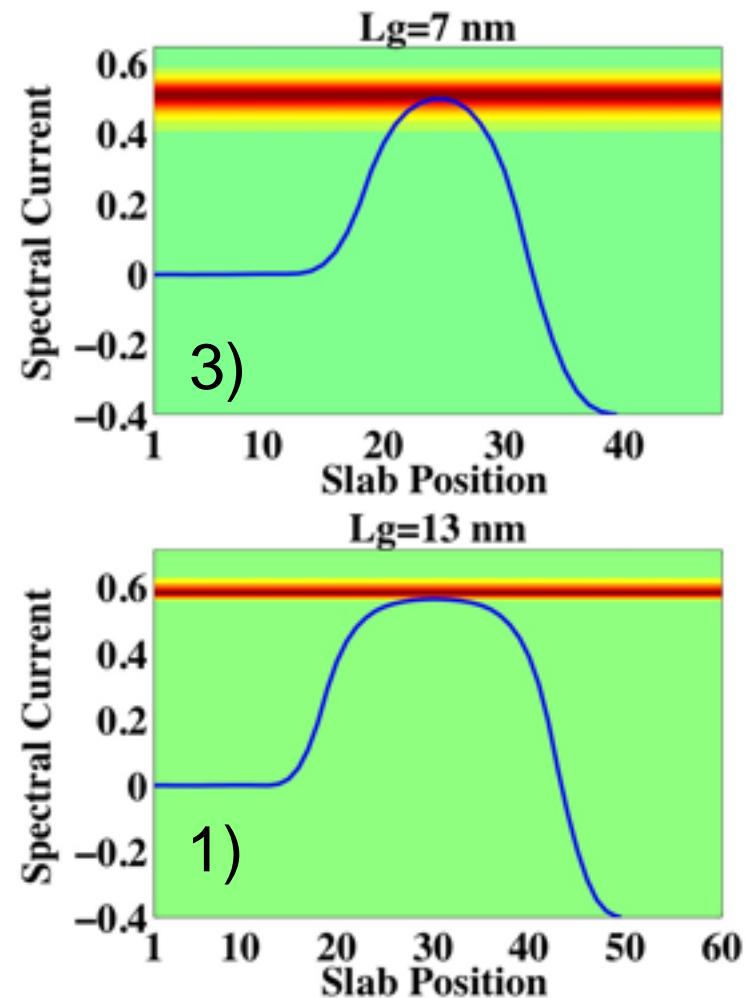
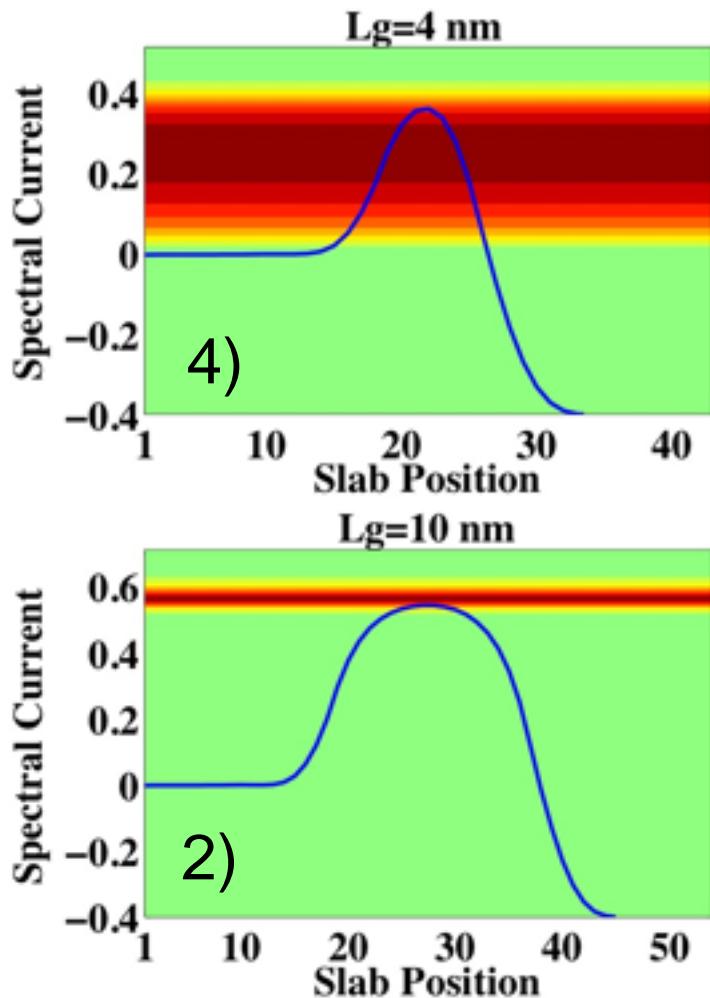
$$\ell \ll L$$



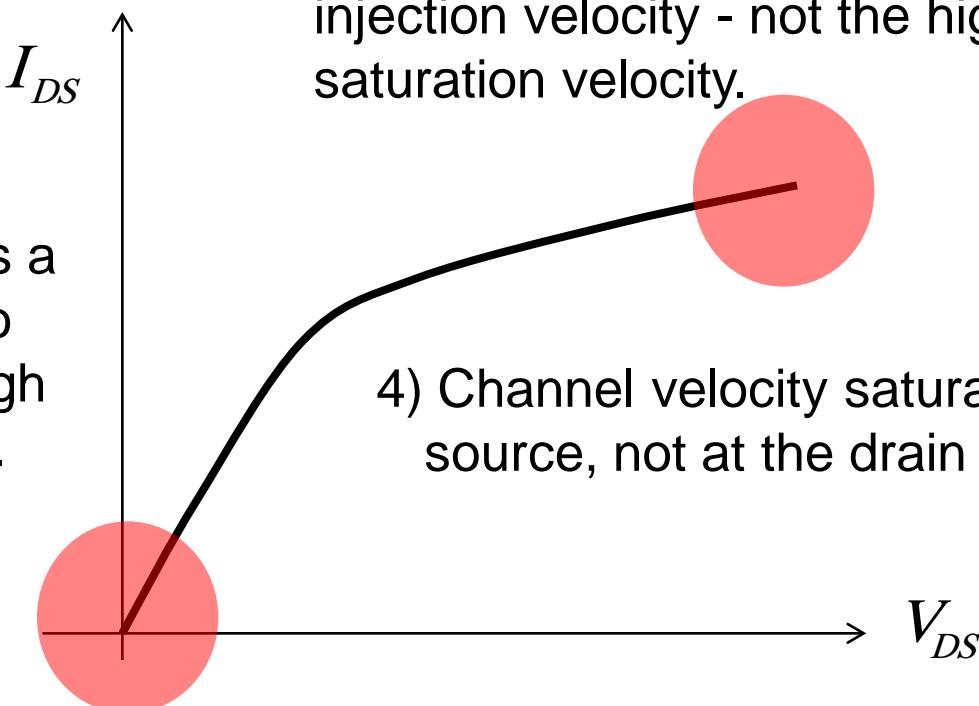
4) Additional increases in  $V_{DS}$  drop near the drain and have a small effect on

$$T \approx \frac{\lambda_0}{\lambda_0 + 1}$$

# limits to barrier control



# physics of nanoscale MOSFETs

- 
- The graph shows a plot of drain current ( $I_{DS}$ ) on the vertical axis against drain-to-source voltage ( $V_{DS}$ ) on the horizontal axis. A curve starts at the origin (0,0) and increases rapidly, then levels off into a saturation region where the current remains constant despite further increases in voltage. Two red circles mark the origin and the end of the saturation region on the curve.
- 1) Transistor-like I-V characteristics are a result of electrostatics.
  - 2) The channel resistance has a lower limit - no matter how high the mobility is.
  - 3) The on-current is controlled by the ballistic injection velocity - not the high-field, bulk saturation velocity.
  - 4) Channel velocity saturates near the source, not at the drain end.

## for more information

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View “Physics of Nanoscale MOSFETs,” a series of eight lectures on the subject presented at the 2008 NCN@Purdue Summer School by Mark Lundstrom, 2008.  
<http://nanohub.org/resources/5306>