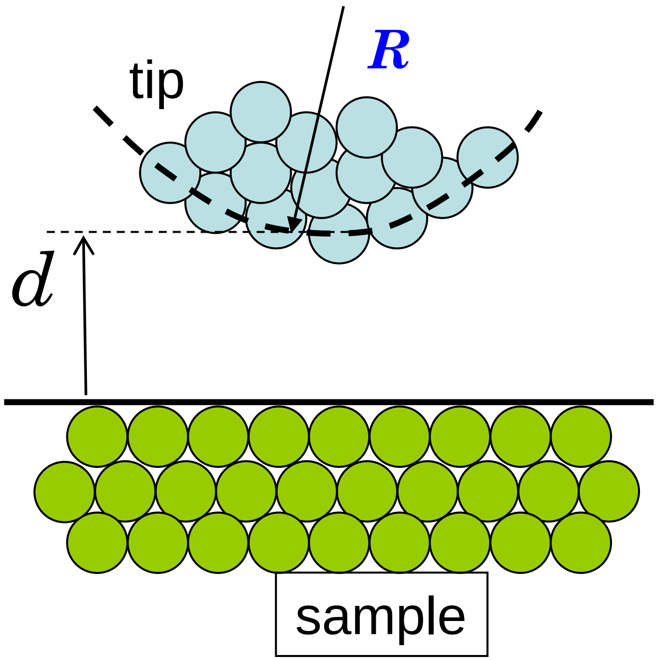
Lecture 9
Force distance curves I

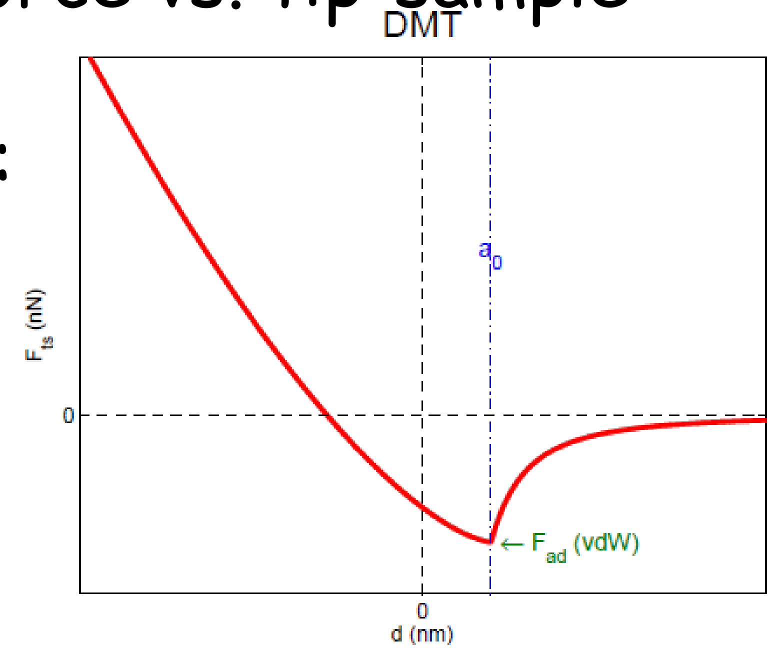
Arvind Raman
Mechanical Engineering
Birck Nanotechnology Center

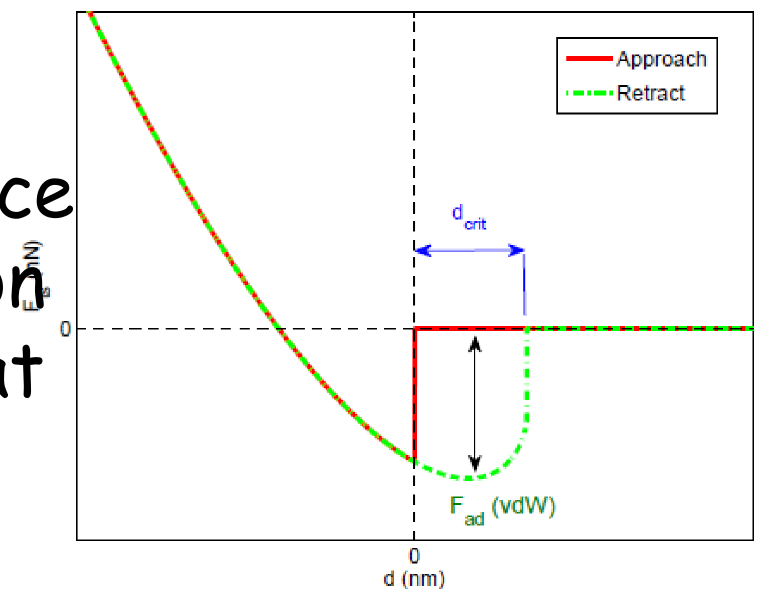
F-d curve

- Is the effective tip sample force vs. tip-sample separation (or indentation)



Examples:





- Usually d=0 chosen at min force
- d<0 indentation, d>0 separation
- Unfortunately, this is not what we measure directly in AFM

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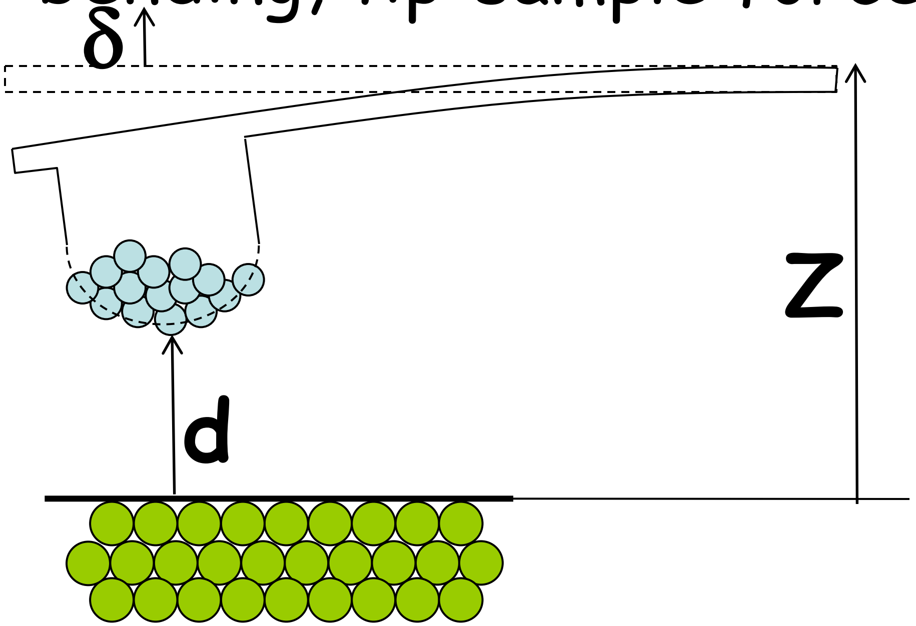
F-Z curve – what we measure in AFM

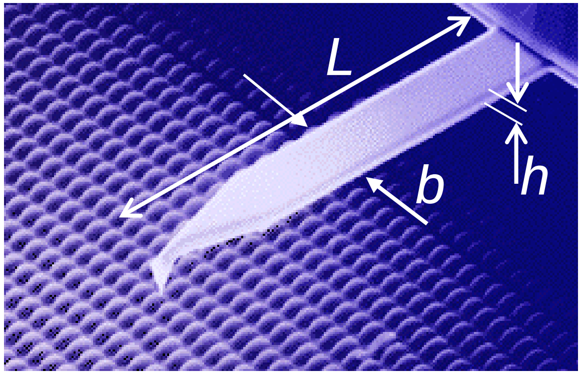
- Z is the Z-piezo displacement, δ is the cantilever bending, tip-sample force is $F_{ts}=k_{cant}\delta$



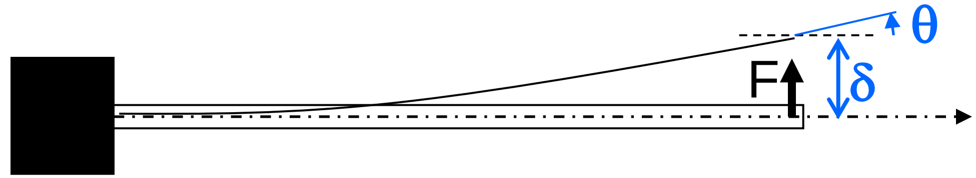
AFM's measure F_{ts} vs. Z !!

- How to convert force-displacement (F vs Z) to force distance (F vs d) and vice versa?
- Collect F-Z data and for every F value, evaluate $d = Z+\delta$, to within an arbitrary constant

The microcantilever – the force sensor



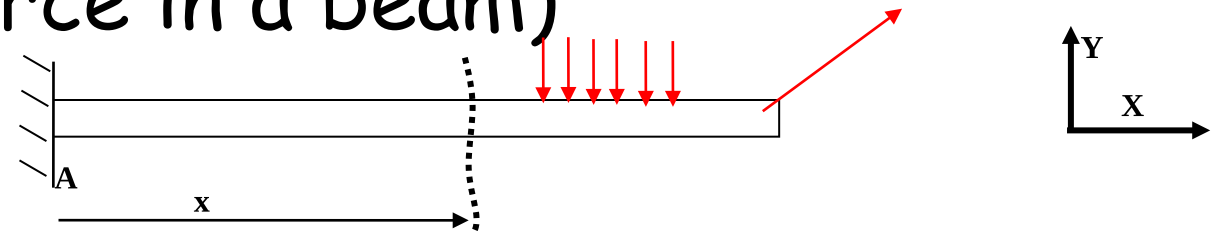
www.olympus.co.jp

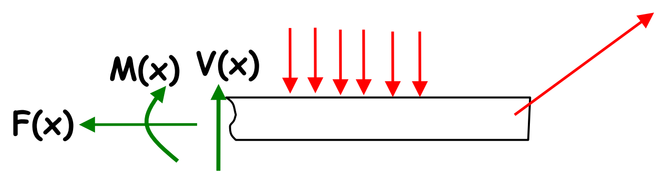


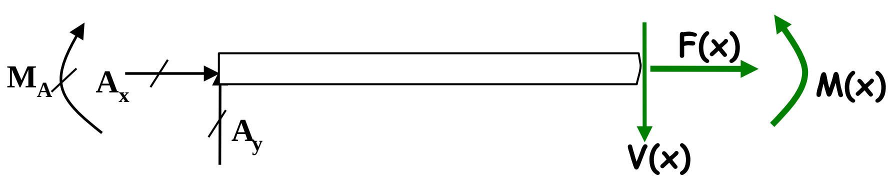
- From elementary beam theory, if E=Young's modulus, $I=bh^3/12$ then
- $\delta=w(L)=F L^3/(3EI)$, and $\theta=dw(L)/dx=FL^2/(2EI)$
- Deflection and slope linearly proportional to force sensed at the tip
- $k=3EI/L^3$ is called the bending stiffness of the cantilever

Classical beam theory

- Understanding internal resultants (shear force, bending moment and axial force in a beam)



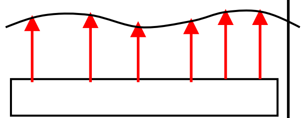
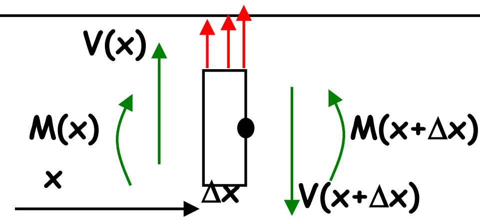




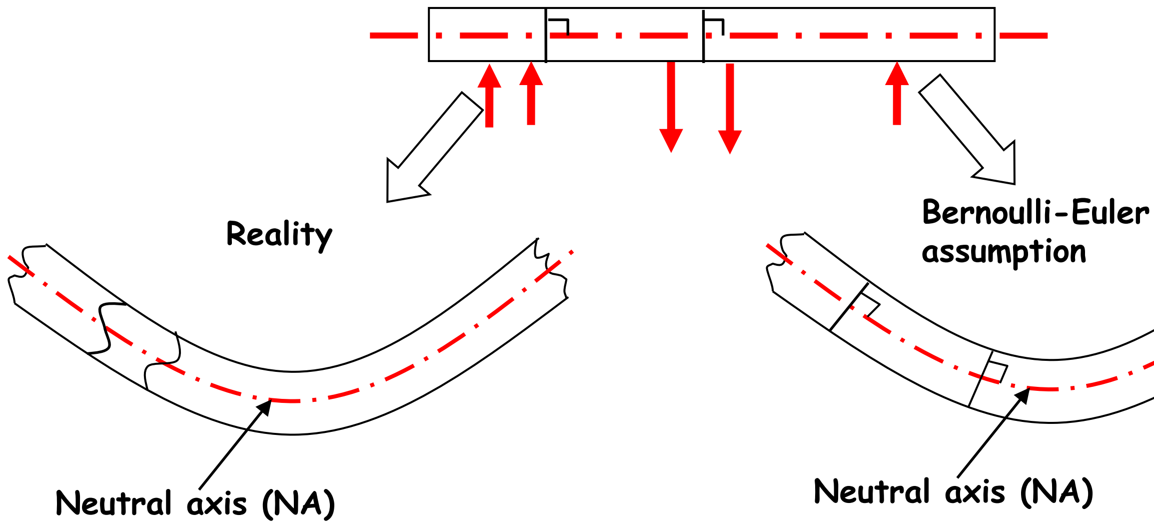
V(x) :Internal shear force (N)
F(x): Internal axial force (N)
M(x): Internal bending moment (N.m)

Classical beam theory

- Relationship between V(x) and M(x)

Key Equation	Applied Load	Derivation
$\frac{dV}{dx} = p(x)$	p(x) N/m 	 $V(x) + p(x)\Delta x - V(x+\Delta x) = 0,$ $as\ \Delta x \to 0\ we\ get\ \frac{dV}{dx} = p(x)$
$\frac{dM}{dx} = V(x)$		$M(x+\Delta x) - M(x) - V(x)\Delta x - p(x)\Delta x\left(\frac{\Delta x}{2}\right) = 0,$ $as\ \Delta x \to 0\ we\ get\ \frac{dM}{dx} = V(x)$

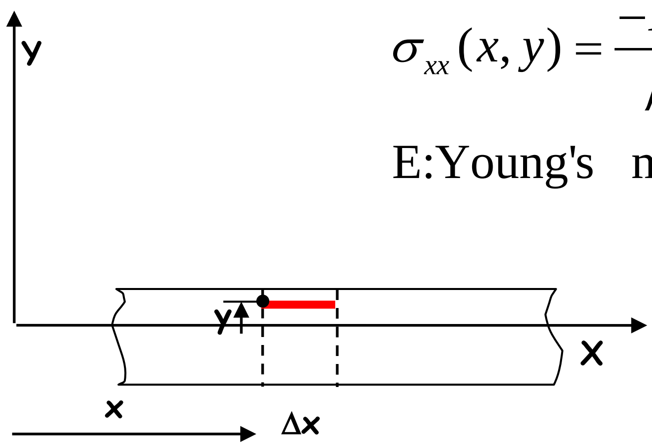
Classical beam theory- stress/strain



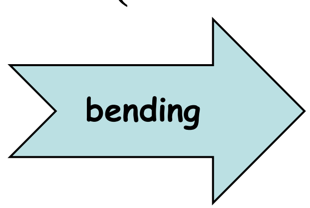
$$\varepsilon_{xx}(x,y) =_{\lim \Delta x \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta} = \frac{-y}{\rho}$$

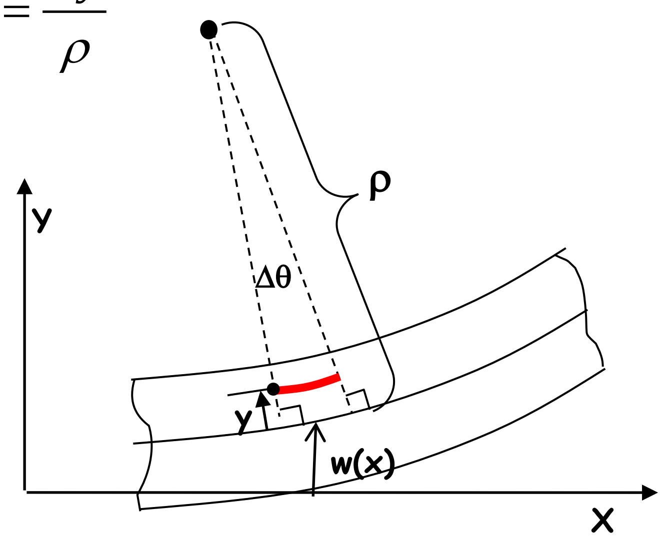
$$\sigma_{xx}(x,y) = \frac{-Ey}{\rho} \sim -Ey\frac{d^2 w}{dx^2}$$

E:Young's modulus (N/m^2 or Pa)



Undeformed beam segment

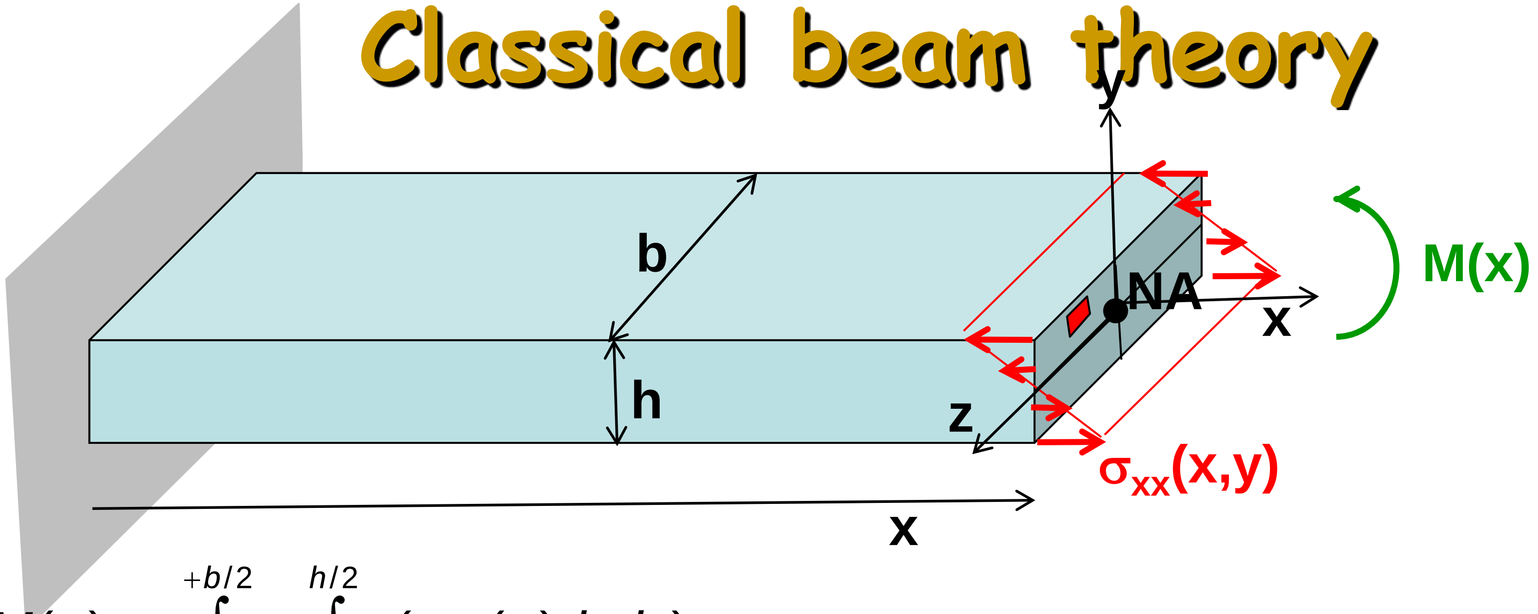




Deformed beam segment

- Key: σ_{xx} varies linearly across section

Classical beam theory



$$M(x) = \int_{z=-b/2}^{+b/2} \int_{y=-h/2}^{h/2} (\sigma_{xx}(x)dydz)y$$

$$\text{or} \quad M(x) = \int_{z=-b/2}^{+b/2} \int_{y=-h/2}^{h/2} Ey^2 \frac{d^2w(x)}{dx^2} dydz = E\frac{d^2w(x)}{dx^2} \int_{z=-b/2}^{+b/2} \int_{y=-h/2}^{h/2} y^2 dydz$$

$$\text{or} \quad M(x) = EI_{zz} \frac{d^2w(x)}{dx^2} \quad \text{where } I_{zz} = \frac{bh^3}{12} \quad (\text{area moment } m^4)$$

$$\text{Likewise} \quad V(x) = EI_{zz} \frac{d^3w(x)}{dx^3}$$

$$\text{Finally} \quad EI\frac{d^4w(x)}{dx^4} = p(x) \text{ to be solved with boundary conditions at } x = 0, L$$

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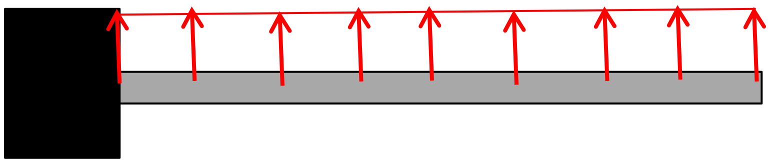
Classical beam theory

- Example1



$$EI\frac{d^4w(x)}{dx^4} = p_0 \Rightarrow \frac{d^3w(x)}{dx^3} = \frac{p_0}{EI}x + c_1$$

$$\Rightarrow \frac{d^2w(x)}{dx^2} = \frac{1}{2}\frac{p_0}{EI}x^2 + c_1x + c_2 \Rightarrow \frac{dw(x)}{dx} = \theta(x) = \frac{1}{6}\frac{p_0}{EI}x^3 + \frac{1}{2}c_1x^2 + c_2x + c_3$$

$$w(x) = \frac{1}{24}\frac{p_0}{EI}x^4 + \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4$$

Boundary conditions

$$w(0) = \theta(0) = 0 \quad EI\frac{d^2w(L)}{dx^2} = EI\frac{d^3w(L)}{dx^3} = 0$$

(*no point moments or force applied at* $x = L$)

$$\Rightarrow c_3 = c_4 = 0, c_1 = -\frac{p_0L}{EI}, c_2 = \frac{1}{2}\frac{p_0L^2}{EI}$$

$$w(L) = \delta = \frac{1}{8}\frac{p_0L^4}{EI}, \quad \theta(L) = \theta = \frac{1}{6}\frac{p_0L^3}{EI} \Rightarrow \frac{\theta}{\delta} = \frac{4}{3}L$$

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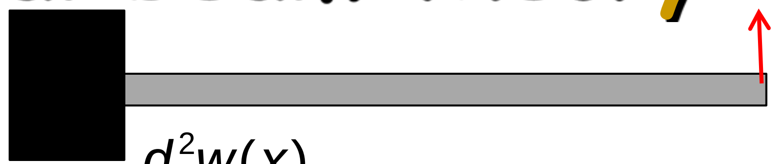
Classical beam theory

- Example2



$$EI\frac{d^4w(x)}{dx^4}=0 \Rightarrow \frac{d^3w(x)}{dx^3}=c_1 \Rightarrow \frac{d^2w(x)}{dx^2}=c_1x+c_2$$

$$\Rightarrow \frac{dw(x)}{dx}=\theta(x)=\frac{1}{2}c_1x^2+c_2x+c_3$$

$$w(x)=\frac{1}{6}c_1x^3+\frac{1}{2}c_2x^2+c_3x+c_4$$

Boundary conditions

$$w(0)=\theta(0)=0 \quad EI\frac{d^2w(L)}{dx^2}=0 \; EI\frac{d^3w(L)}{dx^3}=-F$$

(*no point moment applied at* $x=L$)

$$\Rightarrow c_3=c_4=0, c_1=-\frac{F}{EI}, c_2=\frac{FL}{EI}$$

$$w(L)=\delta=\frac{1}{3}\frac{FL^3}{EI}, \quad \theta(L)=\theta=\frac{1}{2}\frac{FL^2}{EI} \Rightarrow \frac{\theta}{\delta}=\frac{2}{3}L$$

$F=k\delta$, *where* $k=\dfrac{3EI}{L^3}$ *is the static bending stiffness of the cantilever*

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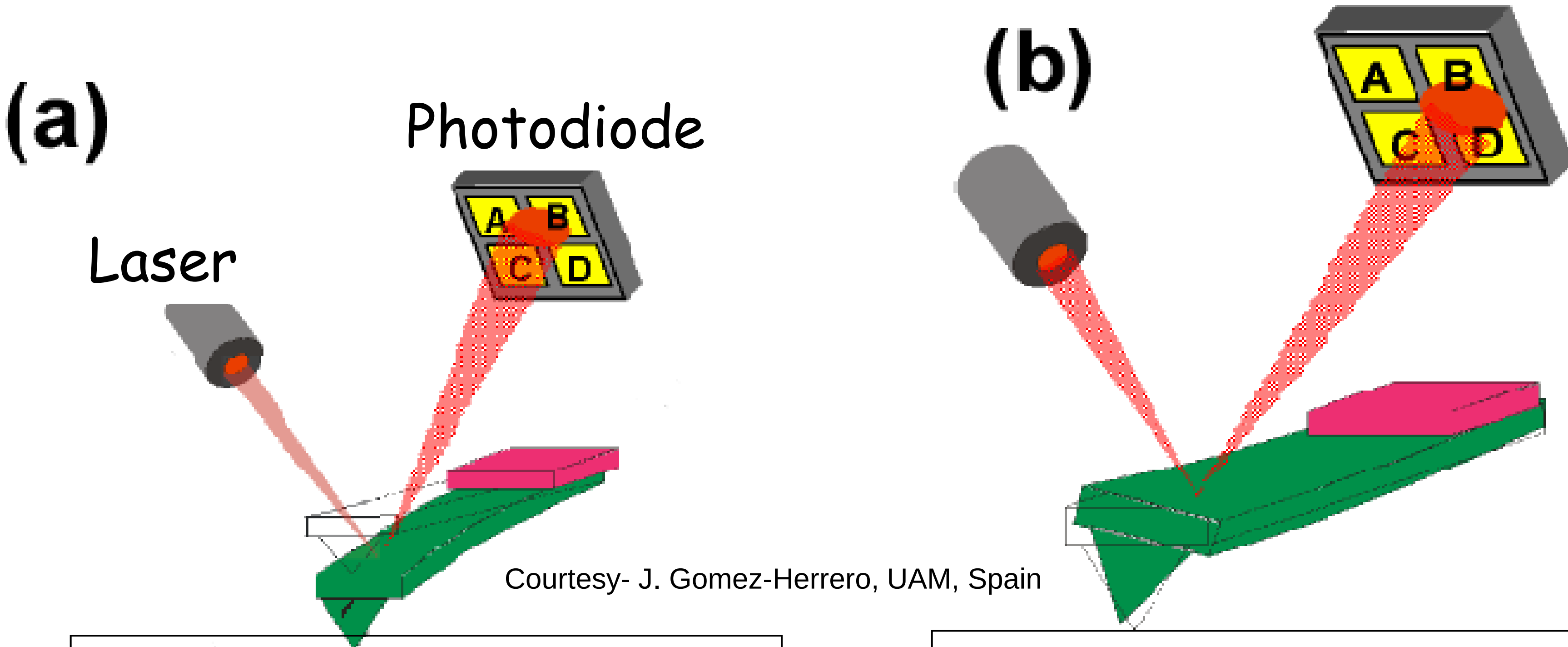
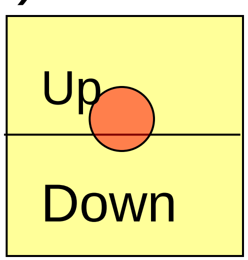
The four-quadrant photodiode



Courtesy- J. Gomez-Herrero, UAM, Spain

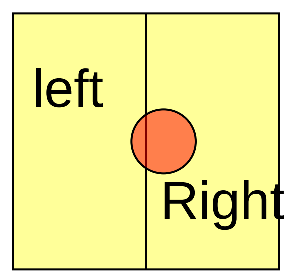
a) Vertical bending



A+B= UP

C+D=DOWN

b)Lateral/torsion motion

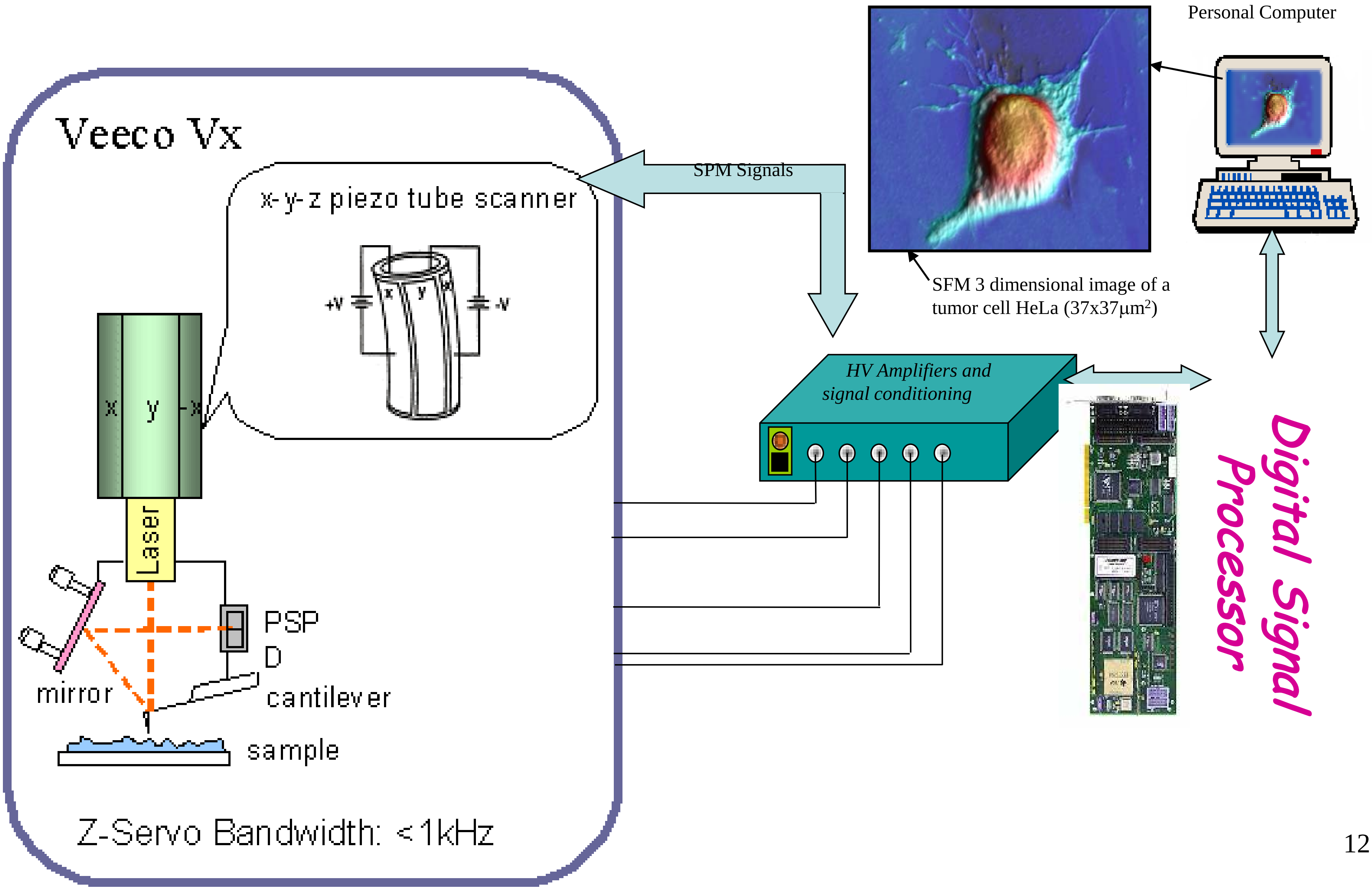


A+C= LEFT

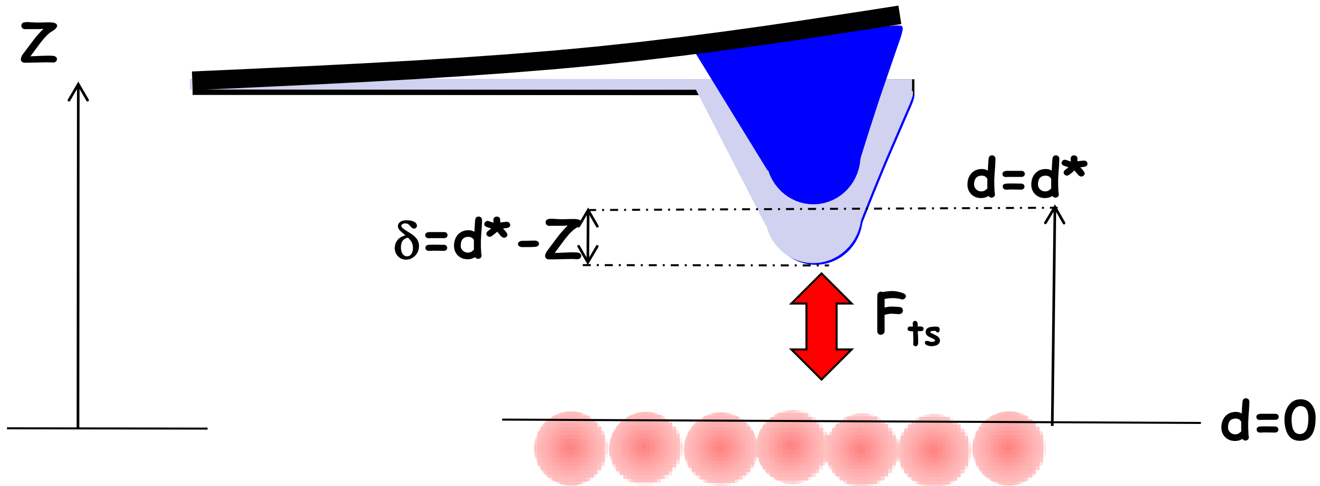
B+D=Right

Commercial AFMs measure the rotation angle!!! (bending or torsion)

AFM Block Diagram

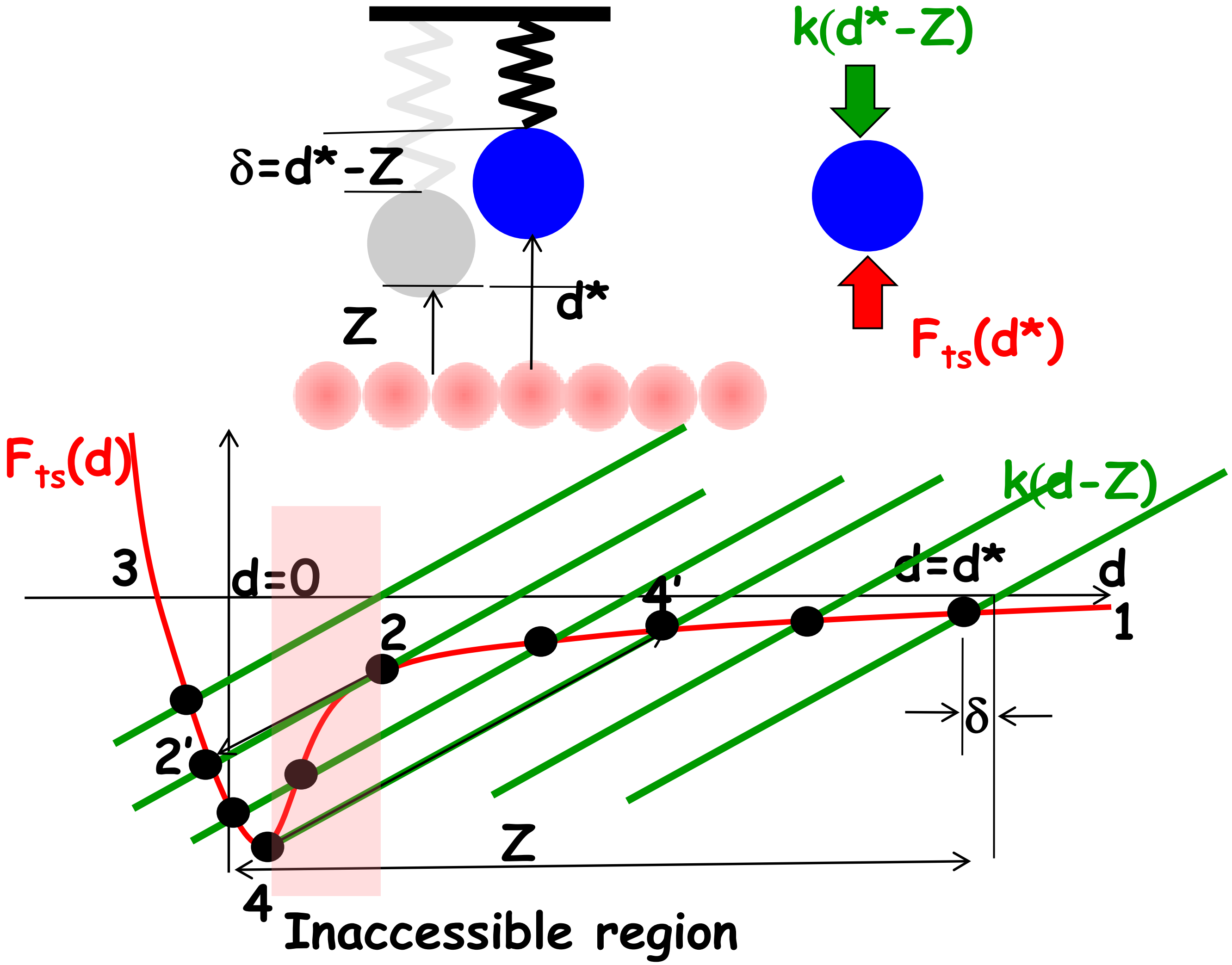


Equilibrium positions during approach and retraction



- How do d* and δ change as Z is reduced during approach and then retracted?
- Note that technically $\delta = d^* - Z - Tip\,height$ but tip height is basically an arbitrary constant

Cantilever instabilities during approach and retraction



- With soft cantilevers (small k) it is not possible to measure entire 'd' range

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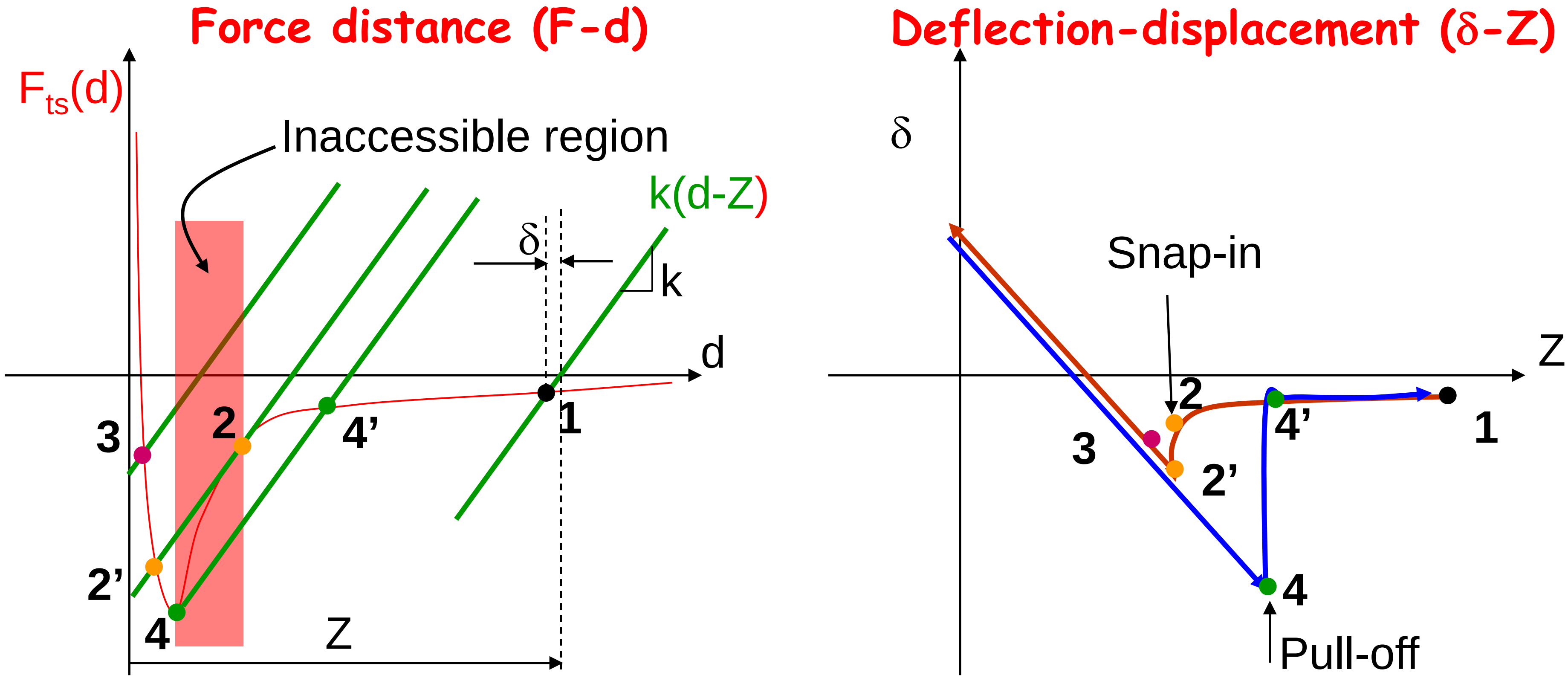
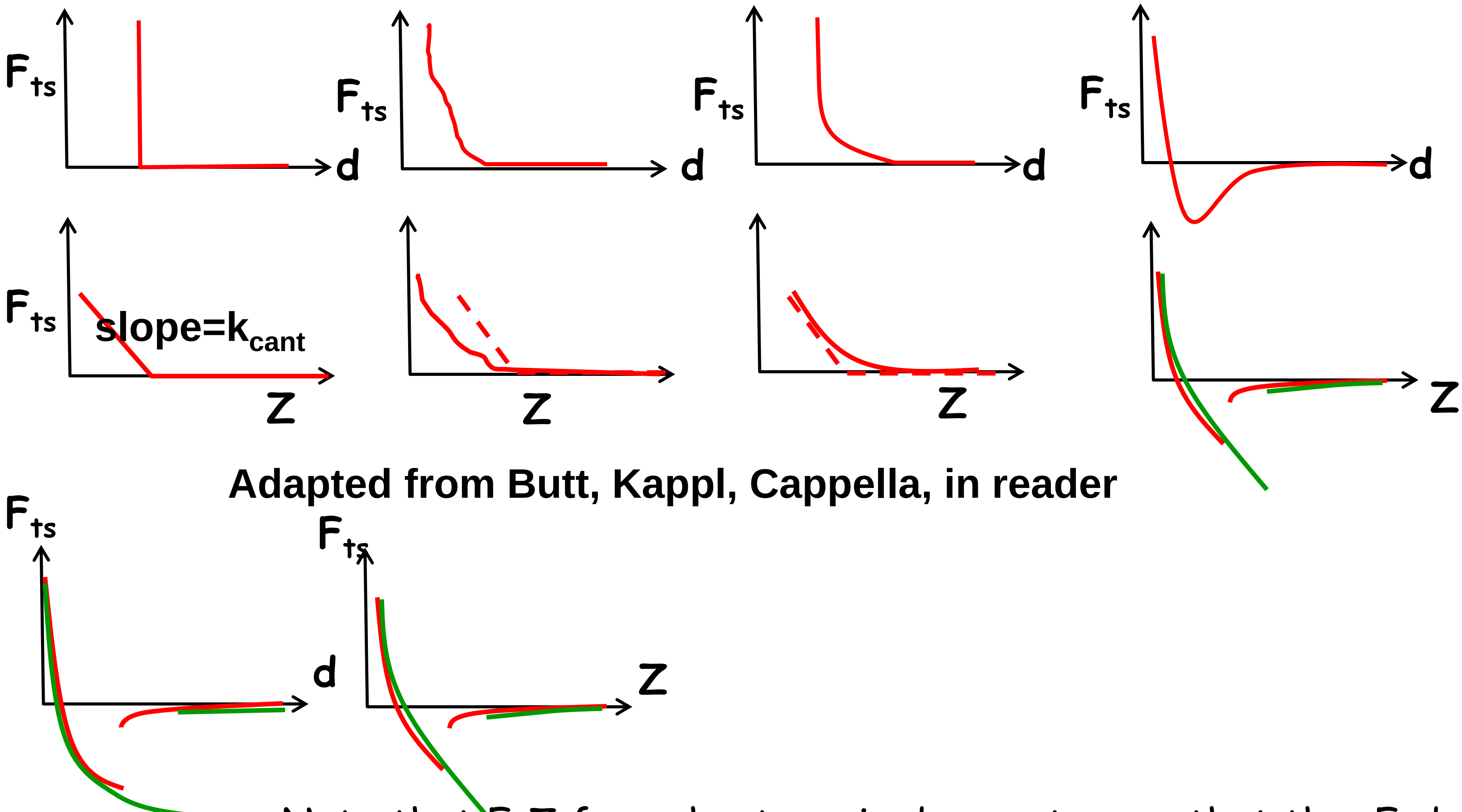
F-d to F-Z conversion



- Note that hysteresis occurs in the δ-Z curve between approach and retraction even though $F_{ts}(d)$ in conservative

Some examples of F-d to F-Z conversions

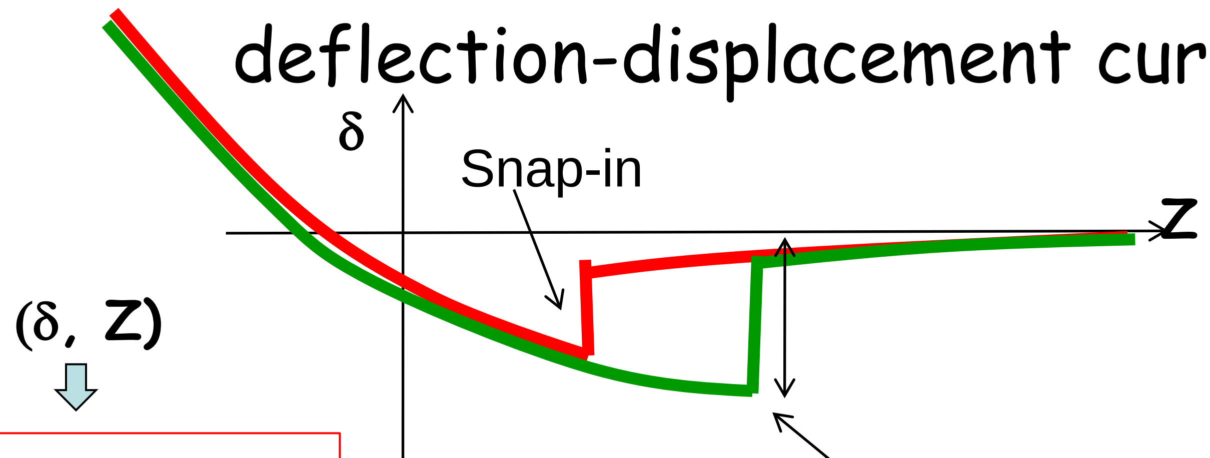


Adapted from Butt, Kappl, Cappella, in reader

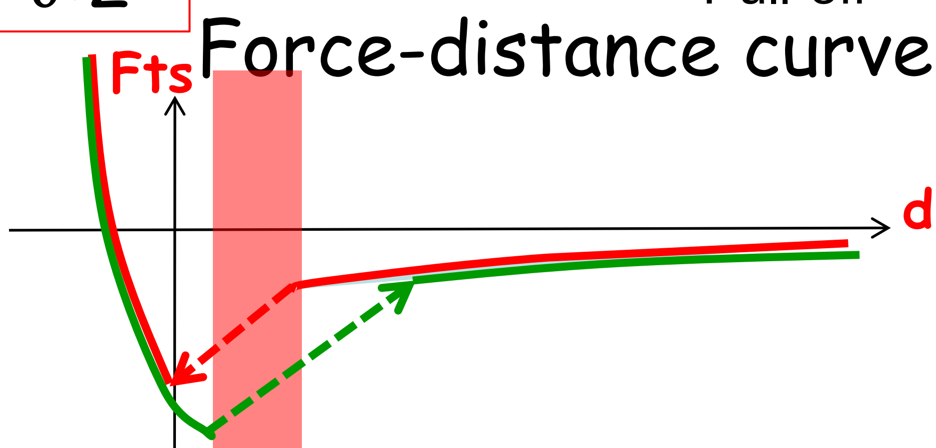
- Note that F-Z force hysteresis does not mean that the F-d is hysteretic (non-conservative)
- F-Z to F-d is actually non-unique when there is hysteresis in F-Z!!

Conversely: F-Z to F-d conversion

- In a typical δ-Z experiment in AFM, the cantilever approaches/retracts from the sample while recording the cantilever deflection.
- However in force spectroscopy we are interested in converting this to a force-distance curve i.e. F_{ts} vs. d. How to convert?

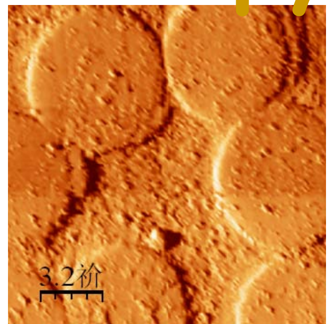


$F_{ts}=k\delta$ $d=\delta+Z$

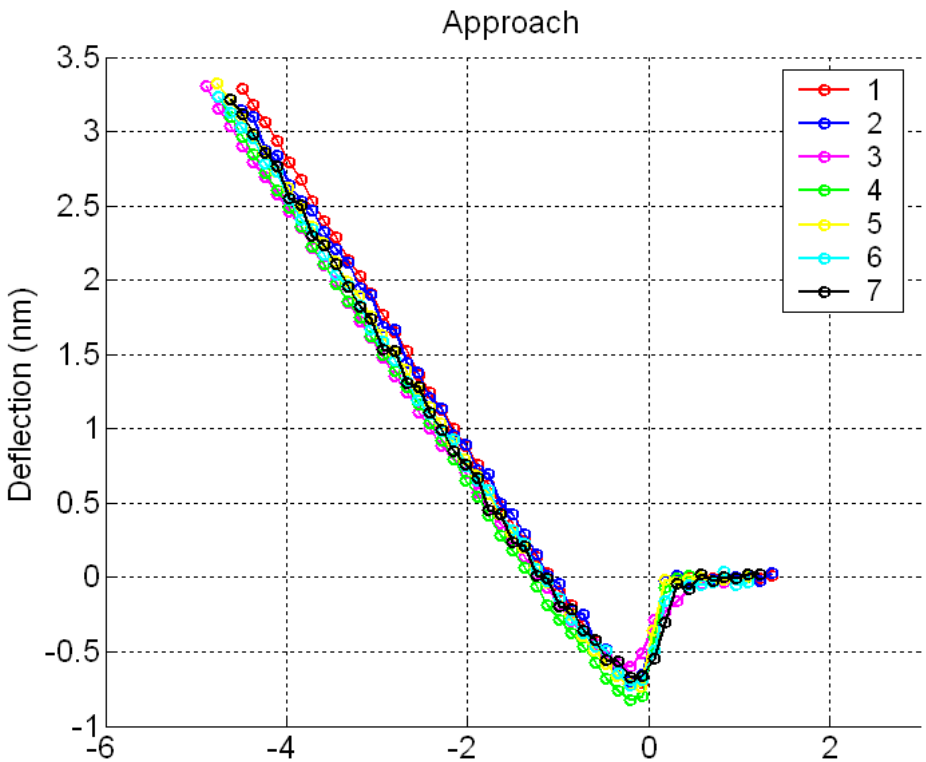


- Important: d is only known to within an arbitrary constant!

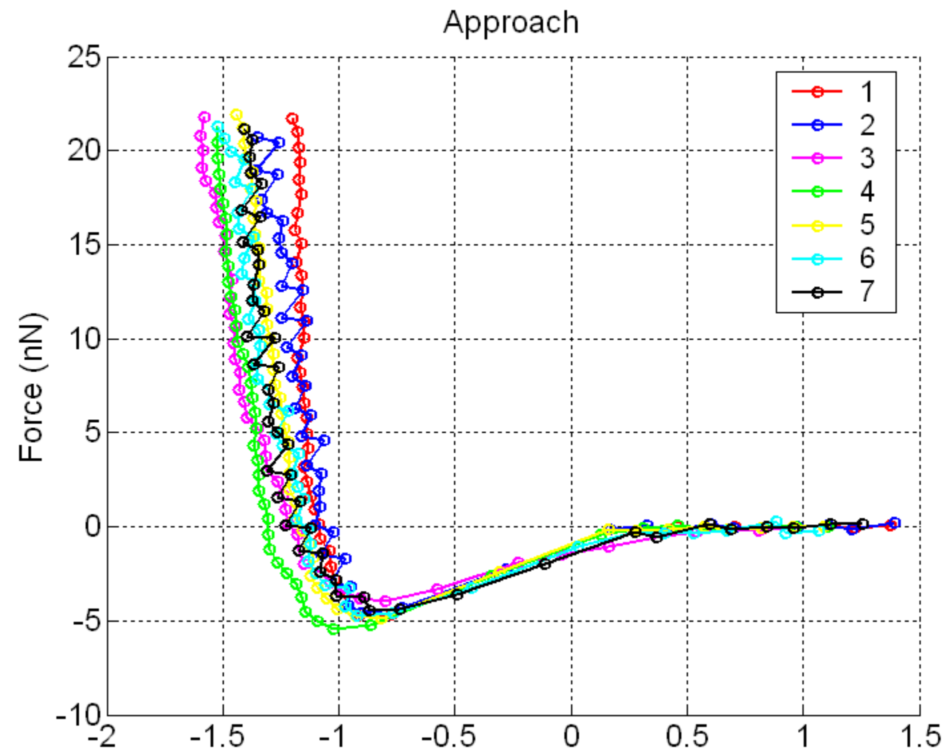
Force spectroscopy – an example



Convert deflection vs. displacement curves to force vs. distance (gap) curves



Piezo displacement Z (nm)



Tip-sample distance d (nm)

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Often d axis is recentered to zero where force is a minimum

Next class

- Practical aspects of force distance curves