1D Transient Heat Conduction CDF Tool

Analytic Solution for 1D Transient Heat Conduction

The problem geometry and boundary conditions are shown below. An initially isothermal ($T_{\text{initial}}$) semi-infinite medium is suddenly subject to a surface temperature $T_h$.

The temperature field can be non-dimensionalized as:

$$\theta (x,t) = \frac{T(x,t)-T_{\text{initial}}}{T_h-T_{\text{initial}}}$$

The governing differential equation (with spatially one-dimensional heat flow) is

$$\frac{\partial \theta (x,t)}{\partial t} = \alpha \frac{\partial^2 \theta (x,t)}{\partial x^2}$$

The solution for all locations $x$ and times $t$ is:

$$\theta(x,t) = 1-\text{erf}\left[\frac{x}{2\sqrt{\alpha t}}\right]$$

where $\alpha$ is the material’s thermal diffusivity.

Graphical CDF Tool

The following is an embedded, active Mathematica CDF tool. The units for $\alpha$ are cm$^2$/sec, with corresponding units of cm and sec for $x$ and $t$, respectively.