

Lectures on Nanoelectronics

Discussion Sessions:
M, Tu, W 330PM

*Please hand your questions
or email them to
Mr. Samiran Ganguly
sganguly@purdue.edu*

1. Introductory concepts (M830A)
1b. The nanotransistor: Lundstrom (M1030A)
2. Semiclassical transport (M130P)
3. Quantum transport (Tu830A)
Beyond voltages and currents
4. Heat flow (Tu1030A)
5. Spin flow (W830A)
6. Entropy flow (W1030A)

Electrical Fluctuations: Vidhyadhiraja (Tu 130P)

Heat transfer: Fisher (W130P)

Field Effect Transistor

Fig.0.1a. The Field Effect Transistor (FET) is essentially a resistor consisting of a “channel” with two large contacts called the “source” and the “drain”.

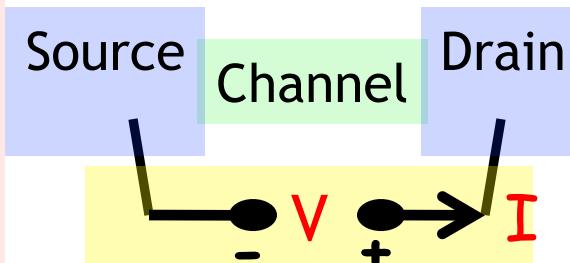
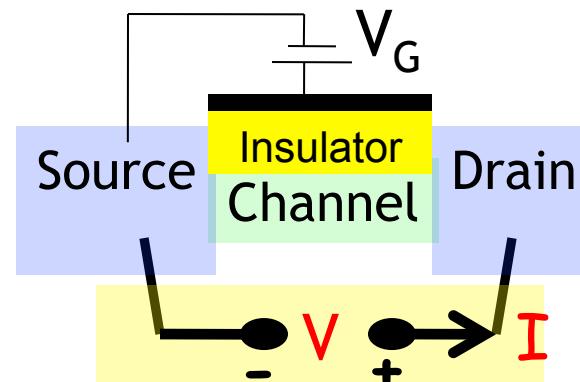
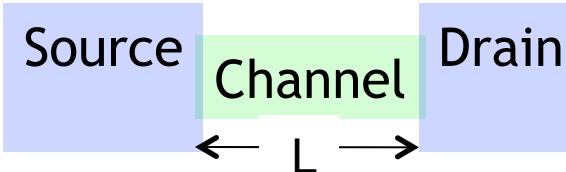


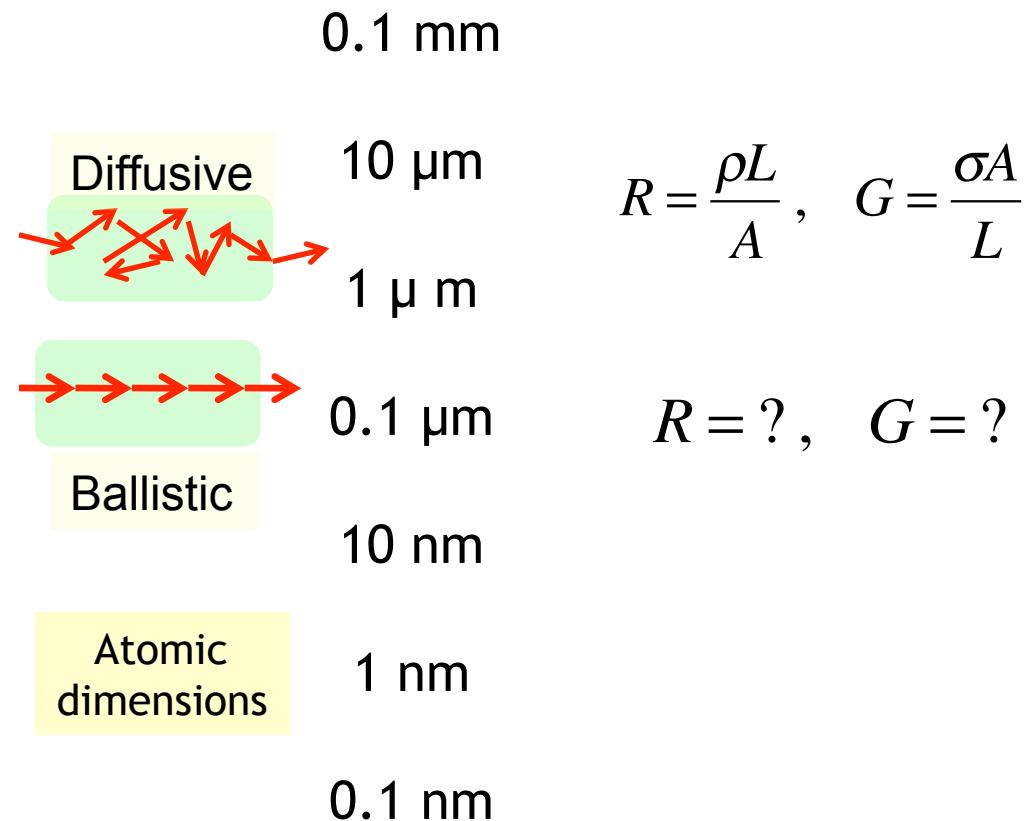
Fig.0.1b. The resistance $R = V/I$ can be changed by several orders of magnitude through the gate voltage V_G .



Top-down and bottom-up

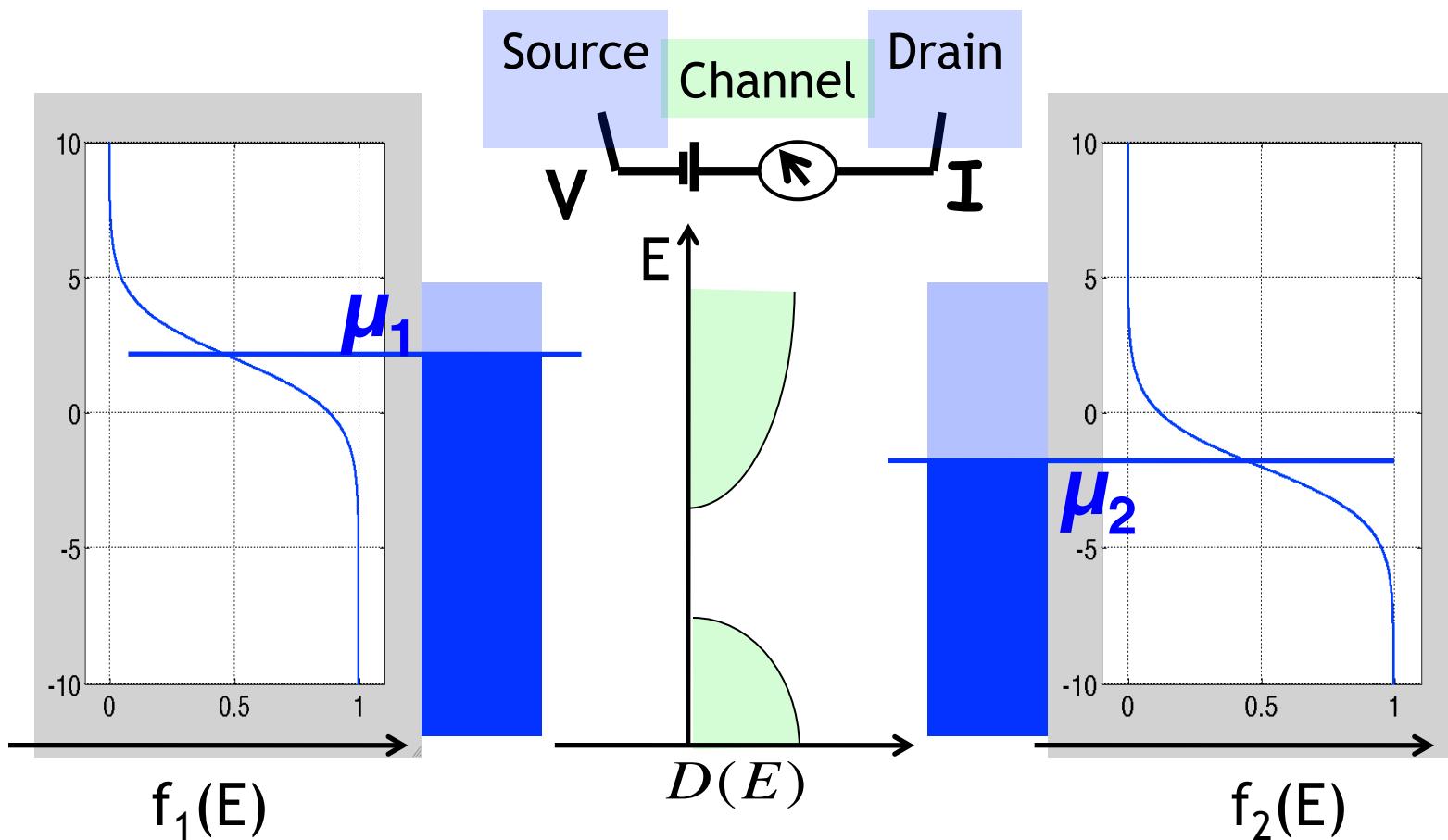


The length of the channel of an FET has progressively shrunk with every new generation of devices ("Moore's Law") and stands today (2010) at ~ 50 nm, which amounts to a few hundred atoms !



Length units:
1 mm = 1000 μm
and 1 μm = 1000 nm

D(E), f(E)



$$f(E) = \frac{1}{e^x + 1}, \quad x \equiv \frac{E - \mu}{kT}$$

Elastic Resistor: Central results

$$I = \frac{1}{q} \int_{-\infty}^{+\infty} dE \tilde{G}(E) (f_1(E) - f_2(E))$$

$$G = \int_{-\infty}^{+\infty} dE \left(-\frac{\partial f_0}{\partial E} \right) \tilde{G}(E)$$

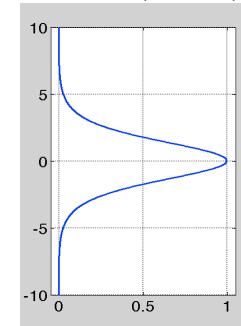
$$\tilde{G}(E) = q^2 \frac{D}{2t} = \frac{q^2}{h} \frac{M \lambda}{\lambda + L}$$

$$\equiv \frac{\tilde{\sigma} A}{\lambda + L}$$

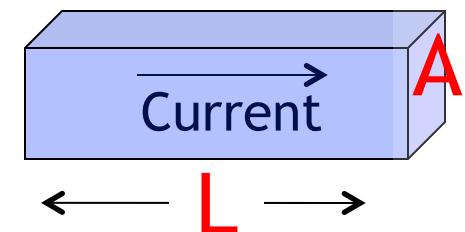
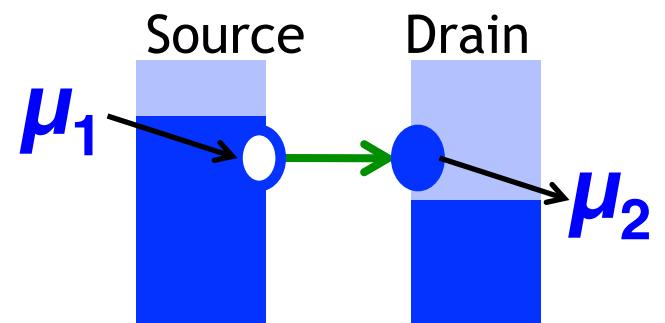
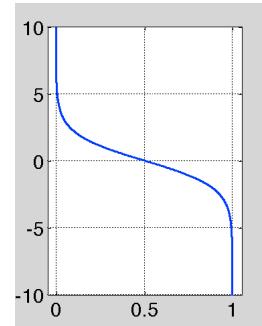
$$\tilde{\sigma} = q^2 \left(\frac{D}{AL} \frac{v^2 \tau}{d} \right) = \frac{q^2}{h} \frac{M \lambda}{A}$$

$$= q^2 \left(\frac{N}{AL} \frac{v \tau}{p} \right)$$

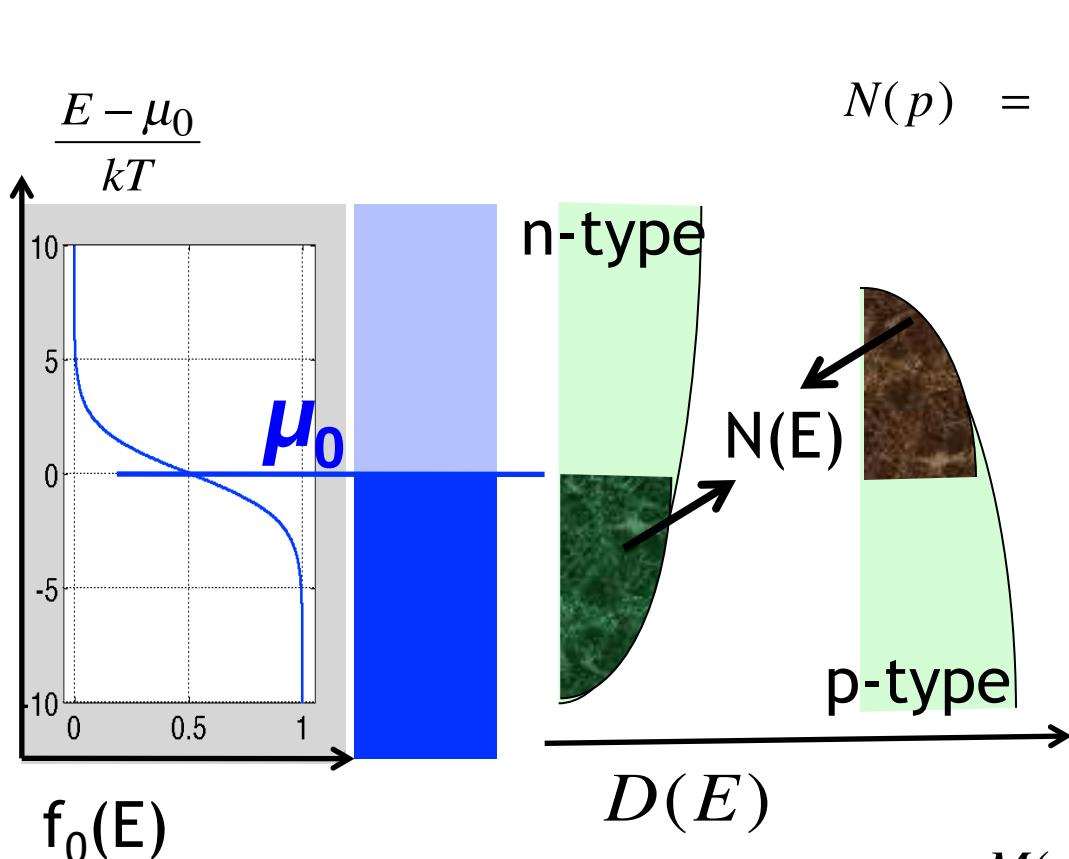
$$\rightarrow 4kT \left(-\frac{\partial f_0}{\partial E} \right)$$



$$\rightarrow f_0(E)$$



D(E), N(E), M(E)



$$N(p) = \left\{ 2 \frac{L}{h/p}, \pi \frac{WL}{(h/p)^2}, \frac{4\pi}{3} \frac{AL}{(h/p)^3} \right\}$$

$$\begin{cases} n \\ p \end{cases} = \int_{-\infty}^{+\infty} dE \left(-\frac{\partial f_0}{\partial E} \right) N(E)$$

$$M(p) = \left\{ 1, 2 \frac{W}{h/p}, \pi \frac{A}{(h/p)^2} \right\}$$

Lectures on Nanoelectronics - I

1. Introductory concepts

1b. *The nanotransistor: Lundstrom*

2. Semiclassical transport

3. Quantum transport

Beyond voltages and currents

4. Heat flow

5. Spin flow

6. Entropy flow

1) $D(E), f(E), M(E), N(E)$

2) *Elastic resistor*

3) *Ohm's law*

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Lectures on Nanoelectronics - II

1. Introductory concepts

1b. The nanotransistor: Lundstrom

2. Semiclassical transport → *2) Where is the resistance?*

3. Quantum transport

1) Point channel model

2) Where is the resistance?

3) Extended channel model

Beyond voltages and currents

4. Heat flow

5. Spin flow

6. Entropy flow

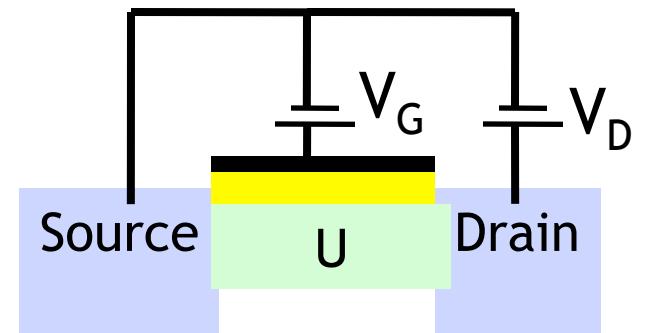
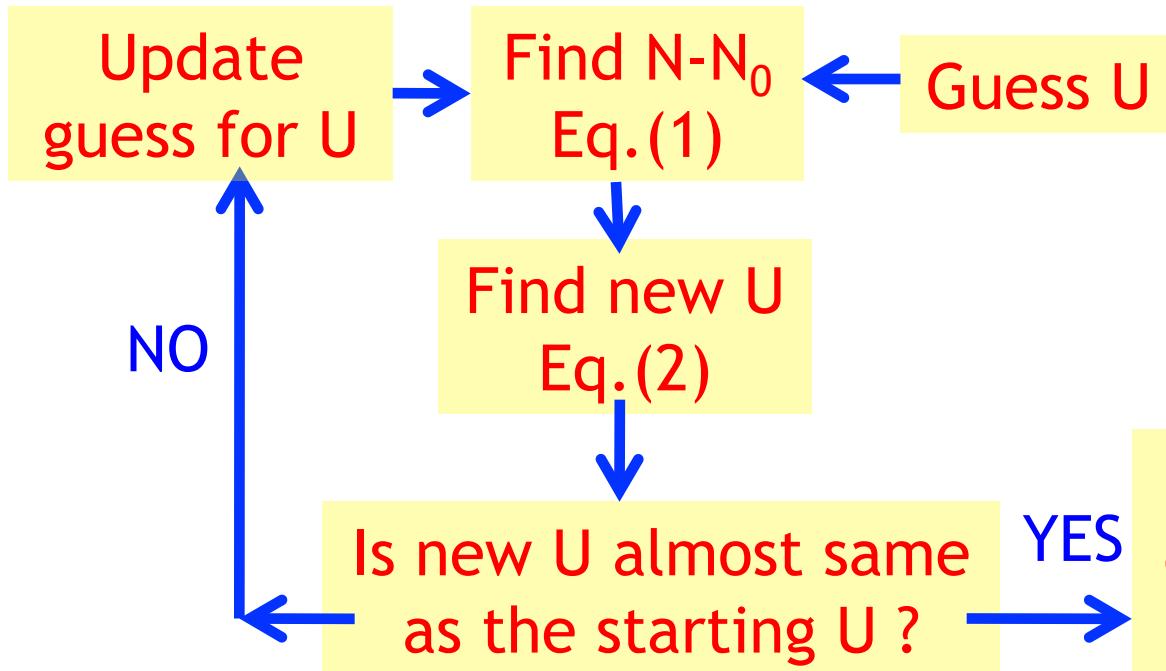
Point channel model

$$N - N_0 = \int_{-\infty}^{+\infty} dE D(E-U) \frac{f_1(E) + f_2(E) - 2f_0(E)}{2} \quad (1)$$

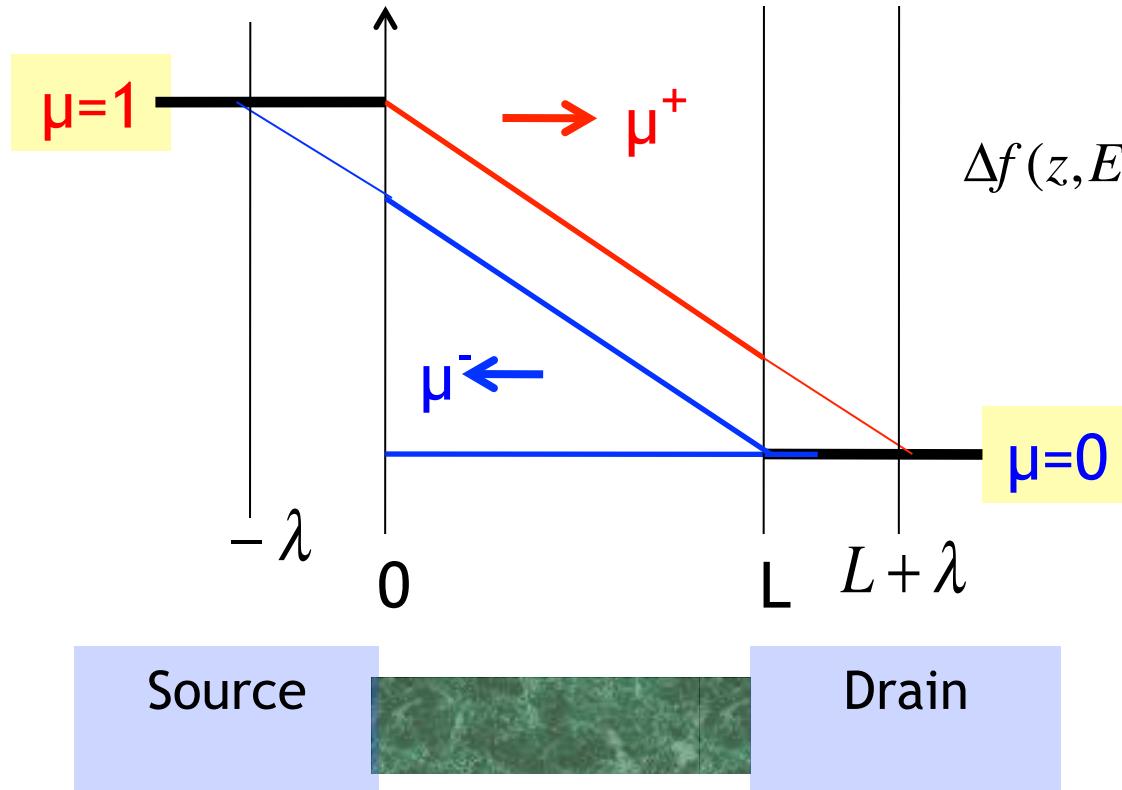
$$U = U_L + U_0(N - N_0) \quad (2a)$$

$$U_L \equiv (1 - \alpha) qV_G + \alpha qV_D \quad (2b)$$

$$I = \frac{1}{q} \int_{-\infty}^{+\infty} dE \tilde{G}(E-U) (f_1(E) - f_2(E)) \quad (3)$$



Where is the resistance?



$$\Delta f(z, E) \approx F_T(E - \mu(z)) \Delta \mu(z)$$

$$\frac{\sigma A}{q} = \frac{q}{h} M \lambda$$

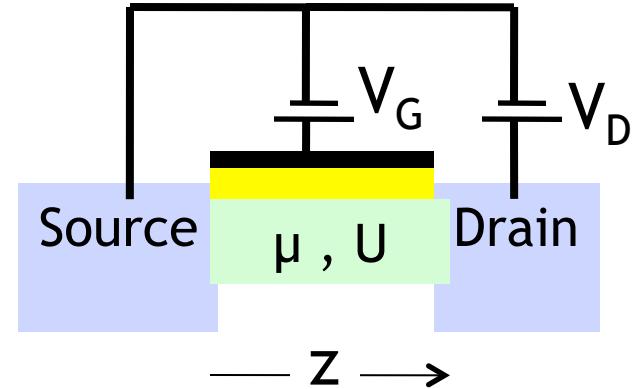
$$\frac{d}{dz} \mu^+ = \frac{d}{dz} \mu^- = -\frac{\mu^+ - \mu^-}{\lambda}$$

$$I = -\frac{\sigma A}{q} \frac{d}{dz} \mu^+ = -\frac{\sigma A}{q} \frac{d}{dz} \mu^-$$

Extended channel model

$$\frac{d}{dz} \left(\sigma(z) \frac{d\mu}{dz} \right) = 0$$

$$\frac{d}{dz} \left(\epsilon(z) \frac{dU}{dz} \right) = q^2 \left(n(z) - n_0(z) \right)$$



$$n(z) = \int_{-\infty}^{+\infty} dE D(z,E) \frac{1}{1 + \exp\left(\frac{E - (\mu(z) - U(z))}{k_B T}\right)}$$

Lectures on Nanoelectronics - I , II

1. Introductory concepts

1b. The nanotransistor: Lundstrom

- 1) $D(E), f(E), M(E), N(E)$
- 2) *Elastic resistor*
- 3) *Ohm's law*

2. Semiclassical transport

3. Quantum transport

Beyond voltages and currents

4. Heat flow

5. Spin flow

6. Entropy flow

- 1) *Point channel model*
- 2) *Where is the resistance?*
- 3) *Extended channel model*

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Elastic Resistor: Central results

$$I = \frac{1}{q} \int_{-\infty}^{+\infty} dE \tilde{G}(E) (f_1(E) - f_2(E))$$

$$G = \int_{-\infty}^{+\infty} dE \left(-\frac{\partial f_0}{\partial E} \right) \tilde{G}(E)$$

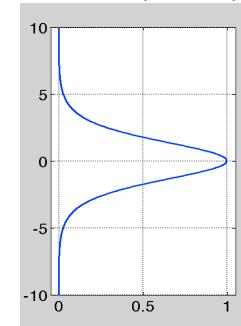
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$$\equiv \frac{\tilde{\sigma} A}{\lambda + L}$$

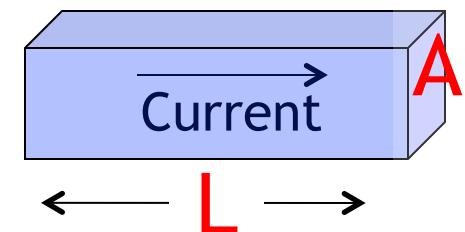
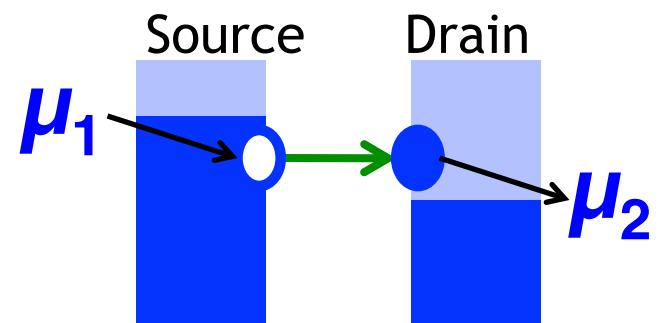
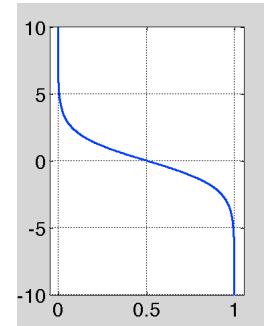
$$\tilde{\sigma} = q^2 \left(\frac{D}{AL} \frac{v^2 \tau}{d} \right) = \frac{q^2}{h} \frac{M \lambda}{A}$$

$$= q^2 \left(\frac{N}{AL} \frac{v \tau}{p} \right)$$

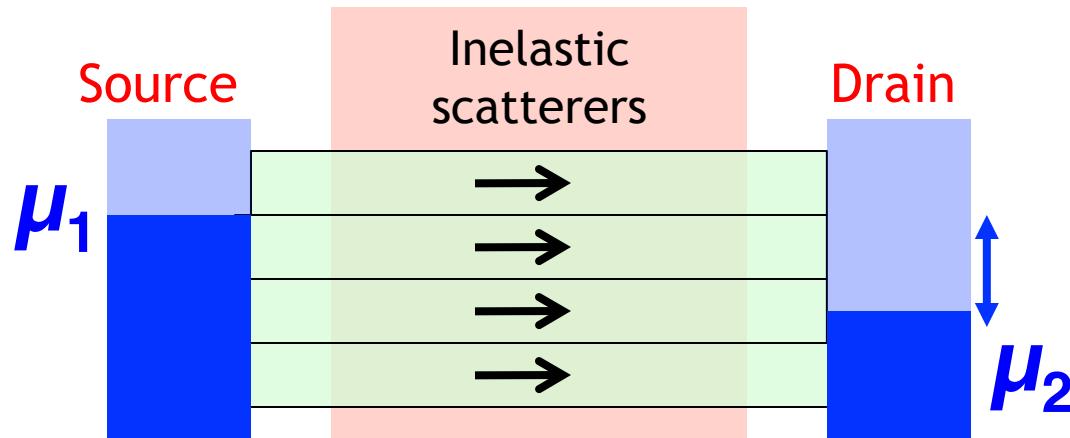
$$\rightarrow 4kT \left(-\frac{\partial f_0}{\partial E} \right)$$



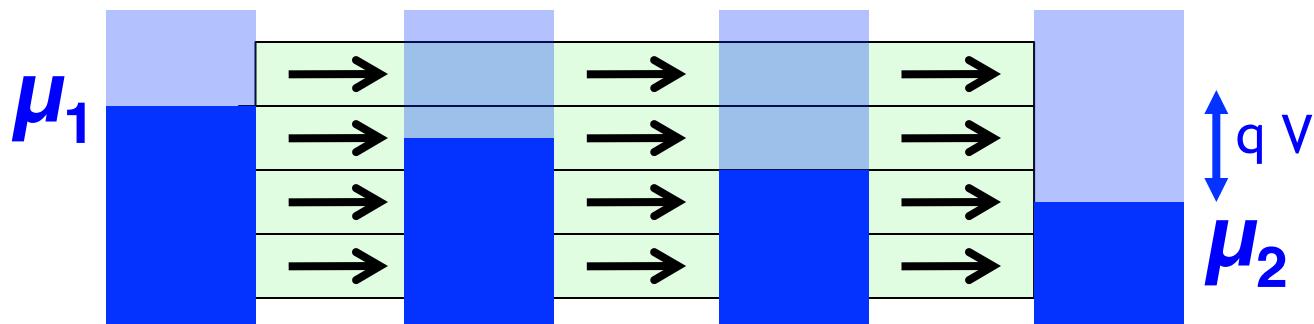
$$\rightarrow f_0(E)$$



Real resistors as a series of elastic resistors

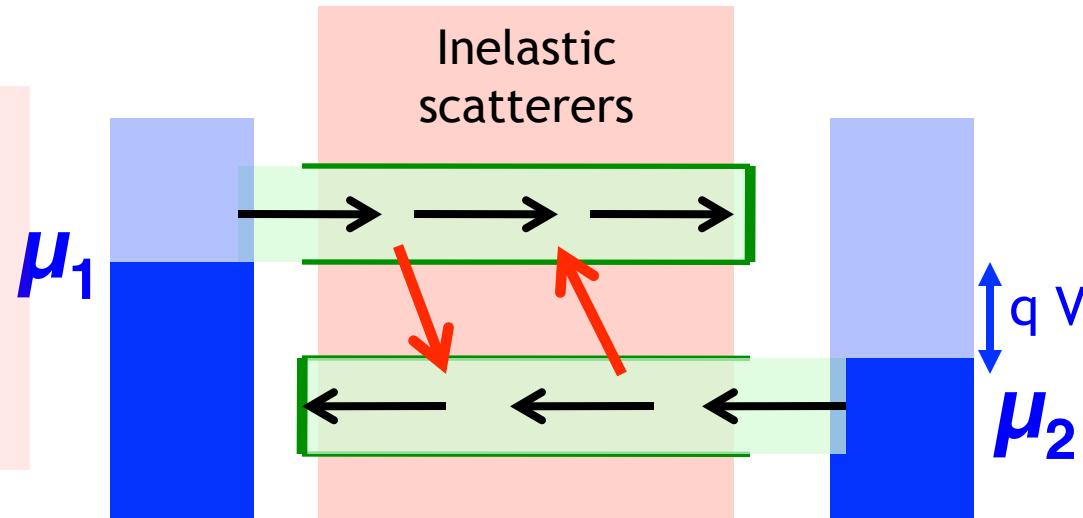


A hypothetical
series of
elastic
resistors:

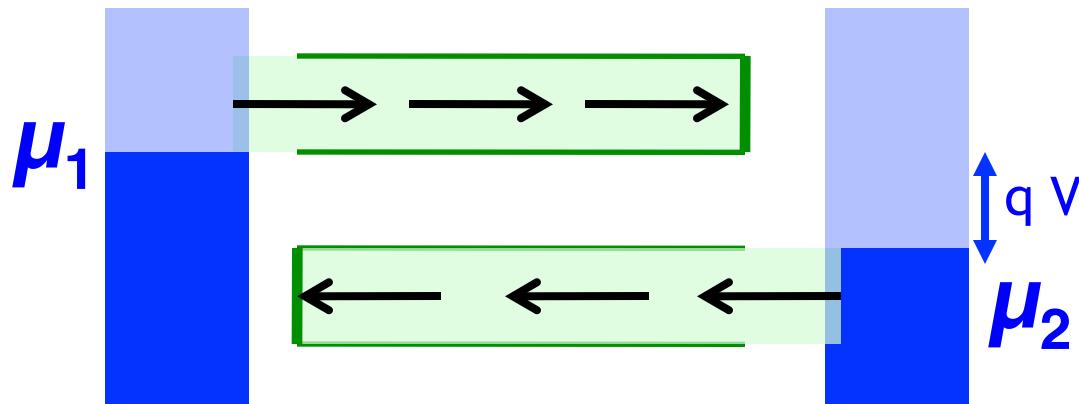


.. But not always ..

An asymmetrically contacted device with distributed inelastic scattering.



The elastic version of this device completely misses the physics.



Elastic Resistor: Central results

$$I = \frac{1}{q} \int_{-\infty}^{+\infty} dE \tilde{G}(E) (f_1(E) - f_2(E))$$

$$G = \int_{-\infty}^{+\infty} dE \left(-\frac{\partial f_0}{\partial E} \right) \tilde{G}(E)$$

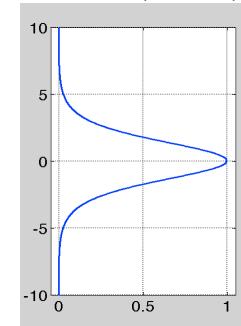
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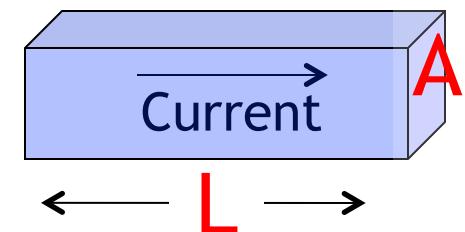
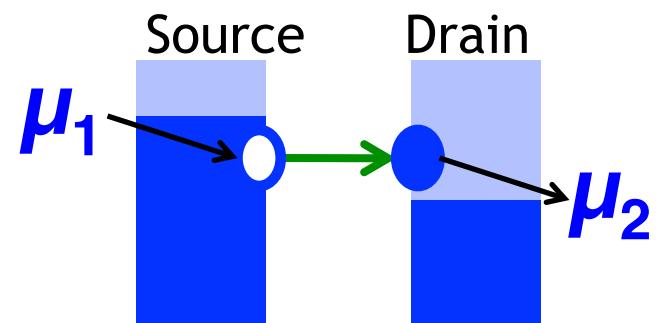
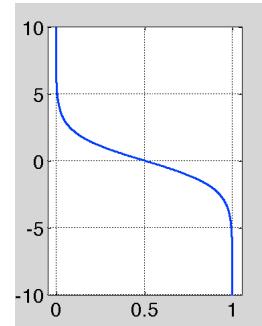
$$\tilde{\sigma} = q^2 \left(\frac{D}{AL} \frac{v^2 \tau}{d} \right) = \frac{q^2}{h} \frac{M \lambda}{A}$$

$$= q^2 \left(\frac{N}{AL} \frac{v \tau}{p} \right)$$

$$\rightarrow 4kT \left(-\frac{\partial f_0}{\partial E} \right)$$



$$\rightarrow f_0(E)$$



Lectures on Nanoelectronics - III

1. Introductory concepts

1b. *The nanotransistor: MSL*

2. Semiclassical transport

3. Quantum transport →

Beyond voltages and currents

4. Heat flow

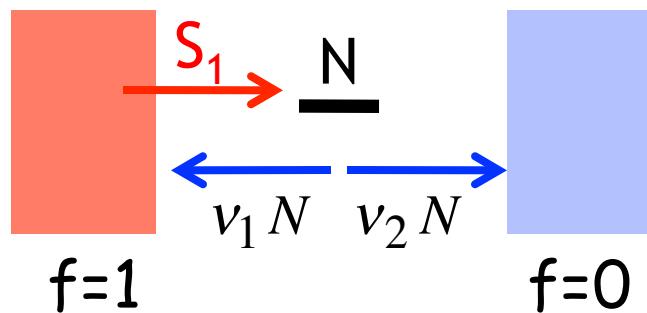
5. Spin flow

6. Entropy flow

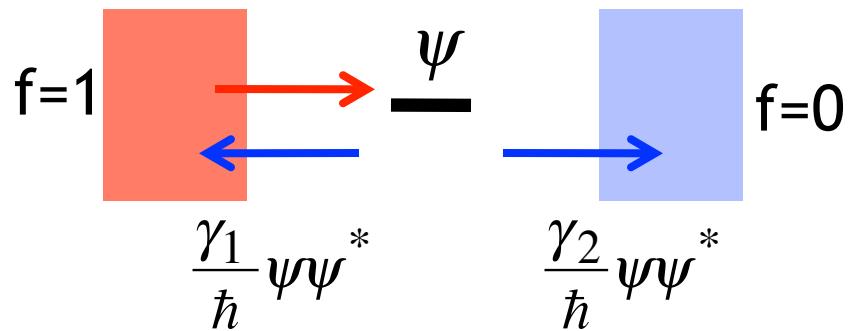
- 1) “Contacting” Schrodinger
- 2) Schrodinger to NEGF
- 3) Localization
- 4) Hall effect

“Contacting” Schrodinger

Semiclassical picture



Quantum picture



$$\frac{d}{dt} N = -(v_1 + v_2) N + S_1$$

$$E\psi = \left(\epsilon - i \frac{\gamma_1 + \gamma_2}{2} \right) \psi + s_1$$

$$S_1 = v_1$$

$$s_1 s_1^* = \gamma_1 / 2\pi$$

NEGF equations with elastic dephasing

Green
function

$$[G] = [EI - H - \Sigma_1 - \Sigma_2 - \Sigma_s]^{-1}$$

"Density of states" $A = i[G - G^+]$

"Electron density"

$$G^n = G\Gamma_2 G^+ f_2 + G\Gamma_1 G^+ f_1 + G\Sigma_s^{in} G^+$$

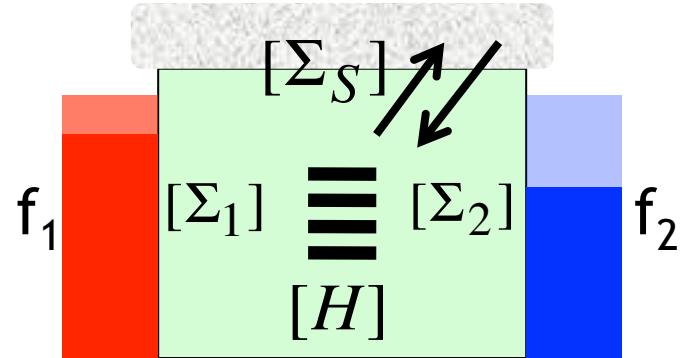
Current

$$\frac{I_1}{q/\hbar} = \text{Trace} \left([\Gamma_1 A] f_1 - [\Gamma_1 G^n] \right)$$

Dephasing model:

$$[\Sigma_s^{in}] = D[G^n]$$

1



$$\Gamma = i[\Sigma - \Sigma^+]$$

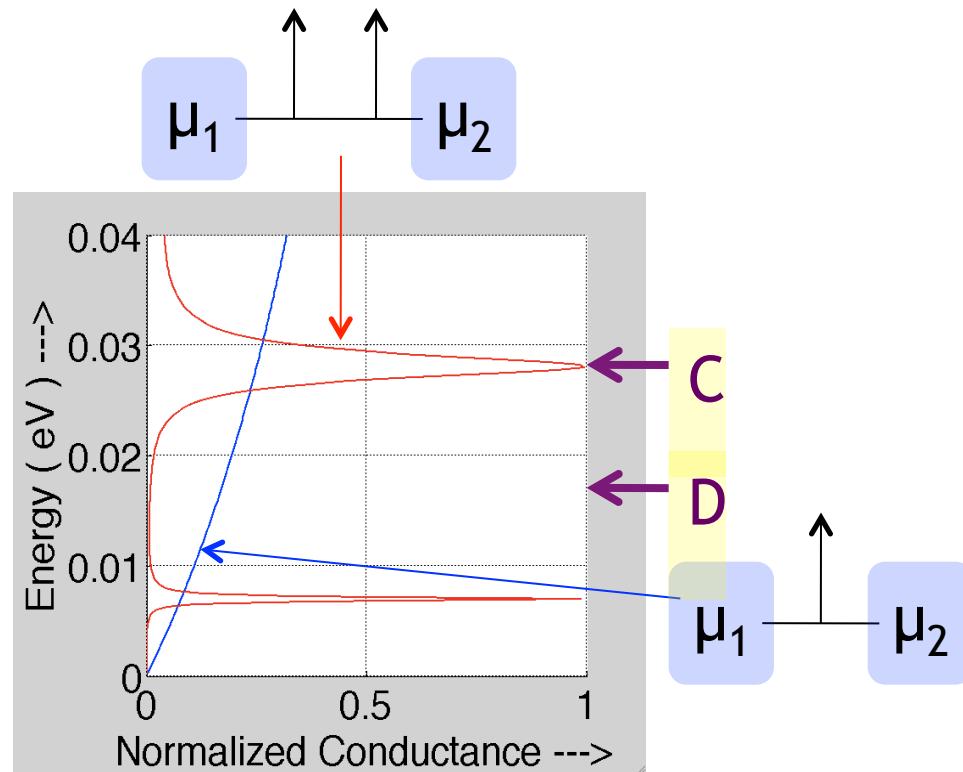
$$\varepsilon \rightarrow [H]$$

$$\gamma \rightarrow [\Gamma], [\Sigma]$$

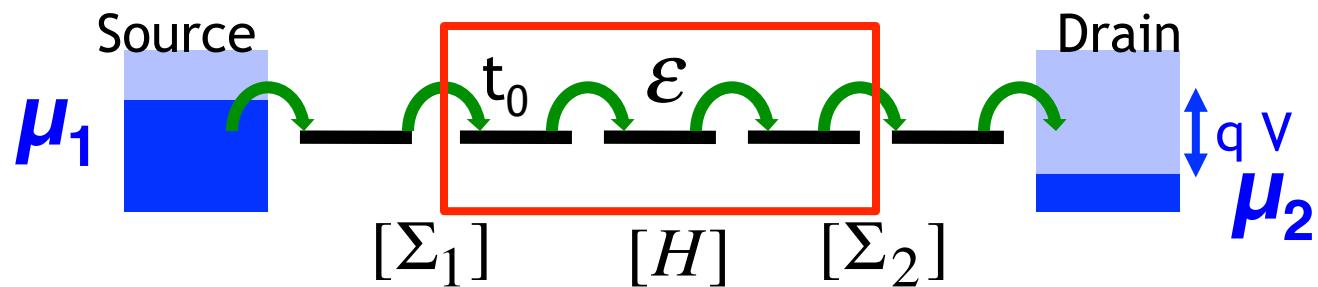
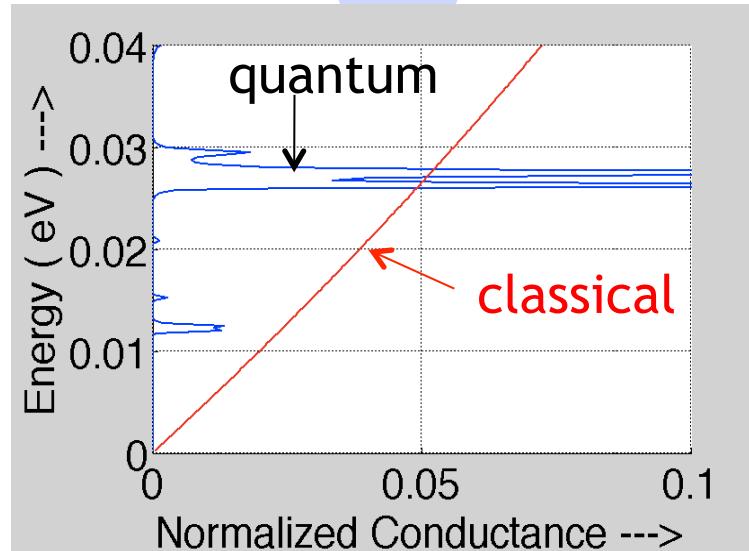
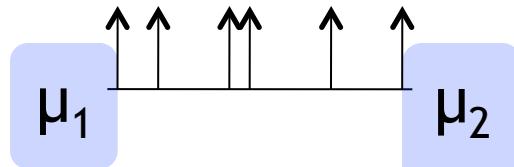
$$n(E) \rightarrow [G^n(E)]$$

$$D(E) \rightarrow [A(E)]$$

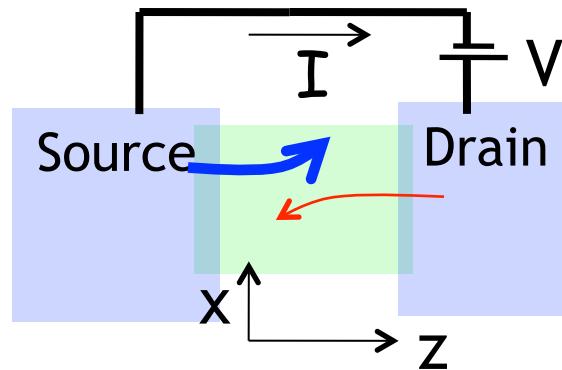
Resonant tunneling



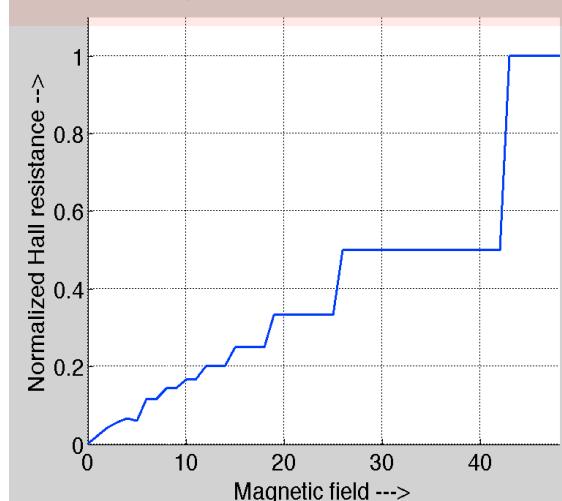
Localization



Hall effect

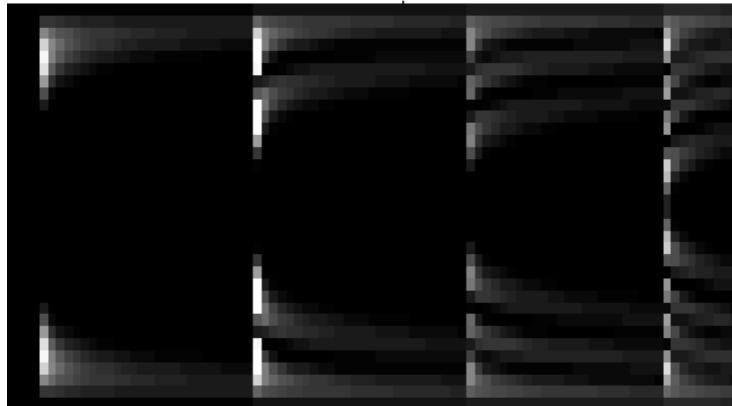


Normalized Hall resistance
versus B-field of ballistic
channel of width $W = 68$
nm, with $E=0.1$ eV

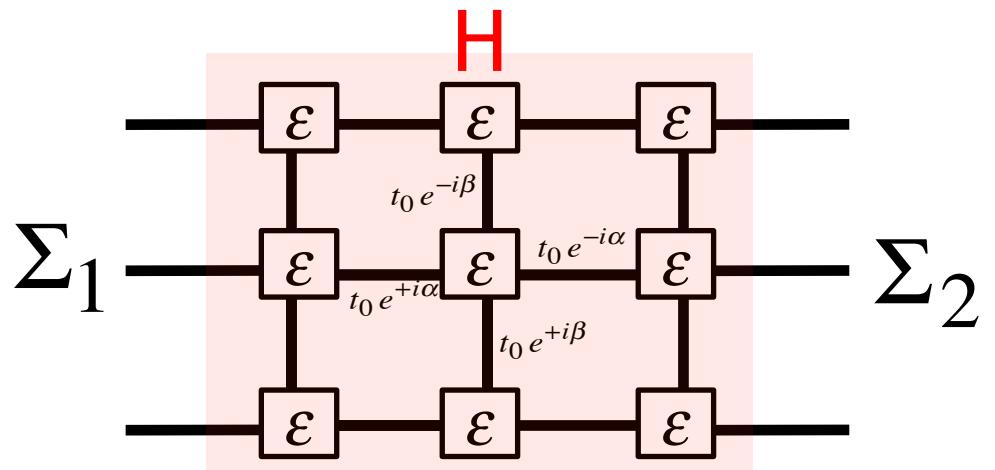


Edge states

Grayscale plot of local density of states, $D(x,E)$. Note the edge states



Energy (eV) --->



Lectures on Nanoelectronics - IV

1. Introductory concepts

1b. *The nanotransistor: MSL*

2. Semiclassical transport

3. Quantum transport

Beyond voltages and currents

4. Heat flow

5. Spin flow

6. Entropy flow

- 1) “Contacting” Schrodinger
- 2) Schrodinger to NEGF
- 3) Localization, Hall effect

Discussion Session: 330PM

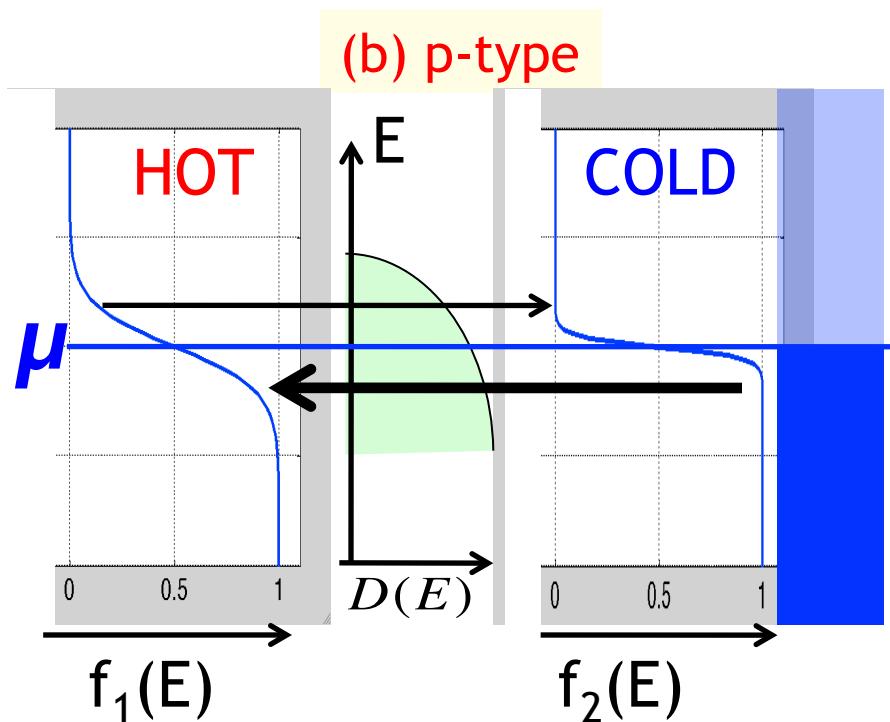
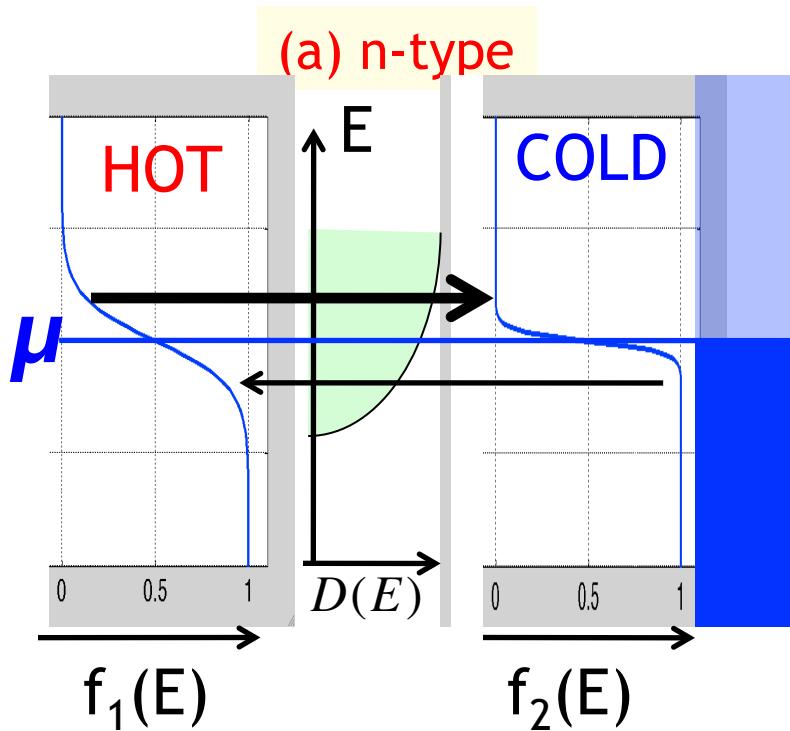
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- 1) Temperature-driven current
- 2) Heat current - Electronic
- 3) Heat current - Phononic

Temperature-driven current

$$I = \frac{1}{q} \int_{-\infty}^{+\infty} dE \tilde{G}(E) (f_1(E) - f_2(E)) \approx G (V_1 - V_2) + G_S (T_1 - T_2)$$

$$G = \int_{-\infty}^{+\infty} dE \tilde{G}(E) \left(-\frac{\partial f_0}{\partial E} \right) \quad G_S = \int_{-\infty}^{+\infty} dE \tilde{G}(E) \frac{E - \mu_0}{qT} \left(-\frac{\partial f_0}{\partial E} \right)$$



Heat current - Electronic

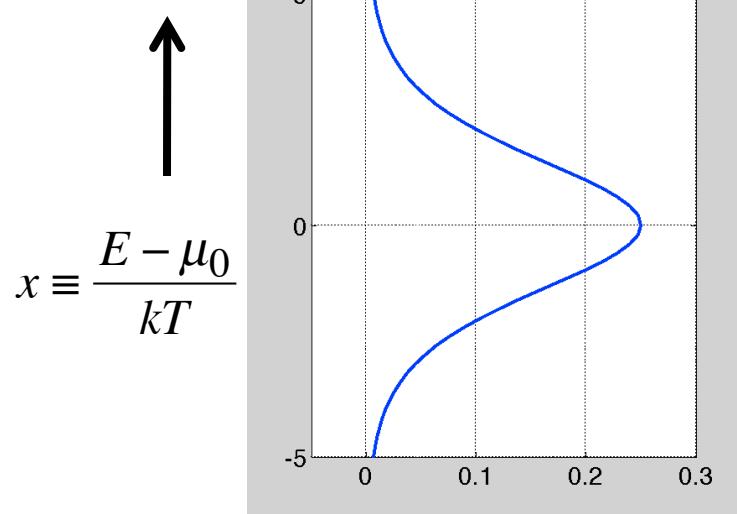
$$I_Q = \frac{1}{q} \int_{-\infty}^{+\infty} dE \tilde{G}(E) \frac{E - \mu_0}{q} (f_1(E) - f_2(E))$$

$$\approx G_P (V_1 - V_2) + G_K (T_1 - T_2)$$

$$G_P = \int_{-\infty}^{+\infty} dE \tilde{G}(E) \frac{E - \mu_0}{q} \left(-\frac{\partial f_0}{\partial E} \right)$$

$$G_K = T \int_{-\infty}^{+\infty} dE \tilde{G}(E) \left(\frac{E - \mu_0}{qT} \right)^2 \left(-\frac{\partial f_0}{\partial E} \right)$$

$$\tilde{F}_T(x) \equiv \frac{e^x}{(e^x + 1)^2}$$



$$G = \int_{-\infty}^{+\infty} dx \tilde{G}(x) \tilde{F}_T(x)$$

$$G_P = TG_S = \frac{kT}{q} \int_{-\infty}^{+\infty} dx x \tilde{G}(x) \tilde{F}_T(x)$$

$$G_K = \frac{k^2 T}{q^2} \int_{-\infty}^{+\infty} dx x^2 \tilde{G}(x) \tilde{F}_T(x)$$

Heat current - Phononic

$$I_Q = \frac{1}{h} \int_{-\infty}^{+\infty} dE \left(\frac{M\lambda}{L+\lambda} \right)_{ph} \hbar\omega (n_1(\omega) - n_2(\omega))$$

$$\approx G_K (T_1 - T_2)$$

$$G_K = \frac{k^2 T}{h} \int_{-\infty}^{+\infty} dx \left(\frac{M\lambda}{L+\lambda} \right)_{ph} \frac{x^2 e^x}{(e^x - 1)^2}$$

$$\equiv \frac{\kappa_{ph} A}{\lambda_{ph} + L}$$

$$n(\omega) = \frac{1}{e^x - 1}, \quad x \equiv \frac{\hbar\omega}{kT}$$

$$\frac{4e^x}{(e^x + 1)^2} \quad \frac{x^2 e^x}{(e^x - 1)^2}$$

$$x \equiv \frac{\hbar\omega}{kT}$$

Lectures on Nanoelectronics - III, IV

1. Introductory concepts

1b. The nanotransistor: MSL

2. Semiclassical transport

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Beyond voltages and currents

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- 1) Temperature-driven current
- 2) Heat current - Electronic
- 3) Heat current - Phononic

Lectures on Nanoelectronics - V

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Beyond voltages and currents

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6. Entropy flow

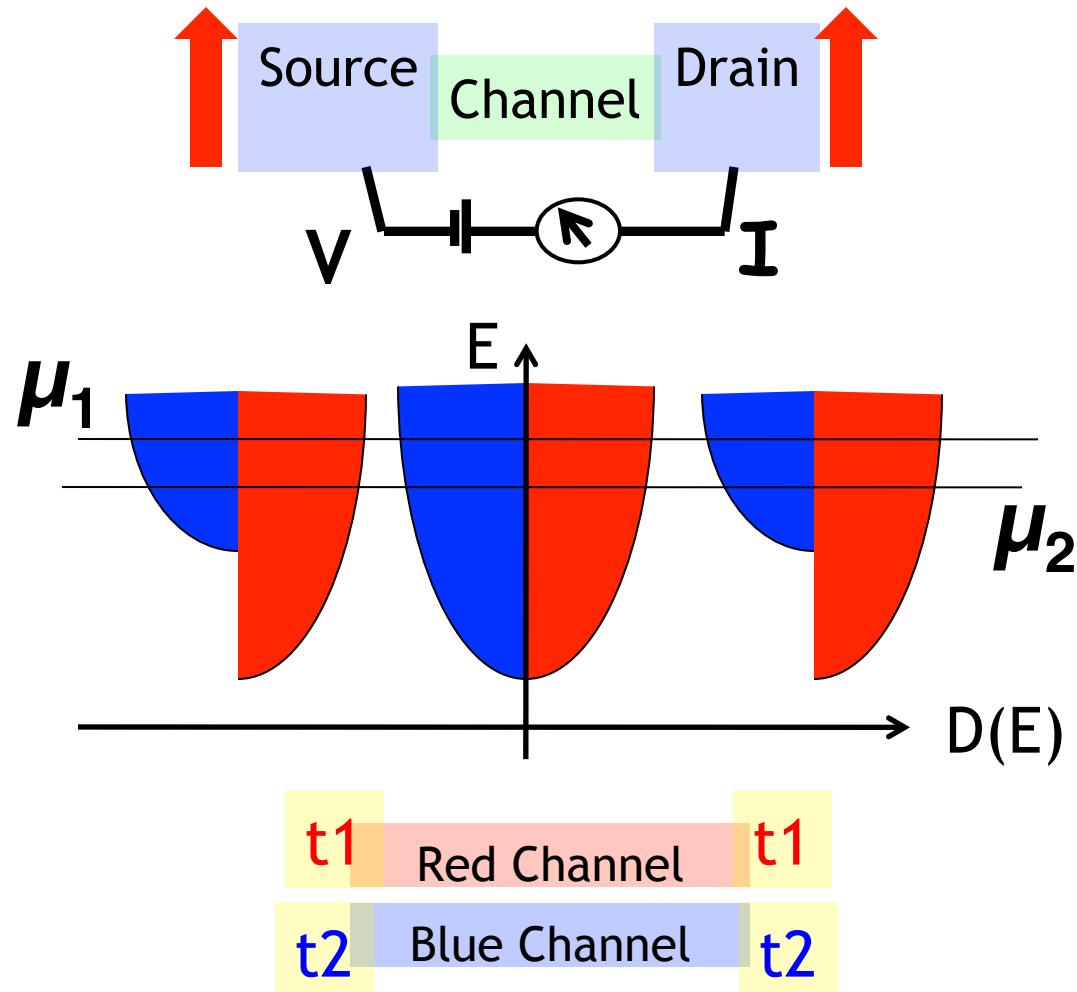
1) Spin valve

2) Nonlocal signal

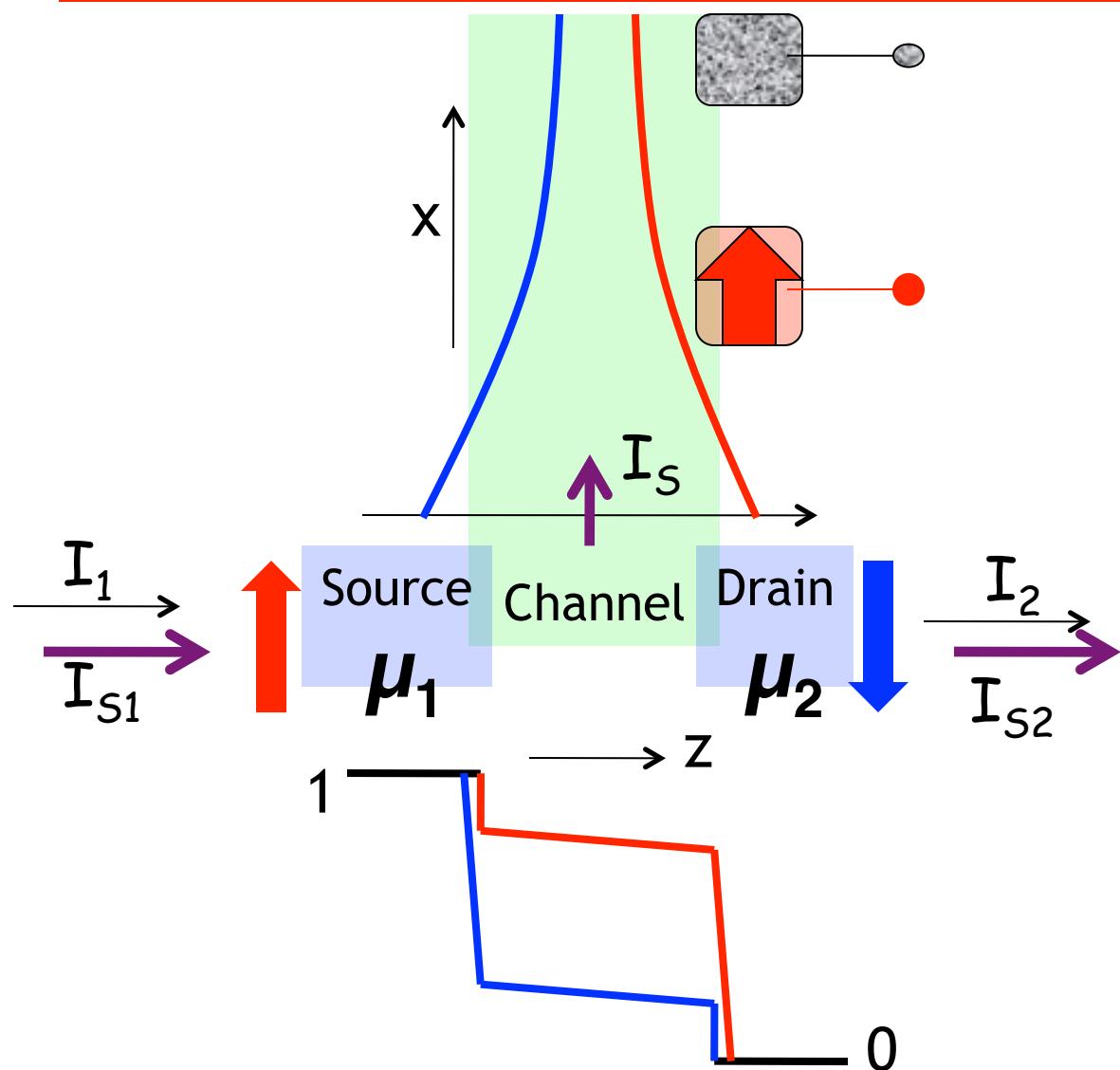
3) Rotating an electron

Spin Valve

A spin valve has magnetic contacts that provide different escape rates denoted "t1" and "t2" for the two spins, color coded as red and blue. The red channel with the shorter escape rate (t1) provides a better transfer path than the blue.



Non-local signal



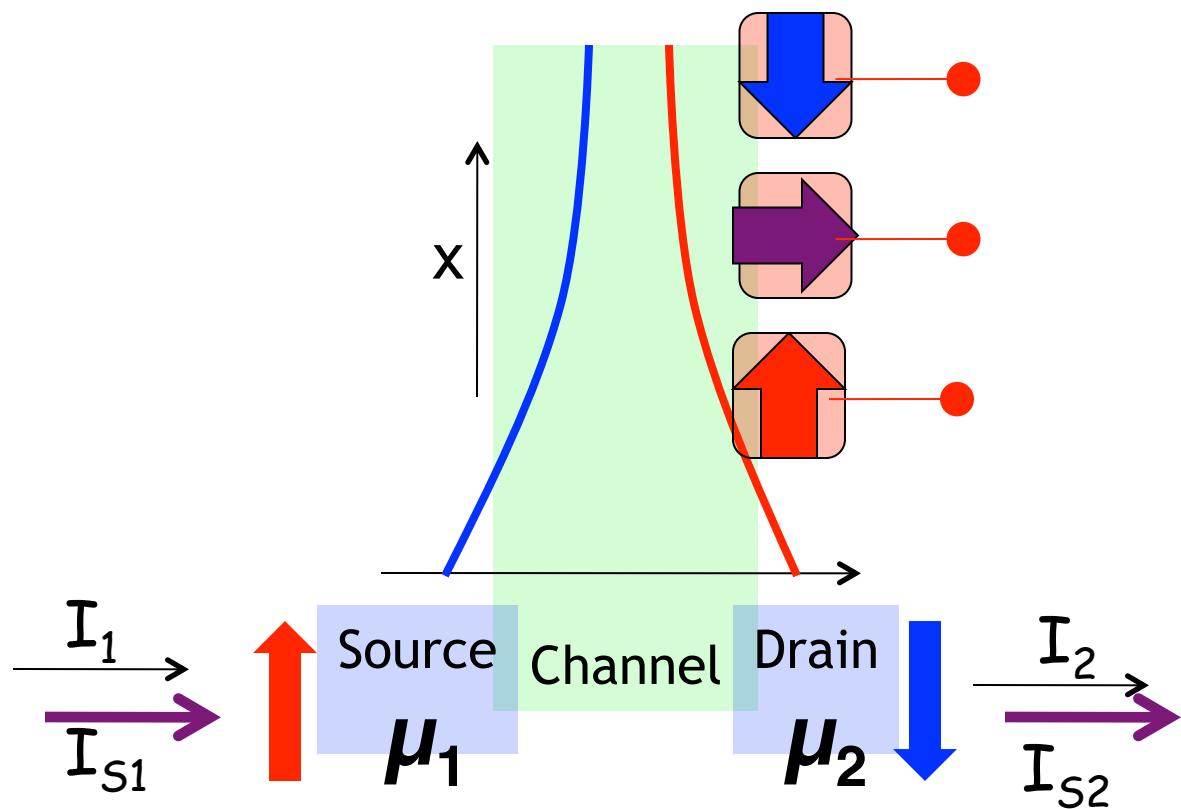
$$\frac{d}{dz} \left(\sigma_{up}(z) \frac{d\mu_{up}}{dz} \right) =$$

$$- g_{sf} (\mu_{up} - \mu_{dn})$$

$$= - \frac{d}{dz} \left(\sigma_{dn}(z) \frac{d\mu_{dn}}{dz} \right)$$

Valet-Fert
Equation

It is not just red and blue



$$\cos^2\left(\frac{\theta}{2}\right)$$

Electrons

Compare

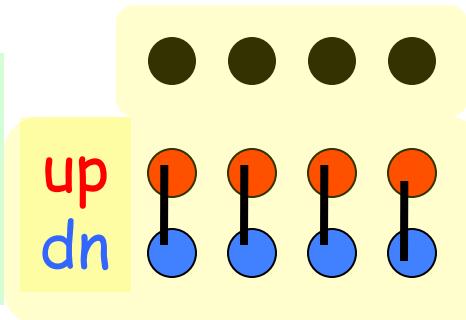
Photons

$$\cos^2 (\theta)$$

Spin in NEGF

NEGF

All matrices double
From $N \times N$
To $2N \times 2N$



G^n

$$\begin{bmatrix} N + S_z & S_x - iS_y \\ S_x + iS_y & N - S_z \end{bmatrix}/2$$

$$\cos^2\left(\frac{\theta}{2}\right)$$

Electrons

Compare

Photons

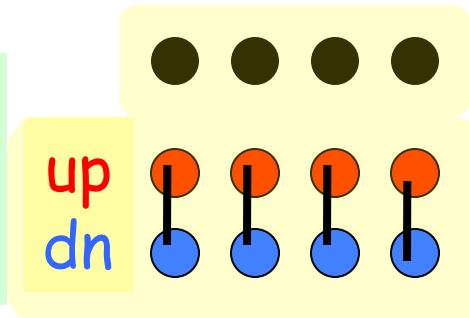
$$\cos^2 (\theta)$$

Misquoting Wigner

It is nice to know that NEGF understands spin,
but I would like to understand it, too.

NEGF

All matrices double
From $N \times N$
To $2N \times 2N$



G^n

$$\begin{bmatrix} N + S_z & S_x - iS_y \\ S_x + iS_y & N - S_z \end{bmatrix}/2$$

$$\cos^2\left(\frac{\theta}{2}\right)$$

Electrons

Compare

Photons

$$\cos^2 (\theta)$$

Lectures on Nanoelectronics - VI

1. Introductory concepts
- 1b. *The nanotransistor: MSL*
2. Semiclassical transport
3. Quantum transport

Beyond voltages and currents

4. Heat flow
5. Spin flow
6. Entropy flow →

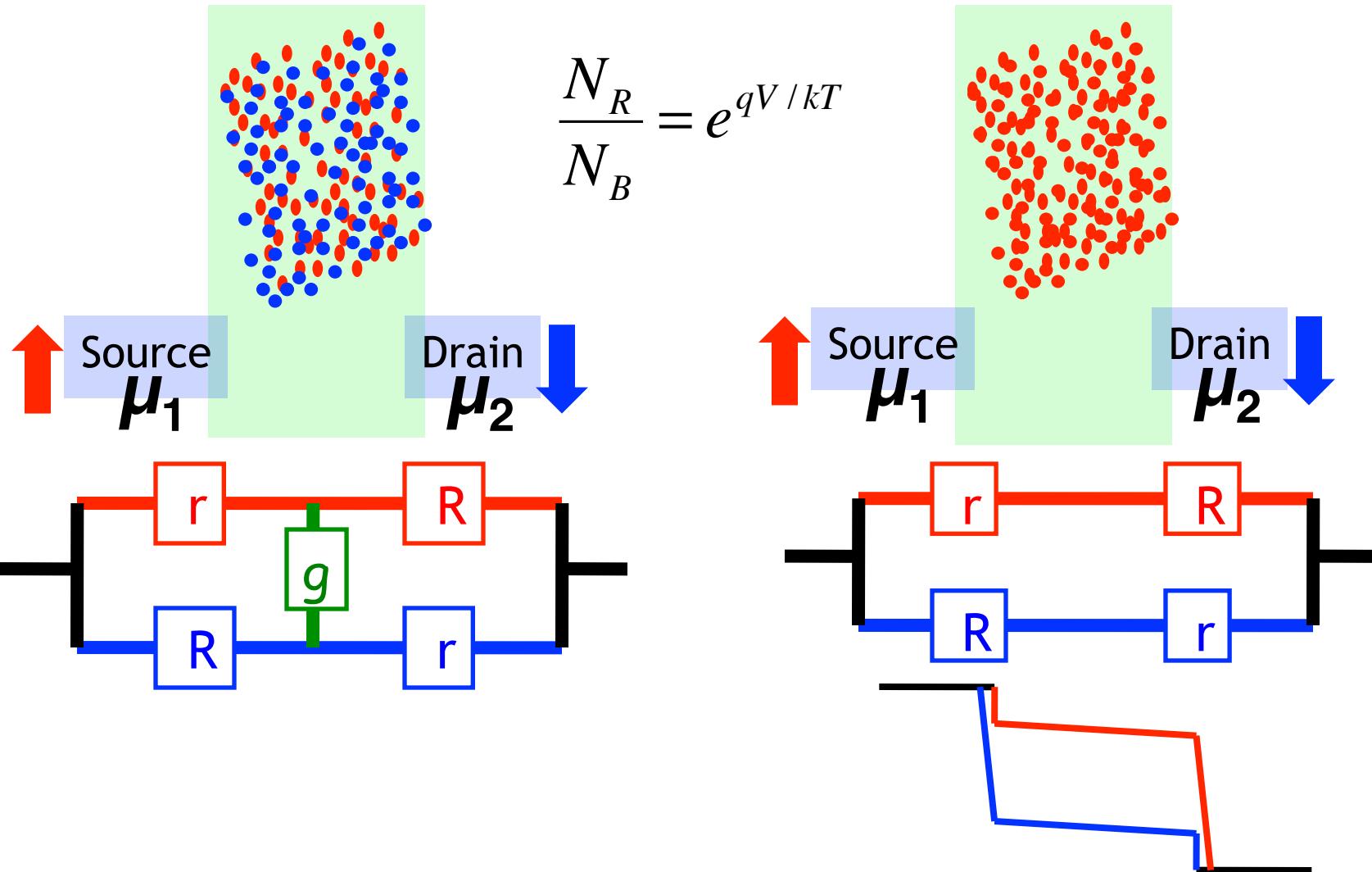
Discussion Session: 330PM

*Please hand your questions
or email them to
Mr. Samiran Ganguly
sganguly@purdue.edu*

- 1) Spin valve
- 2) Nonlocal signal
- 3) Rotating an electron

- 1) Resistor or capacitor?
- 2) Entropy capacitor
- 3) Second law

Resistor or capacitor ?



How much energy can be recovered ?

$$\frac{N_R}{N} = \frac{1}{1 + e^{-x}}, \quad x \equiv \frac{qV}{kT}$$

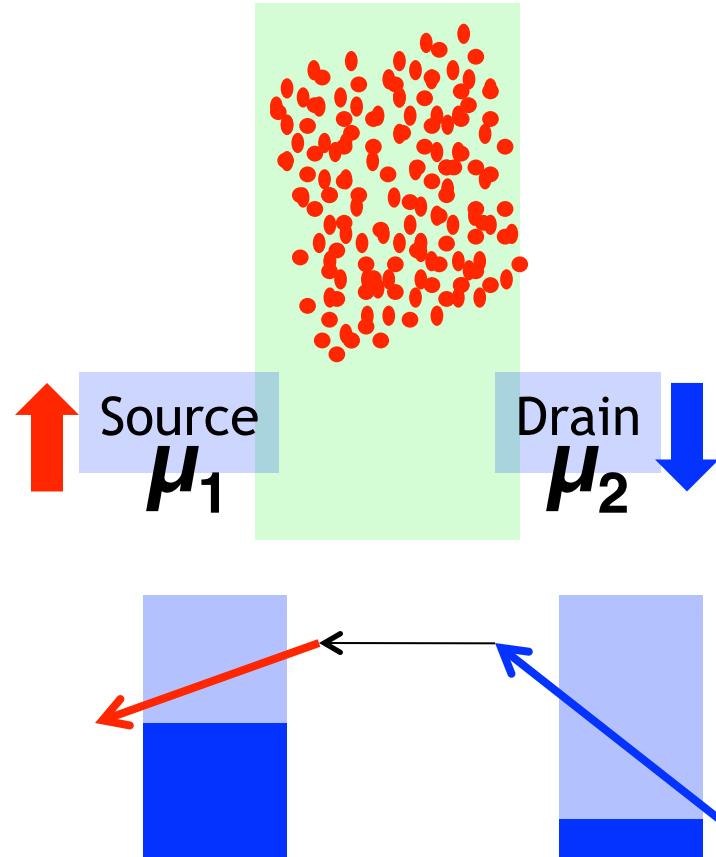
$$dE = -qV dN_R$$

$$E = NkT \int_0^{\infty} dx \ x \frac{e^{-x}}{(1 + e^{-x})^2}$$

$$= NkT \ln 2$$

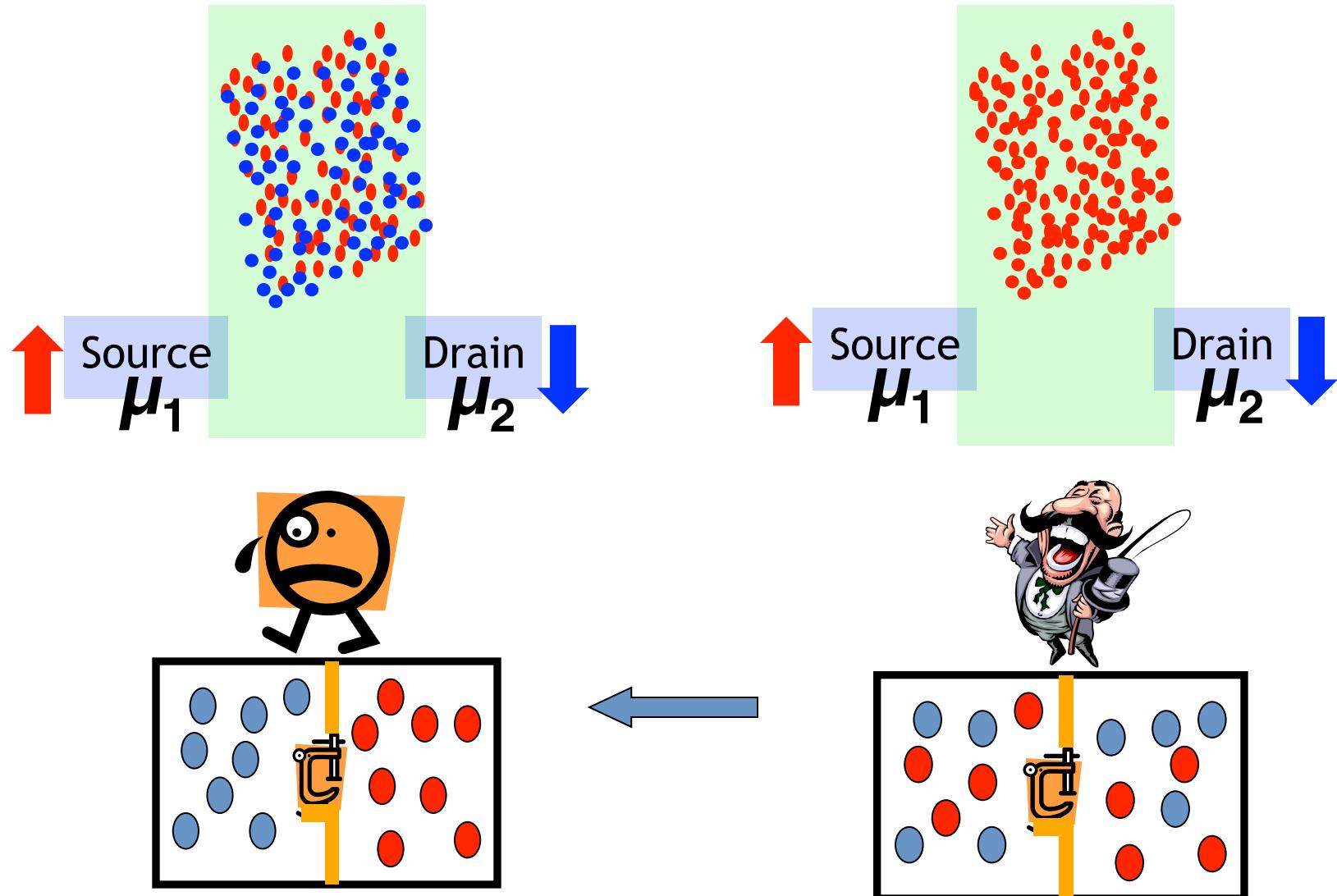
~ 2.5 KJ/mole

Oil ~ 5000 KJ/mole

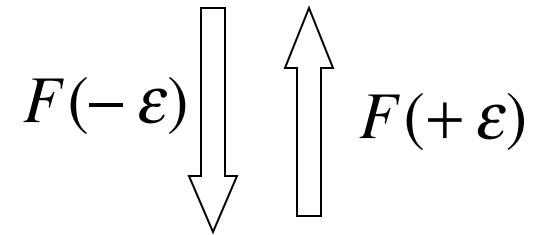
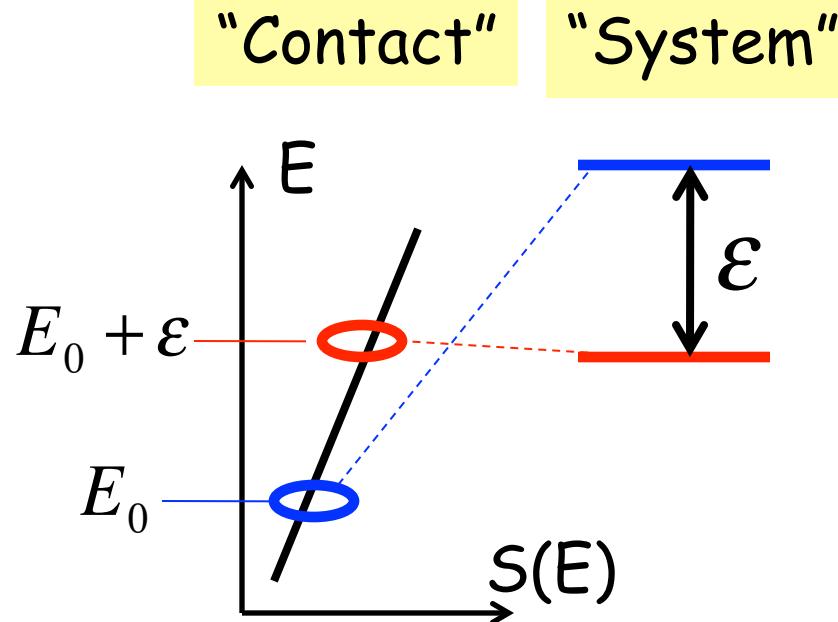


$$|I_{Q1}| + |I_{Q2}| > 0$$

Electronic Maxwell's demon



Why does a system go to its lowest energy?



Down > Up

For any “contact” in equilibrium

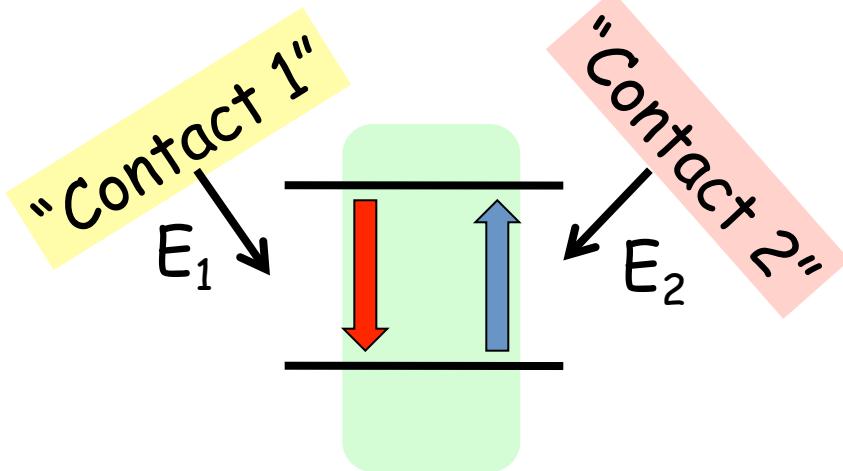
Easier to give energy to it,
than to extract energy from it.

$$\frac{F(-\varepsilon)}{F(+\varepsilon)} = \frac{W(E_0 + \varepsilon)}{W(E_0)}$$

$$= \exp\left(\frac{\varepsilon}{kT}\right)$$

Second law

$$E_1 + E_2 = 0$$



$$\frac{F_1(E_1) F_2(E_2)}{F_1(-E_1) F_2(-E_2)} > 1$$

$$\exp\left(-\frac{E_1}{kT_1}\right) \exp\left(-\frac{E_2}{kT_2}\right) > 1$$

$$\frac{E_1}{kT_1} + \frac{E_2}{kT_2} \leq 0$$

$$\sum_i \frac{E_i - \mu_i}{T_i} \leq 0$$

$$\sum_i E_i = 0$$

$$S \sim k \log W \\ \rightarrow W \sim \exp(S/k)$$

Lessons from Nanoelectronics

Discussion Session: 330PM

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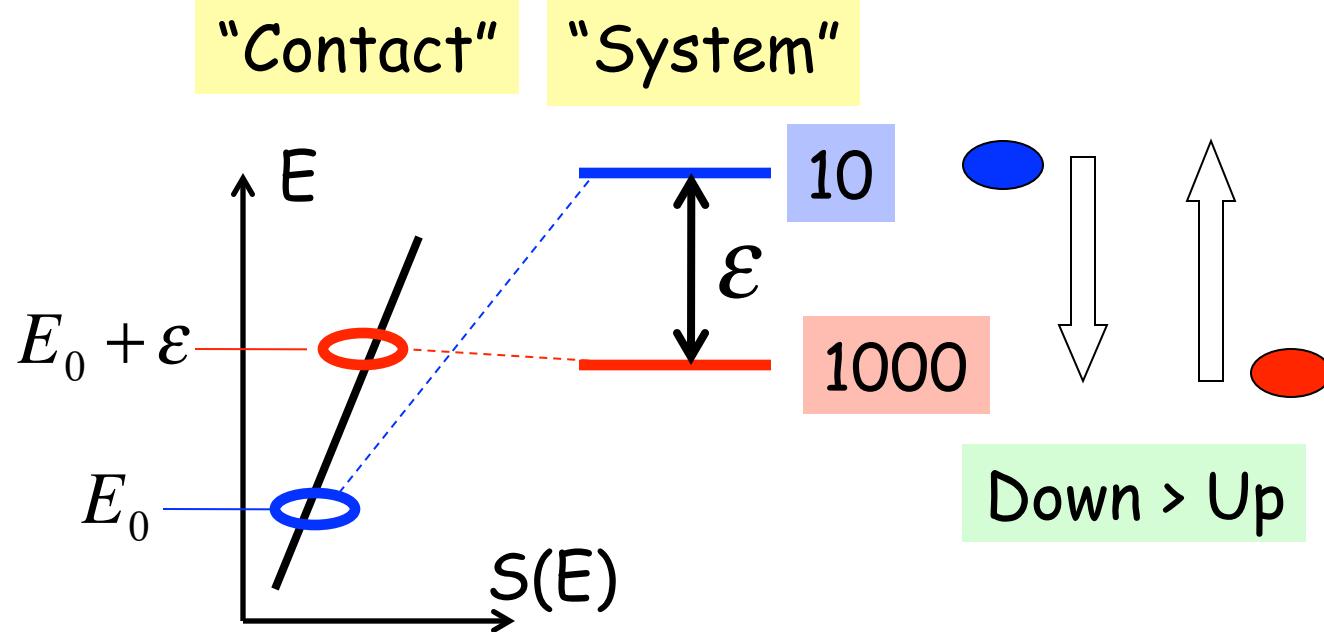
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1. Introductory concepts (M830A)
- 1b. The nanotransistor: Lundstrom (M1030A)*
2. Semiclassical transport (M130P)
3. Quantum transport (Tu830A)

Beyond voltages and currents

4. Heat flow (Tu1030A)
5. Spin flow (W830A)
6. Entropy flow (W1030A)

Modeling the entropic "force"



$$\frac{Down}{Up} = \frac{W(E_0 + \varepsilon)}{W(E_0)} = \exp\left(\frac{S(E_0 + \varepsilon) - S(E_0)}{k}\right) = \exp\left(\frac{\varepsilon}{kT}\right)$$

$$S \sim k \log W$$
$$\rightarrow W \sim \exp(S/k)$$

$$S(E_0 + \varepsilon) - S(E_0) \approx \frac{dS}{dE} \varepsilon = \frac{\varepsilon}{T}$$

Extra

$$\Delta f(E) = F_T(E - \mu_0) \Delta\mu$$

$$\Delta F = \frac{\Delta f(E)}{f_1(E) - f_2(E)} = \frac{\Delta\mu}{\mu_1 - \mu_2} = \frac{\Delta\mu}{qV}$$