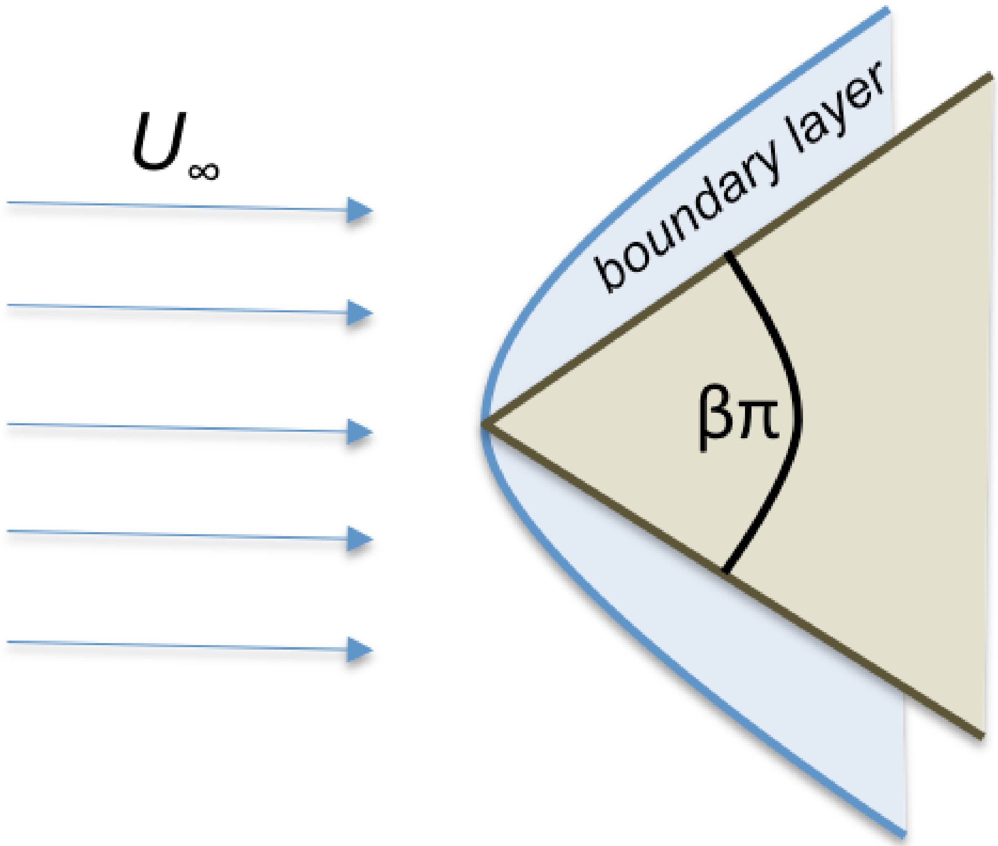
Boundary Layer Flow Solution

Boundary Layer for Flow Past a Wedge

The Blausius boundary layer velocity solution is a special case of a larger class of problems forflow over a wedge, as shown in the following figure in which $$\beta \pi$$ represents the wedge angle.



The general solution is called the Falkner-Skan boundary layer solution, which starts with a recognition that the free-stream velocity will accelerate for non-zero values of $$\beta$$:

$$u_{e}(x)= U_{0} \left(x/L \right) ^{m}$$

where $$ L $$ is a characteristic length and *m* is a dimensionless constant that depends on $$\beta$$:

$$ {\beta} = \frac{2m}{m + 1} $$

The condition *m = 0* gives zero flow acceleration corresponding to the Blausius solution for flat-plate flow.

We then define a similarity variable $$\eta$$ that combines the local streamwise and cross-flow coordinates *x* and *y* (defined relative to the surface of the wedge):

$$ {\eta} = y \sqrt{\frac{U_{0}(m+1)}{2{\nu}L}}\left(\frac{x}{L}\right)^{\frac{m-1}{2}} $$

Then, defining a function *f* that relates to the streamwise and cross-flow velocities, a single ordinary differential equation ensues from boundary layer momentum and mass conservation:

$$ \frac{\mathrm{d}^3 f}{\mathrm{d} \eta ^3}+f\frac{\mathrm{d}^2 f}{\mathrm{d} \eta^2}+ \beta \left[1-\left(\frac{\mathrm{d}f}{\mathrm{d}\eta}\right)^2 \right]=0 $$

This non-linear equation is not amenable to an exact solution (even for the Blausius solution $\beta=0$, which eliminates the last term on the right side). The following Mathematic CDF file solves the equation numerically and provides the streamwise velocity normalized by the local freestream velocity as a function of η.

CDF Tool [1]

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1. ^ Numerical Solution of the Falkner-Skan Equation for Various Wedge Angles, from the Wolfram Demonstrations Project //demonstrations.wolfram.com/NumericalSolutionOfTheFalknerSkanEquationForVariousWedgeAngl/