Equilibrium Carrier Concentrations Lesson

Equilibrium Carrier Concentrations

Once we know how to determine the carrier distribution, we can find the carrier concentration by integrating over all energies:

Electron concentration: $n = \int_{E_c}^{\infty} g_c(E) f(E) dE$

Hole concentration: $p = \int_{-\infty}^{E_v} g_v(E)[1 - f(E)] dE$

After a lot of words and math we derive simple equations we can understand and use:

Electron concentration: $n_o = n_i e^{(E_F - E_i)/kT}$

Hole concentration: $p_o = n_i e^{(E_i - E_F)/kT}$

and finally the $n_o p_o$ product relationship: $n_o p_o = n_i^2$

These equations are only valid when the semiconductor is in equilibrium and nondegenerate $E_v + 3kT \leq E_F \leq E_c - 3kT$. Another way to say a semiconductor is nondegenerate is that the Fermi level, E_F, is more than 3kT from any of the states for which we are counting electrons. The $n_o p_o$ product relationship is one of the most useful equations because once you know one of the carrier concentrations (using the equations for n_o or p_o), the other can be easily calculated.

We typically deal with uniformly doped semiconductors and if they are at room temperature, we also assume total ionization of the dopant atoms. With these assumptions we can use the charge neutrality relationship and the $n_o p_o$ product relationship from above to derive equations for n_o and p_o that take into account the doping concentrations:

Charge neutrality relationship: $p_o - n_o + N_D - N_A = 0$

Electron concentration:

$$n_o = \frac{n_i^2}{p_o} = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{\frac{1}{2}}$$

Hole concentration:

$$p_o = \frac{n_i^2}{n_o} = \frac{N_A - N_D}{2} + \left[\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2\right]^{\frac{1}{2}}$$

These equations can be simplified under a number of situations. Below are the most common:

1. When a semiconductor is not doped, $N_A = 0$ and $N_D = 0$, the semiconductor is intrinsic and $n_o = p_o = n_i$. This also occurs when N_A and N_D are approximately equal, or $n_i >> |N_D - N_A|$.
2. The equations for the carrier concentrations for a *p*-type semiconductor, $N_A >> n_i$ and $N_D = 0$, can be simplified. Since $N_A >> n_i$, we can neglect n_i in the equation for p_o and obtain the carrier concentrations using the following equations:

$$p_o \cong N_A$$

$$n_o \cong \frac{n_i^2}{N_A}$$

3. The equations for the carrier concentrations for an *n*-type semiconductor, $N_D >> n_i$ and $N_A = 0$, can be simplified. Since $N_D >> n_i$, we can neglect n_i in the equation for no and obtain the carrier concentrations using the following equations:

$$n_o \cong N_D$$

$$p_o \cong \frac{n_i^2}{N_D}$$

4. When a *p*-type semiconductor is compensated, doped with both acceptors and donors ($N_A - N_D >> n_i$ and N_D is nonzero), the equations may be simplified similarly to Case 2 because we can still neglect n_i in the equation for p_o. The $n_o p_o$ product relationship can then be used to solve for the electron concentration:

$$p_o = N_A - N_D$$

$$n_o = \frac{n_i^2}{p_o}$$

5. When an *n*-type semiconductor is compensated, doped with both acceptors and donors ($N_D - N_A >> n_i$ and N_A is nonzero), the equations may be simplified similarly to Case 3 because we can still neglect n_i in the equation for n_o. The $n_o p_o$ product relationship can then be used to solve for the hole concentration:

$$n_o = N_D - N_A$$

$$p_o = \frac{n_i^2}{n_o}$$

6. If the doping concentration, or the difference in doping concentrations if the semiconductor is compensated, is comparable to n_i, we cannot simplify the equations. The full expression must

be used.