AQME Exercise: Bound States – Theoretical Exercise

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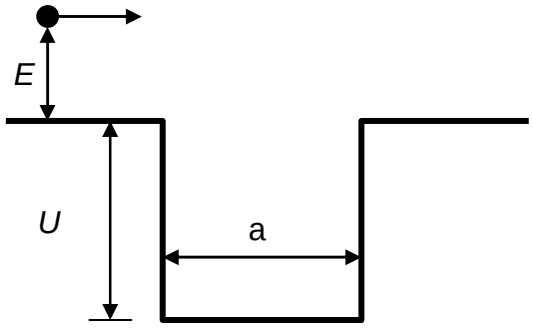
Objective: The objective of this exercise is to teach the students the theory behind bound states in a quantum well.

1. (a) Confirm that $\langle p_x \rangle = 0$ for all energy states of a particle in a 1-D box.
   (b) Verify that the normalization factor for wavefunctions describing a particle in a 1-D box is $A_n = \sqrt{2/a}$.
   (c) Determine <x> for all energy states of a particle in a 1-D box.
2. Consider a particle of mass *m* traveling from left to right over the potential well, as pictured in the figure below. Let *E* be the energy of the particle relative to the top of the well, $-U_0$ the depth of the well, and $-a<x<a$ the position of the well. Designate the three regions to the left, within and to the right of the potential well as I, II, and III, respectively.
   (a) Employing traveling-wave type solutions [$\psi = A\exp(ikx) + B\exp(-ikx)$] in all three regions, establish a relationship for the transmittance (*T*) of the particle across the potential well, where $T = \left|A_{III}/A_I\right|^2$. Note that because the particle will be moving to the right in region III, $B_{III} = 0$.
   (b) What is the limit of your *T*-expression as a→0? What is the physical significance of this limiting case?
   (c) What is the limit of your *T*-expression as E→∞? What is the physical significance of this limiting case?



Relevant Literature: D. K. Ferry, Quantum Mechanics: *An Introduction for Device Physicists and Electrical Engineers* (Institute of Physics Publishing, London, 2001).