Lecture 9
Force distance curves I

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**F-d curve**

- **Is the effective tip sample force vs. tip-sample separation (or indentation)**

**Examples:**

- Usually $d=0$ chosen at min force
- $d<0$ indentation, $d>0$ separation
- Unfortunately, this is not what we measure directly in AFM
F-Z curve - what we measure in AFM

- Z is the Z-piezo displacement, $\delta$ is the cantilever bending, tip-sample force is $F_{ts}=k_{\text{cant}}\delta$

How to convert force-displacement ($F$ vs $Z$) to force distance ($F$ vs $d$) and vice versa?

- Collect F-Z data and for every $F$ value, evaluate $d=Z+\delta$, to within an arbitrary constant
From elementary beam theory, if $E=\text{Young’s modulus}$, $l=bh^3/12$ then

$$\delta=w(L)=F \frac{L^3}{3EI}, \text{ and } \theta=\frac{dw(L)}{dx}=F\frac{L^2}{2EI}$$

Deflection and slope linearly proportional to force sensed at the tip

$k=3EI/L^3$ is called the bending stiffness of the cantilever
Classical beam theory

- Understanding internal resultants (shear force, bending moment and axial force in a beam)

\[ \begin{align*}
V(x) &: \text{Internal shear force (N)} \\
F(x) &: \text{Internal axial force (N)} \\
M(x) &: \text{Internal bending moment (N.m)}
\end{align*} \]
# Classical beam theory

## Relationship between $V(x)$ and $M(x)$

<table>
<thead>
<tr>
<th>Key Equation</th>
<th>Applied Load</th>
<th>Derivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dV \over dx = p(x)$</td>
<td>$p(x)$ N/m</td>
<td>$V(x) + p(x)\Delta x - V(x + \Delta x) = 0$, as $\Delta x \to 0$ we get $dV \over dx = p(x)$</td>
</tr>
<tr>
<td>$dM \over dx = V(x)$</td>
<td>$M(x + \Delta x) - M(x) - V(x)\Delta x - p(x)\Delta x \left( {\Delta x \over 2} \right) = 0$, as $\Delta x \to 0$ we get $dM \over dx = V(x)$</td>
<td></td>
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</tbody>
</table>
Classical beam theory - stress/strain

Neutral axis (NA)

Reality

Bernoulli-Euler assumption

Neutral axis (NA)

\[ \varepsilon_{xx}(x, y) = \lim_{\Delta x \to 0} \frac{(\rho - y)\Delta \theta - \rho \Delta \theta}{\rho \Delta \theta} = \frac{-y}{\rho} \]

\[ \sigma_{xx}(x, y) = \frac{-Ey}{\rho} \sim -Ey \frac{d^2w}{dx^2} \]

E: Young's modulus (N/m² or Pa)

Key: \( \sigma_{xx} \) varies linearly across section
Classical beam theory

\[ M(x) = \int_{z=-b/2}^{+b/2} \int_{y=-h/2}^{+h/2} (\sigma_{xx}(x)dydz)y \]

or

\[ M(x) = \int_{z=-b/2}^{+b/2} \int_{y=-h/2}^{+h/2} Ey^2 \frac{d^2w(x)}{dx^2} dydz = E \frac{d^2w(x)}{dx^2} \int_{z=-b/2}^{+b/2} \int_{y=-h/2}^{+h/2} y^2 dydz \]

or

\[ M(x) = EI_{zz} \frac{d^2w(x)}{dx^2} \text{ where } I_{zz} = \frac{bh^3}{12} \text{ (area moment } m^4) \]

Likewise

\[ V(x) = EI_{zz} \frac{d^3w(x)}{dx^3} \]

Finally

\[ EI \frac{d^4w(x)}{dx^4} = p(x) \text{ to be solved with boundary conditions at } x = 0, L \]
Classical beam theory

Example 1

\[ EI \frac{d^4 w(x)}{dx^4} = p_0 \Rightarrow \frac{d^3 w(x)}{dx^3} = \frac{p_0}{EI} x + c_1 \]

\[ \Rightarrow \frac{d^2 w(x)}{dx^2} = \frac{1}{2} \frac{p_0}{EI} x^2 + c_1 x + c_2 \Rightarrow \frac{dw(x)}{dx} = \theta(x) = \frac{1}{6} \frac{p_0}{EI} x^3 + \frac{1}{2} c_1 x^2 + c_2 x + c_3 \]

\[ w(x) = \frac{1}{24} \frac{p_0}{EI} x^4 + \frac{1}{6} c_1 x^3 + \frac{1}{2} c_2 x^2 + c_3 x + c_4 \]

Boundary conditions

\[ w(0) = \theta(0) = 0 \quad EI \frac{d^2 w(L)}{dx^2} = EI \frac{d^3 w(L)}{dx^3} = 0 \]

(no point moments or force applied at \( x = L \))

\[ \Rightarrow c_3 = c_4 = 0, c_1 = -\frac{p_0 L}{EI}, c_2 = \frac{1}{2} \frac{p_0 L^2}{EI} \]

\[ w(L) = \delta = \frac{1}{8} \frac{p_0 L^4}{EI}, \quad \theta(L) = \theta = \frac{1}{6} \frac{p_0 L^3}{EI} \Rightarrow \frac{\theta}{\delta} = \frac{4}{3} \]
Classical beam theory

Example 2

\[
El \frac{d^4w(x)}{dx^4} = 0 \implies \frac{d^3w(x)}{dx^3} = c_1 \implies \frac{d^2w(x)}{dx^2} = c_1x + c_2
\]

\[
\frac{dw(x)}{dx} = \theta(x) = \frac{1}{2}c_1x^2 + c_2x + c_3
\]

\[
w(x) = \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4
\]

Boundary conditions

\[
w(0) = \theta(0) = 0 \quad El \frac{d^2w(L)}{dx^2} = 0 \quad El \frac{d^3w(L)}{dx^3} = -F
\]

(no point moment applied at \(x = L\))

\[
\Rightarrow c_3 = c_4 = 0, \quad c_1 = -\frac{F}{EIL}, \quad c_2 = \frac{FL}{EIL}
\]

\[
w(L) = \delta = \frac{1}{3}\frac{FL^3}{EL}, \quad \theta(L) = \theta = \frac{1}{2}\frac{FL^2}{EL} \implies \frac{\theta}{\delta} = \frac{2}{3}L
\]

\[
F = k\delta, \text{ where } k = \frac{3EIL}{L^3} \text{ is the static bending stiffness of the cantilever}
\]
The four-quadrant photodiode

(a) Vertical bending

- Up: A+B = UP
- Down: C+D = DOWN

(b) Lateral/torsion motion

- Left: A+C = LEFT
- Right: B+D = Right

Commercial AFMs measure the rotation angle!!! (bending or torsion)
AFM Block Diagram

Veeco Vx
- x-y-z piezo tube scanner
- Laser
- mirror
- PSD
- cantilever
- sample
- Z-Servo Bandwidth: <1kHz

SPM Signals
- HV Amplifiers and signal conditioning
- Digital Signal Processor
- Personal Computer

SFM 3 dimensional image of a tumor cell HeLa (37x37 µm²)
Equilibrium positions during approach and retraction

- How do $d^*$ and $\delta$ change as $Z$ is reduced during approach and then retracted?
- Note that technically $\delta = d^* - Z - \text{Tip height}$ but tip height is basically an arbitrary constant
With soft cantilevers (small k) it is not possible to measure entire 'd' range.
Note that hysteresis occurs in the $\delta$-$Z$ curve between approach and retraction even though $F_{ts}(d)$ in conservative
Some examples of F-d to F-Z conversions

- Note that F-Z force hysteresis does not mean that the F-d is hysteretic (non-conservative)
- F-Z to F-d is actually non-unique when there is hysteresis in F-Z!!

Adapted from Butt, Kappl, Cappella, in reader
Conversely: F-Z to F-d conversion

- In a typical $\delta$-Z experiment in AFM, the cantilever approaches/retracts from the sample while recording the cantilever deflection.
- However in force spectroscopy we are interested in converting this to a force-distance curve i.e. $F_{ts}$ vs. $d$. How to convert?

**Important:** $d$ is only known to within an arbitrary constant!
Convert deflection vs. displacement curves to force vs. distance (gap) curves.

Often d axis is recentered to zero where force is a minimum.
Next class

- Practical aspects of force distance curves